# Investigating incompatibility:
# How to reconcile complementarity with EPR

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ABSTRACT. Incompatibility is either fact-dependent and therefore conditional or else fact-*in*dependent and therefore *un*conditional. If Complementarity (CTY) is to be reconciled with EPR it must evidently belong to the former kind, for the latter allows of no exceptions. In addition, fact-*in*dependent incompatibility (=logical) cannot be the consequence of the *quantum*. But CTY *is* a consequence of the quantum. Therefore, CTY does express conditional incompatibility and hence it can be reconciled with EPR. By contrast, Wave-Particle Duality (WPD), by expressing *logical* incompatibility can do neither of the two. Waves (large) and particles (small) are incompatible also in *classical* mechanics. And classical mechanics does not contain the quantum. Contrary to common opinion, WPD yields the wrong *sort* of incompatibility for CTY.

The uncertainties (UR) are derived from relations $E=h\nu$ and $p=h/\lambda$, without recourse to Fourier analysis: E can only be defined over a period, p only over a distance (contrary to classical suppositions that it can be done at an instant, at a point). Hence, for E defined over $t>0$, $Et=h$; and for p defined over $\lambda$ (or $q>0$), $p\lambda$ $(pq)=h$. Then for shorter periods or shorter distances, E and p will be proportionally less accurately defined, yielding (symmetric) $\Delta E\Delta t=\Delta p\Delta q\geq h$. Thus the UR and CTY are dependent upon quantized action and are impossible without it.

It is then proven that in an EPR environment the *quantum is removed*. (Their argument yields h-h=0.) Hence, UR and CTY are not even *expected* to hold in absence of *h*. However, WPD, whose incompatibility is *un*conditional, is expected to hold everywhere, EPR included. Hence, their example, establishing a *p,q* compatibility, contradicts Duality. But, as shown above, it does not contradict *Complementarity*. It merely rids CTY of the former's presence (thank you very much!) and thereby forms it into shape.

## 1    How Many Complementarities Are There ?

It is generally assumed that the argument by Einstein, Podolsky and Rosen (EPR here-after) is incompatible with the complementary account of QM. The EPR environment warrants the simultaneous measurement of momenta, $p_1, p_2$ and positions, $q_1, q_2$, of their two, correlated particles and so, by warranting their comeasurability it ipso facto warrants the *compatibility* of these concepts. Comeasurability is far stronger than mere compatibility, so, when the former is warranted, so is *a fortiori* the latter. Since, how-ever, the Complementarity of $p$ and $q$ forbids their compatibility, the two accounts, EPR's and Bohr's, stand in diametric opposition.

We must get absolutely clear about this before we proceed. Complementarity properly so-called, denies the simultaneous *existence* of a pair of conjugated variables. Not merely their simultaneous knowledge, as the superficial, 'disturbance by measurement' account of $\Delta p \Delta q \geq h$ by Heisenberg implies, which is an essentially *classical* way of defending the uncertainty.(1) A competent complementarist, P.K. Feyerabend warns us:

This is not just a restriction of our *knowledge*. It is not asserted that we occasionally cannot *know* the position with a precision <1cm. and then conclude, by some kind of positivistic reasoning, that more precise statements are 'meaningless'. Quite the contrary, it is asserted that [...] once these conditions are realized, *there is no such feature in the world*.(2) (Italics in the original. Quotes mine.)

Now compare this (accurate) description of Complementarity with EPR's position:

In a complete theory there is an element corresponding to each element of reality. [..] In quantum mechanics in the case of two physical quantities described by two non-commuting operators, the knowledge of one precludes the knowledge of the other.

Then either (1) the description of reality given in quantum mechanics is not complete or (2) these two quantities *cannot have simultaneous reality*.(3)

The fact that EPR opt for alternative (1) does not change the fact that the other alternative, (2), which they reject, is incorrectly described by them. Quite the contrary; it is rejected *because* it is correctly described and found implausible or extravagant. And alternative (2) denies the very *existence* of either complementary quantity, when its complementary is realized. In consequence, the two accounts, the Bohrian and the Einsteinian, are incompatible because the conjugate *variables* in question are incompatible in the Bohrian account, while they are compatible (and even comeasurable) in the EPR account. Their difference, therefore, is one about *compatibility*.

The traditional picture drawn about the Bohr-EPR debate is that of utter contrast, to which Bohr's own reply to EPR constitutes no exception. Bohr's reply to EPR *is* incompatible with EPR. This is exactly what it was intended to be right from the start and it would not be a reply in his mind if it weren't. But do their *entire* views have to conflict? For my purpose is not to reconcile Bohr's *reply* to EPR with EPR itself, which is out of the question. My purpose is to reconcile *Complementarity* with EPR and this is a totally different matter. Bohr, in his reply to EPR operates with *some* 'complementarity', which does conflict with EPR, but not with *the* Complementarity, which, as I am about to establish, does not. This, of course, introduces us to at least *two, different* types of Complementarity, with different logical properties each. And there are several types more. The discrepancy between the structure of Complementarity in Bohr's reply to EPR as *opposed* to its structure in Bohr's standard doctrine I have presented at length in several works of mine already, (4,5, 6,7) and I will not repeat here.

What I will do, instead, is to shake the confidence of many that, ultimately, all forms of Complementarity essentially reduce to *one* and the one to which they all reduce is incompatible with EPR. This is a mistaken assumption. EPR is incompatible only with *certain* versions of Complementarity, not with Complementarity in *general*. My instrument of demonstration of this claim will be a thorough investigation of our concept of *Incompatibility*. Incompatibility is generally assumed to be a concept which has few secrets for us, if any. This is also a mistaken assumption. Incompatibility is a tricky notion and the myriads(!) of ways of deriving the quantum uncertainties, all largely inconsistent to one another, are witness to this fact: Incompatibility is a *variable* concept.

I need only point out the following difference to the reader: Concepts A and B are, let us suppose, incompatible *all* the time. But concepts C and D are, let us suppose instead, incompatible only *some* of the time. Buying an expensive car and taking an expensive trip to Europe may be incompatible for me right now, when I cannot afford both, but they will both be possible simultaneously, if I win a fortune in the lottery. These two possibilities were incompatible for me *then,* but they are not incompatible for me now. Buying an expensive car and taking an expensive trip to Europe are not incompatible in the way that taking a trip to Europe and staying *home* are incompatible. These are two *different* modes of incompatibility.

There are deep going, logical reasons behind these alternative modes of Incompatibility, which will be brought out in full in due time. For the time being, and for the task at hand, suffice it to say that if a pair of complementary quantities such as $p$ and $q$ are incompatible *all* the time, they will inevi-

tably (and even trivially) conflict with EPR. However, if these two concepts can be shown to be incompatible only *some* of the time instead, the conflict between Complementarity and EPR is no longer inevitable. Momentum and position will not then be incompatible all the time, but only during certain specific conditions. Once these conditions are removed, momentum and position will no longer be incompatible, hence, if EPR present exactly the type of case where these conditions *are* removed, Complementarity and EPR simply cease to conflict.

At this level, the point I'm making is really quite simple to state: Not all forms of Incompatibility are *unexceptional*. In consequence, not all forms of Complementarity need be unexceptional either. So if the type of Incompatibility demanded by Complementarity is not of the **un**exceptional kind, there is no reason to suppose off hand that it will necessarily conflict with EPR. However, the traditional versions of Complementarity *are* of the unexceptional kind, and these versions do conflict with EPR. I will therefore concentrate on these first and show that they are invalid independently of my own case. They are the following two definitions of Complementarity:

(a) Complementarity is a mode of description based on wave-particle Duality.(8, 9)

(b) Complementarity is based on the Fourier treatment of products E=hν, p=h/λ.

(a) and (b) are said to be birds of a feather. (b) is the more precise, mathematical formulation of (a). However, as I will show subsequently, they are inconsistent. Since my argument as a whole depends on the correct understanding of the concept of Incompatibility, I will begin from this: The incompatibility afforded us by the Duality of waves and particles is **un**exceptional. It holds in all possible cases one can think of. To make the point as forcefully as I can, wave-particle incompatibility holds *independently of the truth or falsehood of the quantum*. Waves are large, particles are small. Hence, their incompatibility is tautological and has no need of the quantum! But the two quantum uncertainties, $\Delta p \Delta q \geq h$, $\Delta E \Delta t \geq h$ must result *because* of the existence of the quantum and would vanish for h=0. How can a pair of uncertainties which are *dependent* on the quantum express a kind of incompatibility which holds *independently* of the quantum? Physicists are not too bothered by this ugly mismatch. But they have another thing coming.

If Duality proves something, it proves *too* much. The incompatibility demanded by Complementarity (CTY hereafter) is unthinkable without the quantum. (See below.) But the incompatibility of waves and particles is *self-evident* and so has no need of the quantum. Once remarks like these, which are almost never made, become apparent, it is not too difficult to see why

Complementarity *can* be reconciled with EPR. It is *Duality* which is irreconcilable with EPR, but then again Duality, which has no use of the quantum, is now proved inconsistent with Complementarity, which demands the quantum. And if Duality, which does conflict with EPR, provides a *mismatch* with CTY, there is really no reason to suppose that CTY necessarily conflicts with EPR.

Is then CTY type (a), i.e. Duality, at least compatible with CTY type (b), i.e. Fourier type CTY? Everyone answers "yes" to this question, but once careful analysis takes the place of easy-going, untroubled routine, everything starts to turn sour. Observe de Broglie's relation, they tell us: It says that a wave of wave length $\lambda$ is 'associated' with the momentum of the particle, as in mv=h/$\lambda$. The wave and the particle are joined, they say, in ways demanded by the two quantum uncertainties, E=h$\nu$ and p=h/$\lambda$. E and *p* are the properties of the particle, $\lambda$ and $\nu$ the properties of the 'corresponding' wave. Thus, Rosenfeld assures us that the energy and momentum are concentrated in the *particle,* and the frequency and wave number, $\nu$ and $\sigma$, are defined by the wave.(10)

Concentrated in the *particle,* are they? But if energy is concentrated in the particle, why is it determined by the *frequency,* as E$\approx\nu$ (given *h's* constancy) entails? The universal way of reading E=h$\nu$ is by saying that in QM energy is identified with the *frequency*. D. M.Mackay, for instance, contends throughout his paper that in QM (via the photoelectric effect) we obtain a new and unexpected "empirical identification of energy with the *frequency"*.(11) Hence, energy must be concentrated in the wave; not in the particle. In much more recent years Coveney and Highfield speak identically: this simple Planck relationship between the energy and the frequency in effect says that energy and frequency *are the same thing,* measured in different units.(12)

But if energy is the same thing as the frequency, how is it concentrated in the particle? Indeed, I must suppose that the quantum relation itself, E=h$\nu$ must be on the wrong track no less, insofar as that too, given *h's* constancy, implies that in QM the energy *is* the frequency. And if energy *is* the frequency, it is 'concentrated' in the frequency. And therefore, trivially, not to the particle. So there goes the 'particle' in E=h$\nu$, subdued in unfathomable obscurity due to E$\approx\nu$. Then what about p=h/$\lambda$? Surely, they say, there is a particle there involved in the variable *p*. And this is surely wave-particle Duality.

Strangely, the finer mathematicals details of the Fourier Analysis, i.e. CTY type (b), which is, supposedly, the more formal treatment of CTY type *(a),* turn viciously against what so many tend to regard as but its raw mate-

rial, namely, the empirical presence of Duality. Thus, in Hooker we read: if the quantum relation p=h/λ holds, then the momentum, *p,* can only be determined in the latter, *plane wave* case, where the spatial location, *q,* is completely indeterminate.(13)

But doesn't the spatial location, *q,* normally correspond to a *particle,* which, *q,* however, in the latter, plane wave case, is completely indeterminate? If *q* is indeterminate, the particle should also be. And all this in the determination of *p,* the very variable, supposedly, which is associated with the particle. The problem is that the particle to be 'associated' with the momentum is quite simply *missing,* since the plane wave, now warranting a unique λ value and thus a definite solution to p=h/λ, renders the very idea of a particle applicable at the pain of contradiction. Are we not told by complementarists type (a), i.e. supporters of Duality, that application of any one of the two concepts, wave or particle, absolutely precludes and rules out simultaneous application of the other? We are. Then, when a plane wave is being applied, the particle is absolutely precluded and ruled out according to *their* tenets.

However, on the other hand, when a plane wave is being applied, the momentum, *p, is* determined. According to CTY type (b), we must therefore conclude that the very process which results to the determination of the momentum of a particle, is conducted in necessary absence of the particle itself, whose momentum this *is,* according to CTIES type (a) and (b)! CTY type (b), i.e. the Fourier expansion of the relation p=h/λ, makes introduction of the 'particle', required by CTY type (a) literally impossible. What CTY type (b) provides is a momentum of something *other* than a particle. Though what or *whose* momentum this is, I am the last person responsible for answering. What I am responsible for, is the present argument, not presented ever before, and this argument entails that CTIES types (a) and (b) are inconsistent to one another.

And this is only the half of it. If CTY type (b), i.e. Fourier's, *precludes* the presence of a particle, when it applies the concept of a plane wave in order to determine the momentum, *p,* then, since this process is none other than the process directly leading to the uncertainty ΔpΔq≥h, this uncertainty, thus derived, *eo ipso* precludes the presence of a particle! And therefore concerns no particles of any kind. It only concerns momenta of *would-have-been* particles, were the relations involved only coupled differently than presently are. As they stand, they simply leave the experimentally observed particles a raw experimental fact, beyond the range of the uncertainty. And so beyond the range of QM, with all that that entails. Physicists *assume* that CTIES (a) and (b) are equivalents, as indeed de Broglie *assumed* that in his

p=h/λ the variable *p* is 'associated' with a particle. But, presumably, he did a much better job at associating than even he would have cared to imagine. By associating its momentum with a *wave,* he made it impossible for the particle to even *participate,* just as all particles are known to do with all waves. A particle is a notion contradictory to that of a wave and once a particle's momentum is *ascribed* to a wave, the particle has no choice but to withdraw.

EPR have produced an argument about a thorough *p,q* compatibility. As a complementarist I have no quarrel with it. However, most other complementarists do. They do, because they retain 'CTIES' which I have long dismissed. CTY type (a), i.e. Duality, is clearly incompatible with EPR across the board, for it entails that variables attached to the wave can coexist with variables attached to the particle only at the pain of contradiction. Hence, CTY type (a) is *ex hypothesis* incompatible with EPR. CTY type (b), i.e. Fourier type, though inconsistent with (a), in all other respects manifests a comparable intolerance to EPR. According to the Fourier derivation of $\Delta p \Delta q \geq h$, we require ONE wave to define the momentum (Hooker's point above) but MANY (superposed) waves to define the position. And ONE is as contradictory to MANY as wave is to particle. Hence, no CTY based on Fourier logic can be made compatible with EPR either.

However, since I have shown that CTIES (a) and (b) are incompatible to one *another,* on top of being both incompatible to EPR, and at the same time mutually invalid, since one of them excludes the very particle which they *jointly* demand, they are certainly no serious obstacle. So if we came up with *one* p,q CTY instead, consistent to itself and coherent (for a change), perhaps EPR and Bohr would both be right

## 2   Compatibility: The EPR Made Simple

A fire engine pulling a wagon is moving on the tracks. Then, at a later time the fire engine releases the wagon, which therefore begins to slow down trailing behind. The entire situation after the release is as depicted in Design and as below explained.

Design

|  |  | $q_2$ | $q_1$ |
|---|---|---|---|
| O | x | B, | y | A, |
|  |  | $p_2$ | $p_1$ |
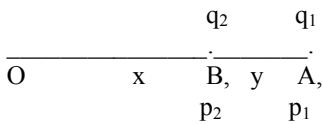
The fire engine is A, the (trailing) wagon is B. $q_1$ is the position of A and $p_1$ its momentum. $p_2$ and $q_2$ are the momentum and position of B. O is the point of their joint departure and $x$ is the distance covered by B at this point in time, namely, now B has travelled a distance OB or, simply, $x$. Since the fire engine, A, moves faster than the wagon after their separation, it has moved ahead of the wagon, B, and has at this point in time covered the distance OA or, simply, x+y. Accordingly, the wagon has travelled a distance OA-OB or OA-y.

The momentum, $p_1$, of the fire engine, plus the momentum $p_2$, of the wagon are trivially the momentum of the entire composite system, AB. Hence, if the momentum of the composite system is named $P$, it trivially follows that $P=p_1+p_2$. The momentum of the composite system, AB, is the sum of the momenta of the two separate systems, A, B.

Now to positions. A's position, $q_1$, given O, is: q1=x+y. But x=q2 and $y$ is BA. Hence, $q_1$=q2–BA. Therefore, $q_1$–q2=BA, where BA is the position of the composite system, AB. We may name this composite position $Q$. B's position, $q_2$, on the other hand, is: $q_2$= x+y–BA. But x+y=$q_1$. Hence, q2=q1–BA. This, in turn, yields, q2+BA=q1. And this, finally, BA=q1–q2, exactly as before. Hence, commencing with (subsystem) A, we get q1–q2=BA and commencing with (subsystem) B we likewise get BA=q1–q2. But BA=Q, therefore, Q (or BA)=q1–q2.

When, therefore, we wish to express the state of the composite system, AB, in terms of action $pq$ at any moment after the separation, this will be action equal with PQ= $(p_1+p_2)(q_1-q_2)$. Then, for curiosity's sake, we may now query whether the action product *converse* to PQ, namely, the product QP, is equal or unequal with PQ itself. Therefore, let us analyze PQ–QP. This is equal with $(q_1-q_2)(p_1+p_2)–(p_1+p_2)(q_1-q_2)$. And this, in turn, is equal with

$$(q_1p_1-p_1q_1) \quad - \quad (q_2p_1-p_1q_2) \quad + \quad (q_1p_2-p_2q_1) \quad - \quad (p_2q_2-q_2p_2)$$
$$\quad\quad a \quad\quad\quad\quad\quad\quad\quad b \quad\quad\quad\quad\quad\quad\quad c \quad\quad\quad\quad\quad\quad d$$

But $a$ is noncommutative in QM, so it yields $h$. $b$, on the other hand, is commutative everywhere, so it yields 0. $c$ is also commutative everywhere, so it also yields 0. And $d$, which is noncommutative, yields $–h$. Hence, in the end we obtain h–h=0. Therefore, the parameters of the composite system, AB, *commute*. And then the action state of the composite system, PQ, is found to commute with QP even on QM standards. Hence, there seems to be no quantum restriction on exactly determining parameters $P$ and $Q$ of the composite system as such. And then the value of the composite product *PQ*

can, at least in principle, be defined even on QM standards. Then, by measuring the momentum $p_1$ of A and the position $q_2$ of B, we can employ $P=p_1+p_2$ to determine B's momentum and $Q=q_1–q_2$ to determine A's position.(Ref.7, p.51) Hence, momenta and positions of both subsystems, A and B, should be known and QM, which denies this, is incomplete.

There is, of course, an objection open to the previous line of reasoning. How do we define $P$ and $Q$ as such, *without* having first defined $p_1$ and $p_2$ for $P$ and $q_1,q_2$ for $Q?$ And to define the individual values of any one of these pairs, respectively, would, according to QM, result to indefiniteness of their direct conjugates. Hence, the job can't be done. This objection is too easy and deserves a bit of trimming. For how can, reversewise, PQ commute with QP, as the mathematics here imply, when the separate parameters individually comprising either of these two products do not *individually* commute with one another also?

Presumably, if the entire product *PQ* commutes with *QP, P* and *Q* can be known with precision, contrary to QM, even if we presently have no idea how. And to then argue as above, that defining $p_1, p_2$ would be incompatible with defining $q_1, q_2$ respectively, would be to posit the truth of QM as warranted in advance, taking for granted the very point in question and, indeed, for granted against an argument showing that it should not be granted at all. It should not be forgotten that the argument, as stands, already concedes too much to QM and QM still can't handle it. It concedes that subtraction NR. *a* is *h* and it concedes that subtraction NR.*d* is –*h*. It's just too bad for QM that the two *h's* cancel each other out, isn't it? Hence, rather than circularly supposing that the variables involved are incompatible, as they would be if *h* was present, why not suppose instead that the case is classical? And then all four parameters would be simultaneously knowable *eo ipso*. It's not on to turn against me, what I only allowed for *your* convenience.

EPR, therefore, have definitely established a *p,q* compatibility in their argument, even if they,(3) or rather Einstein, was then dragged into what can be observed and what not, vitiating their own point. Sheer EPR compatibility, with no premises added, is perfectly sufficient for making dire trouble for at least two types of proposed CTIES, types (a) and (b). That it spells trouble for type (a), Duality, should be evident to all. Variables attached to the particle compared to variables attached to the wave, *eo ipso* assume between them the very type of incompatibility which particles and waves themselves possess. And this sort of incompatibility is utterly uncompromising. Nothing can be extended and nonextended at the same time. Hence, in the case of waves and particles, the mere supposition that an entity can be both, is self contradictory eo ipso and *without* measurability. And is there-

fore equally contradictory for any two variables (putatively) connected to them. Hence, EPR require no simultaneous 'measurability' of $p$ and $q$ to contradict Duality, if only they have established the *compatibility* of $p$ and $q$.

Strangely, however, they require no simultaneous measurability to contradict CTY type (b) either, and this is far more alarming than what befell picturesque Duality. The Fourier expansion of relation $p=h/\lambda$ shows us that we require ONE (plane) wave for fixing a value for $p$ and MANY (superposed) waves for fixing a value for $q$. And "one" is to "many" what wave is to particle. To even suppose that you can obtain *many* of what you must simultaneously obtain only *one,* results to immediate verbal contradiction. Hence, if EPR have established a *p,q* compatibility in their argument, they flatly contradict the relation $p=h/\lambda$, which entails that $p$ and $q$ are *incompatible,* not just "non-comeasurable", and therefore, since $p=h/\lambda$ was directly deduced from $E=h\nu$, they flatly contradict $E=h\nu$ also. And this conclusion, kept cautiously quiet about by EPR themselves, reaches far beyond the fate of CTY *or* the "completeness" of QM, as they chose to put it. It renders QM as such impossible. Hence, a reconciliation between Complementarity and EPR is not an option. It is a necessity.

Quantum theorists in general and complementarists in particular must therefore begin to learn from their mistakes, rather than incorrigibly continue to demand comeasurabilities of $p$ and $q$ before they are prepared to question the plausibility of CTY and QM as such. Because in view of the preceding analysis neither of these is barely plausible, if only $p$ and $q$ are shown to be at least –unobservedly- compatible. CTY and QM entail the *incompatibility* of $p$ and $q$ not just their empirical disjunctiveness, and Duality is a key witness to this fact. So unless CTY and QM are shaped up to meet and satisfy EPR requirements they will go on suffering in their credibility. What is incumbent upon us to show is that, although $p$ and $q$ are compatible in the EPR case, still this is no threat to either CTY or QM. And this brings us to my previous remark.

If $p$ and $q$ are compatible in an EPR context *and* also incompatible in a complementary context, then they are *sometimes* compatible, sometimes *in*compatible. A type of incompatibility, in other words, which is *unlike* wave-particle incompatibility *or* Fourier incompatibility for that matter. Unlike, one is tempted to suppose, what we expect of incompatibility to even be.

## 3   Incompatibility

What the sequel of the foregoing remarks points to is that we are here concerned with two distinct types of incompatibility between a pair of concepts or states. A type which is unexceptional, e.g. that of Duality, and a type which is not. The latter type, incompatibility admitting of exceptions, could well be the type required to explain how EPR manage to obtain a compatibility between $p$ and $q$ in their setting, one however which is not general enough to warrant their compatibility all *around*. And here the contribution of EPR ends, so far as I'm concerned. EPR have actually reversed the mistake of wave-particle complementarists; just as much as the latter erroneously assumed that, if $p$ and $q$ are incompatible somewhere, they are eo ipso incompatible everywhere, so did EPR assume that, if $p$ and $q$ are compatible somewhere, they are eo ipso compatible everywhere, hence it is good bye to CTY. I'm afraid not. So here goes.

The propositions "A>B" and "A<B" are incompatible by definition. This is one kind of incompatibility. But the bandit's command, "your money *or* your lives!", is another, and hardly at all like the former. The former incompatibility is self evident, the latter not. Nothing can be greater and at the same time smaller than something else. But people *can* have their money and their lives at the same time, and most of them do, *unless* as previously specified. No self evidence is involved in this latter type of incompatibility. Consequently, no self *sufficiency* in it either. The latter type requires something extrinsic to the concepts involved, if to ever result. And since something extrinsic to them cannot be warranted by concept analysis, it invariably turns out to be an additional *fact*.

Logical incompatibility between two concepts, in being self sufficient, results through the mediation of nothing, save the two concepts themselves, by directly comparing their semantics. We need not and, for that matter, can not consistently attribute it to anything other than the two incompatible concepts *per se*, solitarily compared to one another. Their antithetic definitions suffice to the task. Hence, logical incompatibility, in being utterly self-sufficient and intolerant to supplementation, is thereby **un**conditional. To say that "A>B" is incompatible with "A<B" on *condition* that "so and so" is a verbal contradiction. If incompatible on *condition* that "so and so", then, in "so and so's" absence, the two statements would turn up compatible. Which they don't. Consequently, logical incompatibility is unconditional, which simply means

fact **in**dependent. And this, in turn, means that this type of incompatibility cannot consistently relate to *any* fact[1]

By stark contrast, factual incompatibility is trivially fact-dependent. It may never result irrespective of an interfering fact (in our case the bandit), which fact we may call the "prohibitive fact".(14) And will be withdrawn, if the prohibitive fact is itself withdrawn. To sum up: I have distinguished between two, diametrically opposed types of incompatibility. The first type is self evident (=logical), therefore self-sufficient, therefore fact- **in**dependent, therefore **un**conditional. The second type, by contrast, is not self evident, therefore not self-sufficient, therefore fact-*dependent* (=factual), therefore merely conditional. Conditional, that is, on a certain prohibitive fact. The formal definitions of these two types are comparably distinct and contrastive. They are as follows:

$$\textbf{[a]} \ (p \rightarrow -q) \ \& \ (q \rightarrow -p), \ \textbf{[b]} \ [(p \rightarrow -q) \ \& \ q \rightarrow -p)] \leftrightarrow r.$$

For the value assignment –r to relation [b] the conjunction "p *and* q" is immediately derivable, something which is altogether precluded in relation [a]. For the value assignment –r to relation [b], [a] and [b] automatically assume incompatible truth tables. There is there- fore the greatest possible contrast between these two types of incompatibility. Now the following guideline becomes of essence: *If CTY and EPR are to be reconciled, CTY can only express factual (=conditional) incompatibility. There can be no consistency between CTY and EPR, if CTY is to express unconditional (=logical) incompatibility.*

With this in mind, let us now turn to see which interpretation of the two is the one which the two quantum uncertainties really support: Do the two quantum uncertainties, (UR hereafter), $\Delta E \Delta t \geq h$, $\Delta p \Delta q \geq h$, express conditional or do they express unconditional incompatibility between their two related sets of variables, E with *t* and *p* with *q?* In view of the preceding reasoning the answer to this question comes naturally. The reciprocal uncertainties in the values of the two pairs of conjugated variables, E with *t* and *p*

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[1] The duality between waves and particles is also logical. Hence, likewise incapable of relating to *any* fact. But people say this incompatibility *is* the incompatibility of the two URs. If so, then the two UR could not even *relate* to the quantum, *h,* which *is* a fact and of which they are the consequence. The trouble this distinction creates for wave- particle 'CTY' is now more than evident. This would be a CTY without the quantum!

with *q,* as presently joined, obviously express *conditional* incompatibility between these variables. Conditional, evidently, on *h* itself. Clearly, for h=0 both clusters of related uncertainties would vanish. On the other hand, they do emerge for h>0. Consequently, ΔEΔt, ΔpΔq are uncertainties which are there because and only because of *h*. And would be removed in its absence. This reads, respectively,

$$\Delta E \Delta t \leftrightarrow h>0 \text{ and, accordingly, } \Delta p \Delta q \leftrightarrow h>0$$

But not otherwise. Hence, both formulae precisely correspond to logical formula [b] at the exclusion of [a]. Evidently, then, the two UR express conditional incompatibility between their related variables, conditional, that is, on *h*. On this reading alone, the defects inherent in EPR immediately disclose themselves. *h* was *removed* in their argument and *that's* how CTY was evaded! And now the prospects of a compatibility between CTY and EPR are openly announced, at EPR's expense. EPR have simply left out the "prohibitive fact". And ended up with a *p,q* compatibility instead of their CTY.

Plausibly, the two pairs of conjugate classical variables, E and *t, p* and *q,* yielding the two action products Et and pq of the corresponding uncertainties, are rendered incompatible in QM because, simply, the latter theory incorporates an *additional fact,* hitherto unacknowledged and unanticipated by the classical theory, namely, action quantization. And it is the intervention of this precise and prohibitive fact, absent in classical assumptions, which is responsible for the incompatibility in their joint determinations below its limit, *h*. The two sets of incompatibilities are therefore *fact dependent,* that is to say, conditional *on* a fact: *h*. And therefore, trivially, express conditional incompatibility *only*. C. Hooker says, with excellent reason:

Bohr believes that while it has seemed to us at the macro-level of classical physics that the conditions were in general satisfied for the joint applicability of all classical concepts, we have discovered this century that this is not accurate and that the conditions required for the applicability of some classical concepts are actually incompatible with those required for the applicability of other classical concepts. This is the burden of the doctrine (B4). [(B4)=Complementarity.]

This conclusion is necessitated by the discovery of the quantum of action *and only because of its existence*. It is not therefore a purely *conceptual* discovery that could have been made *a priori* through a more critical analysis of classical concepts. It is a discovery of the *factual absence* of the condi-

tions required for the joint applicability of certain classical concepts. (Ref. 13, p.137. Last italics the author's.]

This, therefore, is exactly as foretold. The incompatibility above referred to is factual, because it is not the product of concept analysis, disclosing a logical discrepancy between the disjunctive concepts (and as such available a priori) and, therefore, as being fact *dependent,* it is eo ipso conditional. Conditional, that is, on the fact itself upon which it *is* dependent, and which I have previously labelled "the prohibitive fact". In other words, the quantum. And will be *removed,* if the quantum is removed. (No wonder EPR thought they had it made, when they removed it!)

On the whole, therefore, at first it seems a safe bet that the two pairs of classical variables of QM, when featuring pairwise in the two corresponding quantum uncertainties, should express conditional incompatibility between the thus related concepts and nothing but. On closer inspection, however, the situation appears a great deal more complex than initially assumed. Closer inspection in fact reveals that, when analyzed and examined all across the logico-conceptual board, the quantum uncertainties manifest and force upon us an incompatibility which is *both;* conditional *and* unconditional for one and the same pair of classical concepts. In my concluding sections I will remedy this remarkable imperfection. Presently, however, I will proceed to establish it.

## 4    Applying the Distinction

*Wave-Particle Duality:* Once the Conditional *vs* Unconditional Incompatibility contrast is applied to specific quantum arguments, familiar to all and so far considered as but a natural extension of the basic premises, these arguments suddenly start to look logically grotesque. Wave-particle Duality is just one such case. As a rule observed by nearly all physicists, the quantum uncertainties and Duality are treated as if intimately associated. But we have seen enough of the results of this practice. Particles are local entities, so particles are small. By contrast, waves are nonlocal entities, so waves are large. And the opposition between large and small is logical, that is to say, fact *in*dependent. Hence, waves and particles are *self-sufficiently* incompatible. This is why, besides, waves and particles are incompatible also in ***classical mechanics***. And classical mechanics does not contain the quantum.

Well, then. If the two uncertainties (UR) are a consequence of Duality, one set of variables belonging to the wave, the other set to the particle, then, since waves (large) and particles (small) exclude one another *self-sufficiently,* and hence without any help from the quantum, the variables appearing in the two UR, as derived from Duality, would *also* exclude one

another self-sufficiently, and hence without any help from the quantum. In fact, they do not need any help from anything at all, except of course the *self contained* opposition between "large" and "small" itself. Which opposition, as remarked, obtains independently of the quantum. Consequently, either the two UR have nothing to do with Duality, as I have been arguing for two decades now (15,16, 14,17) or else they have to do with Duality, but then they have nothing to do with the... *quantum,* on which, however, they are supposed to depend!

In other words, how can the incompatibility contained in Duality, which *qua* self sufficient obtains full force even in *classical* mechanics, ever be responsible for the incompatibility between the classical conjugate variables, which latter results only on the basis of *quantum* assumptions? Or, to put the point differently, how can a fact **in**dependent incompatibility, as that belonging to Duality, ever be responsible for a fact dependent incompatibility, as that demanded by the two quantum uncertainties?

If some are prone to wittingly retort here that WPD *is* the quantum, then I strongly advise against it. If it is, then the quantum, in itself but *one* entity, would now have to be *two* entities, in fact two incompatible entities, and so be two entities instead of one *and* incompatible to itself! Plus making the incompatibility resulting from such a 'union' logical *and* factual –and all the rest– at the same time. I'd take EPR consistency any day, if I had to choose, though thankfully I do not.

Some people still believe that wave-particle Duality is the epitome of the quantum uncertainties, if not indeed the epitome of QM as such. But once the Conditional/Unconditional Incompatibility contrast is applied to it, it simply proves to be an incoherence. The uncertainties, exactly as Hooker stressed, must absolutely depend on the quantum or be nothing at all. But if the uncertainties are constructed upon the logical model afforded by Duality, they will thereby express a *self-sufficient* type of incompatibility and, as we have seen, such incompatibility –trivially– has no need of the quantum. To be precise, cannot even make room for the quantum, except contradictorily. People think that wave-particle Duality furnishes *the* right sort of quantum incompatibility required by the UR. I have just shown that it furnishes *the* wrong sort, if there ever was one. And this conclusion now crosschecks fully with EPR expectations.

*Application to* $\Delta E \Delta t \geq h$: But the real trouble starts here. Duality, though hammered deep in the heads of many physicists, is not really formally accurate. It is just a collection of intuitive imageries and 'pictures'. The Fourier Analysis of the quantum relation $E=h\nu$, however, is nothing of the sort. It is, rather, a precise mathematical treatment and expansion of the said relation.

(18,19,20) Yet the problem is still the same. For that too is equally open to both accounts, the conditional *and* the unconditional.

Consider how the Fourier reasoning is applied to the quantum relation E=hν. Fourier's known relation, $\Delta\nu\Delta t\geq1/2\pi$, was based on the observation that it is a *logical* impossibility to determine the frequency *at* an instand dt=0. Frequency is by definition a repetitive phenomenon and hence by definition such as requires a time latitude to be exemplified, if at all. Obviously, I cannot define the regular reoccurrence of a certain event over even time intervals within a time dt=0, i.e. a time so narrow that won't allow the event to occur even once. As D.M. Mackay has remarked almost fifty years ago, the idea of defining a frequency *at* an instant dt→0 is self contradictory. *"This is not physics but logic",* he says. (Ref.11, p. 107.)

Quite so. But E=hν itself is not logic. It is physics; or at least it should be. Once the quantum relation, E=hν is therefore (factually!) established, by simply substituting for ν=E/h in Fourier's above mentioned relation, we immediately obtain $\Delta E/h\Delta t\geq1/2\pi$ and, finally, $\Delta E\Delta t\geq h$. Now, what sort of incompatibility does $\Delta E\Delta t\geq h$ express, if derived in this way? Well, it should express precisely the sort of incompatibility which ν itself, the frequency, does in Fourier's relation. Are we not constantly reminded that "energy" *is* the frequency in QM? Mackay, for one, Coveney and Highfield for two more (Refs.11 & 12 respectively) have most explicitly told us that energy and frequency are *one*. So to the task of specifying the syllogistic mechanism involved on the basis of this oneness:

Premise 1: Energy is logically equivalent with the Frequency.
Premise 2: Frequency is logically incompatible with an exact time.
Conclusion: Hence, *Energy* is logically incompatible with an exact time.

When two concepts are logically equivalent, they are coextensional. *Everything* which is true of the one must be true of the other or else their logical equivalence is contradicted. Hence, in the most straightforward and valid of manners, energy is above shown to be *logically* incompatible with an exact time, just as frequency previously was. But concepts incompatible in this sense are *self-sufficiently* so. And concepts which are self sufficiently incompatible are concepts whose incompatibility is fact *in*dependent. And therefore such that cannot even *relate* to a fact, e.g. *h*. Hence, in accordance with the Fourier treatment of the relation E=hν, we obtain an uncertainty ΔEΔt, due to a fact, *h,* with which it cannot even relate. The situation is even worse than EPR themselves had ever imagined. For rather than merely "incomplete" QM actually turns up incoherent on my distinction.

The protests against this conclusion are not too difficult to imagine. Since, I will be told, E=hν is a premise necessary to the derivation of the uncer-

tainty, and since E=hν explicitly incorporates the quantum, the intimate connection of the resulting uncertainty with the quantum is eo ipso warranted, as can be verified from the fact that the quantum, *h,* does after all appear in the uncertainty as much as all other requisite elements. Hence, none of what I claim follows. But a single word spoils all that, (though not the fun).

Substitution. Indeed, what is the true essence of the entire Fourier derivation? It is, in a word, the *substitution* of ν in ΔνΔt≥1 by E/h in order to derive ΔEΔt≥h. But being at all entitled to substitute E/h for ν presupposes that the two of them, the substituted and the substitute, just have to be identical, equal, equivalent or what have you. You name it. They have to be it. In consequence, E/h, which replaces ν, the frequency, *is* the frequency or else the substitution is illegitimate and has no business being there in the first place. And then, since E/h *is* the frequency, what is true of the frequency must be true of its substitute, E/h. And then, since what is true of the frequency is that it is unconditionally incompatible with time, E/h is also unconditionally incompatible with time. It is either that or else the substitution is sheer bogus and no ΔEΔt≥h will ever follow.

Hence, by right of mathematical law, the law of substitutions, E/h (=ν) is unconditionally incompatible with time, *even* if it deceitfully contains *h* in the fraction just to mislead (some of) us. The conclusion can now be denied at the pain of contradiction. By putting E/h in the place of ν in ΔνΔt≥1, we make E/h whatever ν is, thus deriving a *logical* E,t uncertainty and, therefore, a fact-independent one that cannot even relate to the very *h,* which it has itself put there! In the face of my distinction, the Fourier treatment of E=hν leads to incoherence and absurdity. Valid reasoning is reasoning which transmits the logical properties of the premises down to the last conclusion. And the logical properties of premise ΔνΔt≥1 is that it incorporates a self sufficient type of incompatibility, rendering *h* redundant. So I guess the best way out of the dilemma is to deny that the whole reasoning *is* valid!

The essence of the problem here encountered stems from the fact that, in view of the distinction here introduced (and hitherto absent in all quantum theorizing), E=hν proves a full scale logical hybrid. Taken in one context, i.e. as a factual truth, E=hν should lead the variables E and *t* to factual incompatibility in this context. However, taken now in a different context, i.e. that demanding the identity of ν with E/h, should lead the same variables to logical incompatibility in this other context. When, in other words, E=hν is considered in its *outward* relation to reality, it must in this capacity be a factual truth. But when considered per se, i.e. *inwardly,* in this other capacity

it incorporates a logical truth. What should we say then? That what E=hν really asserts is that, on its basis, E and *t* are unconditionally incompatible concepts on *condition* that E=hν is true? On the basis of the distinction here introduced this is exactly what we have to say. Though, of course, in its absence, we wouldn't have to.

## 5    Making Proper Use of the Quantum

We have seen that Fourier's ΔνΔt≥1 creates a host of undesirable problems and logical riddles that threaten the coherence of QM, unnecessarily. But QM, in this case E=hν, is self-*sufficient* and has no need of Fourier's relation to produce the desired results. So how can we obtain them, as demanded, at the exclusion of ΔνΔt≥1? (21)

The answer, once the question is put thus, is not too difficult to visualize and prescribe: We must simply confine the deep going association of energy with frequency to only *certain* properties of the frequency rather than, as so far is done, extend it to every known property of frequency instead. This, besides, is exactly what the quantum relation itself demands. For it nowhere says that energy is identical with the frequency. What it says is, simply, that it is identical with the frequency times the *quantum*. Hence, the implied association between them, rather than extending throughout the entire conceptual board, should be confined to just those aspects of frequency, which can at the same time make room for the atomicity of action. In this way, and once only *certain* properties of frequency are to be directly related with energy, the door to Fourier's (catastrophic in this case) ΔνΔt≥1 will not be opened as widely as before.

I have to propose just such a way which, in a nutshell, is the following: *Confine the energy-frequency identification to a unique wave-period.* And do away with all the rest of them, first because they make nothing but trouble, making us say that the uncertainty expresses both a fact-dependent *and* a fact-independent incompatibility; secondly, because this is much closer to the basic quantum relation, E=hν, and so to the truth. E=hν does not speak of an out and out identication of energy with frequency at all but only of one such by *means* of *h*. If the derivation is confined to a single wave period, then the relation ΔνΔt≥1, which is the root of all the problems, will no longer dominate the argument. For the reasoning lying at its basis, "how can frequency be defined *at* an instant, when there are so many occurrences to consider?" will clearly cease to apply to a unique wave period. For a unique wave period this question cannot even come up.

Yet the otherwise warranted association between energy and frequency, via E=hν, will not be abandoned, only modified and preserved in the fact that energy is still, after all, associated with a *period*. Last, but by no means least, what is also expected of the argument to establish is that the said period, *t,* with which energy is to be associated, itself corresponds to a quantum of action, of dimensions Et. So that the ensuing uncertainty be attributed *to* that unit of action Et, directly resulting from the said association.

Time to show how the idea of confining the energy-frequency association to a single period instead is supposed to work. I will begin with an interesting passage by Olivier Darrigol, in itself faintly containing this conception and which, once adapted to match my own proposal, will yield the exact required results. Darrigol writes:

Einstein's form of the quantum condition fitted well with de Broglie's idea that action played the role of a *phase*. Langevin called the action variables "the cyclic *periods"* of the action integral. This denomination implied that Langevin regarded action as a *periodic* function.(22)

The meaning of this, when confined, as I have demanded, to a single wave period instead, is that in QM the least possible exemplification of a dynamical quantity, E or *p,* can only be recorded *over a period*. A wave period. If this period is expressed in terms of the frequency, ν*,* we obtain E=hν and so Et=h. If it is expressed in terms of the wavelength, λ*,* we obtain p=h/λ and so pλ=h, as two expressions of minimal action, *h*.

The idea, stated in more precise terms, discloses that the basic quantum relation, E= hν, signifies that E, exactly like ν, can no longer be defined *at* an instant dt→0, as was classically assumed, but only over a period t>0, whose boundary instants $\{t_1,t_2\}$ are here given by the overall frequency of this period, ν. Correspondingly, *p* also cannot be determined *at* a point location, dq→0, but only over a *distance,* whose boundary points $\{q_1q_2\}$ are analogously given by the wavelength of this period, λ. Only, remember, to be both defined over *that* period, t>0 or *that* distance, $\{q_1q_2\}$. The argument nowhere asserts that there are *many* such periods or many such distances required for the said determinations. In fact, the rest of them are nothing but a surplus, because in a plane wave they are one and all *stereotype*. So any single one of them would do, at exclusion of the (redundant) rest.

And here the association between energy and frequency, or between momentum and wavelength, comes to an end. The Fourier headache is no longer applicable to *this* understanding of quantized action. But the uncertainties certainly are! In fact, they are openly announced in the limitations themselves, that energies can only be defined over *periods,* hence ΔE=∞ at

an instant Δt=0, momenta only over *distances,* hence Δp=∞ at a point Δq=0, and, indeed, conversely. (But of this a little later.)

Now to the relevance and, indeed, the very emergence of the quantum out of this limitation. It is *because* E cannot be defined *at* an instant dt→0, but can only be 'mapped' over a period, that the resulting product of action Et is not arbitrarily reducible to a diminishing value, Et→0, so yielding a *quantum* of action instead. I.e. it is *because* E can only be determined over a limiting period t>0, rather than within a t→0, that the product Et is likewise placed under the same limit, yielding a quantum of action, *h*. Clearly, in the case that E could be recorded *at* an instant, taking that instant as narrow as we might wish, leading to a t→0, the product Et would be a vanishing quantity and no quantum of action would ever result. On this premise, and on that alone, i.e. that E cannot be defined *in* an instant, the energy time uncertainty relation can be shown to result.

1. ΔEΔt≥h

If there is such a thing as a shortest time permissible, i.e. a time *limit,* imposed on the conditions warranting the very manifestation of E, this being a time limit of dimensions Δt={$t_1 t_2$}or ν, then any subsequent *narrowing* of this interval, of the order, say, Δt'= {$t_1 t_2$}/2 can only mean that the overall energy determination will only be *reciprocally* affected and, therefore, reciprocally inaccurate. For if we require *at least* a time length of dimensions Δt, if we are to determine the energy with an accuracy ΔE=n, where *n* is sufficiently small to stand for a high E approximation, then, all other conditions being identical, at *half* the time formerly allowed, i.e. within Δt/2, we can only expect to end up with an uncertainty ΔE=2n if the action product itself, of which E and *t* are the components, is to always remain *constant,* i.e. *h*. And so on, reciprocally, for any other diminution.

In other words, if energy can never be defined within an instant t→0, and hence is only to be defined over the boundaries {$t_1 t_2$}=Δt, as is dictated by the assumption that the action unit which is the product of components E and *t* is to remain constant (or minimal) at all times, then the optimal definitions of these two action components, E and *t,* cannot themselves be any sharper than the said limiting product, Et. And therefore that the joint errors in the definitions of these two action components, E and *t,* can at best be equal, or if not, then greater than this limiting product Et. Hence, in symbols, ΔEΔt≥Et. But Et=h. So ΔEΔt≥h.

This proposal has, on the one hand, incorporated the requisite energy-frequency association, since it associates the exemplification of energy with

a period. And it has, on the other, shown that this association results to a *minimal* such period, t>0. Namely, a quantum of action of dimensions Et. And so now the concept of the quantum action is used in very explicit form. Only now it represents the association of energy with a *period;* not with *periodicity*. Hence, the trouble maker, $\Delta\nu\Delta t\geq 1$, is no longer there with it. This derivation is dependent on the quantum but *not* dependent on $\Delta\nu\Delta t\geq 1$. And by being so it only expresses conditional incompatibility between E and *t*. Conditional, as can be seen, on the product Et=h>0 itself, now *inclusive* of E=h$\nu$, only currently confined to a period. Which is exactly as it should be and exactly what I was after.

## 2. $\Delta p\Delta q\geq h$

By strict analogy with energy before, if momentum can only be defined *over* a distance and therefore not *at* a point location $\Delta q\rightarrow 0$, hence only over a distance of dimensions $\Delta q=\{q_1q_2\}(=\lambda)$, it equally follows that any attempt at its definition within spatial boundaries narrower than those specified, will be as inaccurate as the distance itself employ-ed for its definition is taken shorter. Same as before, if we need at least a distance of dimensions $\{q_1q_2\}=\lambda=\Delta q$, if we are to determine the momentum within an accuracy $\Delta p=n$, where *n* is sufficiently small to stand for a high momentum approxi-mation, then, all other conditions being identical, at *half* the distance formely allowed, i.e. at $\{q_1q_2\}/2$, we can only expect to end up with an uncertainty $\Delta p=2n$, if the action product itself, of which *p* and *q* are the components, is to remain constant, i.e. *h*.

So, in general, if momentum cannot be defined *at* a space point, but is only to be defined over the spatial boundaries $\{q_1q_2\}=\Delta q=\lambda$, as is dictated by the assumption that the product p$\lambda$ is to remain constant (or minimal) at all times, then the optimal definitions of these action components, *p* and *q,* cannot themselves be any better than the limiting product p$\lambda$.[2] And then, consequently, the joint inaccuracies in the definitions of *p* and *q* can at best be equal, or if not then greater, than this limiting product p$\lambda$. Hence, in sym-bols, $\Delta p\Delta q\geq p\lambda$. But p$\lambda$=h. So $\Delta p\Delta q\geq h$.

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[2] For the demonstration of the point that p$\lambda$=pq, so that action quantization as p$\lambda$=h can be directly applicable to the product pq, and thereby be subjected to the relevant uncertainty, see Antonopoulos, ref.21, Appendix.

Here distances have taken the place of periods but other than that the two derivations are thoroughly symmetric. We just substitute distances for durations and rerun the argument. And distances commit one to wavelengths no more than durations, or periods, previously committed one to periodicities. Once wavelengths give their place to plain lengths, periodicities theirs to plain periods, then momenta definable over the former will yield pq, and energies definable over the latter will yield Et, both equal to *h,* and then all separate requirements are joined in thorough consistency. The quantum relations, E= hν and p=h/λ are utilized, to the extent, however, that they are confined to periods for defining E, and to distances (lengths) for defining *p*. Any *further* application of the "energy is frequency" idea, and all the rest, thereby leading to a straightforward introduction of waves proper, will only invite trouble.

The notion that "energy is associated with the frequency", without being abandoned in my argument, was simply cut down to size just enough to escape the contradiction that the UR express both inconsistent types of incompatibility at once, *and* the related contradiction, via monopolizing of the argument by ΔνΔt≥1, that the two UR are due to a quantum, *h,* with which they cannot even relate. All of the previous anomalies have been effortlessly avoided, yet without further departing from the initial conception. Come to think of it, what *is* the initial conception? Take de Broglie's pλ=h. How do theorists describe it? They describe it by saying that momentum relates to a wavelength, i. e. ***a*** wavelength. *p* does not relate at once with *all* the wavelengths involved in a uniform wave. It just relates to any *one* of them. No uniform wave in existence *has* more than one wavelength. They all have a single wavelength in repetition.

Hence, by conception, pλ=h is confined to *one* wavelength and one wavelength is all it ever takes to define it. Hence, it nowhere follows therefrom that *p* necessarily relates to a wave. Why not to a wave *length?* And hence to a length. You'll say that no wave is like that? Quite. But it is now quantum waves that we are dealing with, not just any wave. In other words waves whose intrinsic constitution must always be capable of relating to a *quantum* of action, of dimensions pλ=h. Include, then, *all* wavelengths available and in no time it all results to an infinity of those, and all of them to be associable at once with a *single* quantum=h. Or else with an infinity of action quanta, right where there should only be a single quantum=h instead. Either way you end up with one element too many. Rather than introducing an arbitrary curtailing of infinite waves, my method provides exactly what the quantum situation per se demands. And so, instead of *infinite* waves, we must simply cut down to a finite wave period.

Then, since energy can only be defined *over* that period, *t,* and in no less than *that* period, *t,* the resulting product Et will be a unit of quantized action, Et=h, leading exactly to ΔEΔt≥h for any E determination attempted over a period sooner than *t*. And, accordingly, since *p* can only be defined over a *distance,* λ or *q,* and in no less than that distance, *q,* the resulting product pq will be a unit of quantized action, pq=h, leading exactly to ΔpΔq≥h, for any *p* determination attempted over a distance shorter than *q,* both of these E, *p* uncertainties growing proportionally larger, as periods and distances grow proportionally smaller; and vice versa.

A couple of pages ago it took me one brief paragraph for deriving each of these two symmetric uncertainties. Now it has taken me one paragraph for both. The trained reader need only take a look at the Fourier analysis of the two quantum relations, without counting its several noted contradictions, or at the unfathomable abstractions of matrix mechanics, both of which methods are alike in all their dizzying complexity, and compare them with the simplicity and the elegance of the two derivations here proposed. They essentially but unpack the initial idea, that energies can only be defined over periods, momenta over distances, with reciprocal uncertainties resulting, if the limiting periods or distances are simply transcended. No more than that, no less. Simplicity is one feather in their cap.

Here, then, is the other. As presently derived, the quantum uncertainties express fact- dependent and only fact-dependent incompatibility. This can be shown to follow from two, converging routes; First, since infinite waves, or infinite periodicity, is now reduced to a *single* period instead, the door to the logical, hence fact *in*dependent, relation Δ**v**Δt≥ 1, is closed conclusively shut. There is no longer a way of turning the uncertainties into logical ones in its absence. Second, crosschecking with first, now the uncertainties have been (jointly) shown to depend on a contingent *fact*. The fact being that, in opposition to the classical supposition, energies can no longer be defined *at* an instant, momenta *at* a point. And it is clearly a *factual* truth, and a startling one at that, that energy cannot be defined *at* an instant, dt→0, but only over a time *limit*. This is a limit placed by reality. Not a limit placed by logic. Hence, that energies cannot emerge *at* an instant, is a factual truth and so, therefore, are the uncertainties resulting on its basis.

## 6    Back to EPR; Conclusion

CTY has been shaped up, if it really needed shaping, because Bohr's own views were themselves quite shapely no matter how others may have twisted them to shapelessness. And so the answer to EPR has been furnished in truly

feedback process. My derivations made no use of $\Delta v \Delta t \geq 1$, and certainly no use of Duality, either of which had made $p$ and $q$ unconditionally incompatible. And if no longer **un**conditionally incompatible, their logical status no longer conflicts with EPR. When the prohibitive fact responsible for the resulting incompatibility, i.e. $h$, is withdrawn, and in this argument (or EPR's!) it *can* be withdrawn, then a pair of concepts incompatible on *condition* of its presence will *eo ipso* turn compatible in its absence. This is exactly what my formula [b] predicts. My argument nowhere demands that E with $t$ or $p$ with $q$ will continue to remain incompatible still after the crucial, and limiting period $t>0$ (or the corresponding limiting distance $q>0$) is over. Energy and momentum *have* been defined over that period or that distance and there are no further limitations placed on simultaneous accuracies in their values since, when all is said and done, they were all defined over a period or a distance not *smaller* than the quantum. *Beyond* its boundaries the conjugated variables are fully compatible.

This is exactly what 'went down' in the EPR case. A quick look at my reconstruction of their case shows that the numerical operations named $a,b,c$ and $d$ (amazingly!) yield h–h=0. And thereby lead to a straightforward $p,q$ commutativity. Is this perhaps supposed to mean that EPR attack CTY by throwing at it an utter irrelevancy, fighting windmills rather than dragons? Far from it. I don't know how much of Bohr's philosophy Einstein had really grasped and from what little of his I've read in this connection I am convinced he understood horribly little. But none of this ever makes EPR irrelevant. EPR, as I have been at pains to emphasize, fight real quantum dragons: Wave-Particle Duality and $\Delta v \Delta t \geq 1$, imposed upon an otherwise rather innocent E=hv. Their argument is not just relevant but valuable.

One (pseudo)derivation of $\Delta p \Delta q \geq h$, that based upon Duality, which attaches one variable to the wave, the other to the particle (which to which, pray?), makes the (quasi) resulting $\Delta p \Delta q \geq h$ an unconditional uncertainty and therefore, by right of birth, an uncertainty which holds for *all* possible cases, that of EPR included. Due precisely to its uncontainable claims, exempting nothing, Duality is all too pertinent to EPR and EPR, as I have shown, refute it. Since my doctoral days, twenty five years ago, I have always found Duality an inelegant, facile and, what is of essence, an incoherent method for deriving mutual exclusion. If derived from the fact **in**dependent incompatibility between waves and particles, such mutual exclusion could not even relate to the quantum. This is why I am perhaps one of the few Bohrians in the market taking EPR so seriously and perhaps the sole there is, who actually supports its validity.

For the exact same reason, indeed all the more so, EPR is highly relevant to the basic skeleton of QM, the two quantum relations, E=hν and p=h/λ. The Fourier expansions of those two likewise imply unconditional incompatibility between the associated variables. The essence of Fourier logic is: *one* wave for the momentum, *p, many* waves for the position, *q.* Which is a logical incompatibility identical to that of Duality. And then, if *p* and *q* are incompatible in all possible cases, they should, trivially, be incompatible in the EPR case, which makes the latter very much relevant, the difference being, however, that EPR refute the incompatibility. So EPR, in having warranted a *p,q* compatibility, refute not the relations E=hν and p=h/λ *themselves* really, but only their uncontainable Fourier over-flow all the way to infinite waves, at evident contempt of the quantum. It is *this* practice which renders *p* and *q* unconditionally incompatible and hence EPR, in having established their occasional compatibility, come by to fully crosscheck and support my own requirement, that under no circumstances are ν and λ to belong to infinite waves. And here their independent findings are literally priceless for my case. And Bohr's. And QM's.

In stark opposition, it is not too difficult to show that my deduction of ΔpΔq≥h is perfectly compatible with EPR. *My* derivation utterly depends on a quantum pλ=pq=h and is impossible without it. And in my reconstruction of EPR we obtain h–h=0, and so the quantum is now absent. Hence, no further incompatibility between the classical concepts should be expected. Their argument cannot affect a *quantum* dependent CTY and, since Bohr's CTY is exactly that, it cannot affect it, though it does affect the other two, quantum *in*dependent 'CTIES', (a) and (b), Duality and the Fourier analysis.

This conclusion points to two kinds of world behaviour, the micro and the macro. It would perhaps be natural to compare the *quantitative* dimensions of overall action *PQ* involved in their example as opposed to the single quantum of dimensions pλ=h involved in mine. But the difference, really, is other than quantitative. It is, to be precise, a difference of *quality*.(23)Action in their context is **sub**divisible and action in mine is **in**divisible. And this is the greatest difference there is. There can be no micro/macro *monism*.

There is in quantum theorizing a method called "the correspondence principle", consisting in the admonition to design quantum equations in such a way as to always yield an asymptotic convergence and so an asymptotic return to classical mechanics for large quantum numbers. But in the case I'm making and, I believe, in Bohr's thought permanently, the correspondence principle proves inadequate on both scores; the asymptocy as such and the convergence. Bohr was implicitly a *dualist* (Ref. 2, p.315) and so is, implicitly, my own argument. Hence rather than an asymptotic convergence to

classical requirements, which essentially extends quantum laws into the macro world, what we have in-stead is a *discontinuous* but *total* return to such requirements. This is what dualism is all about. A simple numerical model of CTY I have devised 25 years ago explains how:

Consider the case of a simple equation, A+B=x, where all three variables are to receive values exclusively from the field of naturals –for evident reasons– i.e. positive integers without zero. (Some say because zero is not a 'number'; I would say because zero is not a positive number.) Then, for any value ascription to x, such that x>1, both the other two variables, A and B, can receive definite and simultaneous value assignments. However, for the value ascription x=1 this is no longer possible. Given 1's elementary *indivisibility,* our sole two and mutually exclusive options are: A=1 and B=indeterminate or B=1 and A=indeterminate. "1" is nondistributive in the field of naturals and hence drives A and B to mutual exclusion. Now compare this with the words of Bohr:

The fundamental postulate of the *indivisibility* of the quantum of action [...] forces(!) us to adopt a new mode of description designated as complementary.(24) [Or] Complementarity is a term suited to embrace the feature of *individuality* of quantum phenomena. (Ref. 19, p.39)

The first thing to observe is that, if this model reflects the true state of Atomicity and the words of Bohr, then the incompatibility here resulting is strictly and uniquely conditional. Conditional on x>1 *or* x=1. In the former case no incompatibility results, in the latter it does. A and B are not incompatible to one *another,* as wave is to particle or Δv=0 is to Δt=0. They are only incompatible on *condition* that x=1. In the field of rational numbers, where "1" is subdivisible, no mutual exclusion would result at all. The second thing to observe is that, if the model truly reflects Atomicity, the 'description' provided is *complete*. There can never be a better description for the case x=1. The fact that we can assign "1" to *either* variable, before committing ourselves to any assignment, does not mean that we can assign it simultaneously to both.

And the third thing to observe is that, for any other assignment x>1, we get back to simultaneous and definite value assignments to both A and B *directly*. The return to a 'classical' analogy is not asymptotic and gradual, tending to an ever receding and inaccessible limit. It is total and discontinuous. This is not a difference of quantity. It is a difference of quality, never attainable on the (dispensable for Bohr) principle of correspondence. I don't see why something of this sort cannot happen between classical and quantum mechanics, warranting the *radical* return to macro experience just as radically, indeed, as it can conversely warrant a comparable return to the

atom. And this, I tentatively propose, is exactly what has happened with EPR. EPR have returned to a macro situation *qualitatively*. Obviously, they themselves would have wished to have done so quantitatively, so that, if sause for the goose is sauce for the gander, the *micro* would then have to be a miniature of the macro instead. And thus turn the tables on "correspondence". I do not think they have succeeded. Why, indeed, do they use *two* correlated systems for establishing a *p,q* compatibility and not just one? This is a question I have never seen asked of them and one they certainly have not paused to ask themselves.

My guess is because what they *could* do for x>1, in their case corresponding to an ascription x≥2 of ours, they simply couldn't do for x=1, the Atom. That aspect of CTY, not Wave-Particle 'CTY' nor Fourier 'infinities', but *authentic* CTY as it emerges in its direct dependence on the Atom, still remains out of their reach.

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