Gravitation from a Gauge like Formulation

B.G. Sidharth

International Institute for Applicable Mathematics & Information Sciences
Hyderabad (India) & Udine (Italy)
B.M. Birla Science Centre, Adarsh Nagar, Hyderabad - 500 063 (India)

ABSTRACT. After many fruitless decades of trying to unify electromagnetism and gravitation, it is now being realized that this can be done only in discrete spacetime, as indeed the author had demonstrated. In this context, a unified description of gravitation and electromagnetism is provided within the framework of a gauge like formulation.

1 Introduction

In an earlier communication [1], based on a discrete spacetime noncommutative geometrical approach, we had shown that it was possible to reconcile electromagnetism and gravitation. It is of course well known that nearly ninety years of effort has gone in to get a unified description of electromagnetism and gravitation starting with Hermann Weyl's original Gauge Theory. It is only in the recent years that approaches in Quantum Gravity and Quantum Super Strings, amongst a few other theories are pointing the way to a reconciliation of these two forces. These latest theories discard the differentiable spacetime of earlier approaches and rely on a lattice like approach to spacetime, wherein there is a minimum fundamental interval which replaces the point space time of earlier theories. Indeed as Hooft has remarked, "It is some what puzzling to the present author why the lattice structure of space and time had escaped attention from other investigators up till now...." [2,3,4] Infact we had recently shown that within this approach, it is possible to get a rationale for the de Broglie wavelength and Bohr-Sommerfeld quantization relations as well[5]. Nevertheless, the link with the gauge theories of other interactions, based as they are, on spin 1 particles, is not clear, because the graviton is a spin 2 particle (or alternatively, the gravitational metric is a tensor).

2 A Gauge like Formulation

In this latter context, we will now argue that it is possible for both electromagnetism and gravitation to emerge from a gauge like formulation. In Gauge Theory, which is a Quantum Mechanical generalization of Weyl's original geometry, we generalize, as is well known, the original phase transformations, which are global with the phase λ being a constant, to local phase transformations with λ being a function of the coordinates [6]. As is well known this leads to a covariant gauge derivative. For example, the transformation arising from $(x^\mu) \to (x^\mu + dx^\mu)$,

$$\psi \to \psi e^{-\imath e\lambda} \tag{1}$$

leads to the familiar electromagnetic potential

$$A_\mu \to A_\mu - \partial_\mu \lambda \tag{2}$$

The above transformation, ofcourse, is a symmetry transformation. In the transition from (1) to (2), we expand the exponential, retaining terms only to the first order in coordinate differentials.
Let us now consider the case where there is a minimum cut off in the space time intervals. As is well known this leads to a noncommutative geometry (Cf.ref.[1])

$$[dx_\mu, dx_\nu] = O(l^2) \tag{3}$$

where l is the minimum scale. From (3) it can be seen that if $O(l^2)$ is neglected, we are back with the familiar commutative spacetime. The new effects of fuzzy spacetime arise when the right side of (3) is not neglected. Based on this the author had argued that it is possible to reconcile electromagnetism and gravitation [7,8,9,10]. If in the transition from (1) to (2) we retain, in view of (3), squares of differentials, in the expansion of the function λ we will get terms like

$$\{\partial_\mu \lambda\} dx^\mu + (\partial_\mu \partial_\nu + \partial_\nu \partial_\mu) \lambda \cdot dx^\mu dx^\nu \tag{4}$$

where we should remember that in view of (3), the derivatives (or the product of coordinate differentails) do not commute. As in the usual theory the coefficient of dx^μ in the first term of (4) represents now, not the gauge term but the electromagnetic potential itself: Infact, in this

noncommutative geometry, it can be shown that this electromagnetic potential reduces to the potential in Weyl's original gauge theory [6,7]. Without the noncommutativity, the potential $\partial_\mu \lambda$ would lead to a vanishing electromagnetic field. However Dirac pointed out in his famous monopole paper in 1930 that a non integrable phase $\lambda(x, y, z)$ leads as above directly to the electromagnetic potential, and moreover this was an alternative formulation of the original Weyl theory [11,12].
Returning to (4) we identify the next coefficient with the metric tensor giving the gravitational field:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (\partial_\mu \partial_\nu + \partial_\nu \partial_\mu) \lambda dx^\mu dx^\nu \tag{5}$$

Infact one can easily verify that ds^2 of (5) is an invariant. We now specialize to the case of the linear theory in which squares and higher powers of $h^{\alpha\beta}$ can be neglected. In this case it can easily be shown that

$$2\Gamma^\beta_{\mu\nu} = h_{\beta\mu,\nu} + h_{\nu\beta,\mu} - h_{\mu\nu,\beta} \tag{6}$$

where in (6), the Γs denote Christofell symbols. From (6) by a contraction we have

$$2\Gamma^\mu_{\mu\nu} = h_{\mu\nu,\mu} = h_{\mu\mu,\nu} \tag{7}$$

If we use the well known gauge condition [13]

$$\partial_\mu \left(h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} h_{\mu\nu} \right) = 0, \text{ where } h = h^\mu_\mu$$

then we get

$$\partial_\mu h_{\mu\nu} = \partial_\nu h^\mu_\mu = \partial_\nu h \tag{8}$$

(8) shows that we can take the λ in (4) as $\lambda = h$, both for the electromagnetic potential A_μ and the metric tensor $h_{\mu\nu}$. (7) further shows that the A_μ so defined becomes identical to Weyl's gauge invariant potential [14].
However it is worth reiterating that in the present formulation, we have a noncommutative geometry, that is the derivatives do not commute and moreover we are working to the order where l^2 cannot be neglected. Given this condition both the electromagnetic potential and the gravitational potential are seen to follow from the gauge like theory. By retaining coordinate differential squares, we are even able to accommodate apart from the usual spin 1 gauge particles, also the spin 2 graviton which otherwise cannot be accommodated in the usual gauge theory. If

however $O(l^2) = 0$, then we are back with commutative spacetime, that is a usual point spacetime and the usual gauge theory describing spin 1 particles.

We had reached this conclusion in ref.[1], though from a different, non-gauge point of view. The advantage of the present formulation is that it provides a transparent link with conventional theory on the one hand, and shows how the other interactions described by non Abelian gauge theories smoothly fit into the picture.

Finally it may be pointed out that the author had argued that a fuzzy spacetime input explains why the purely classical Kerr-Newman metric gives the purely Quantum Mechanical anomalous gyromagnetic ratio of the electron [15,16], thus providing a link between General Relativity and electromagnetism. This provides further support to the above considerations.

References

[1] B.G Sidharth, Annales de la Fondation Louis de Broglie, 27 (2), 2002, pp.333ff.

[2] B.G. Sidharth, "Chaotic Universe: From the Planck to the Hubble Scale", Nova Science Publishers, Inc., New York, 2001.

[3] G. 't Hooft, ArXiv:gr-qc/9601014.

[4] T.D. Lee, "Particle Physics and Introduction to Field Theory", Harwood Academic, 1981, pp.383ff.

[5] B.G. Sidharth, "Geometry and Quantum Mechanics", to appear in Annales de la Fondation Louis de Broglie.

[6] M. Jacob (Ed.), "Gauge Theory and Neutrino Physics", North Holland, Amsterdam, 1978.

[7] B.G. Sidharth, Il Nuovo Cimento, 116B (6), 2001, pg.4 ff.

[8] B.G. Sidharth, Il Nuovo Cimento, 117B (6), 703ff, 2002.

[9] B.G. Sidharth, Found.Phys.Lett., August 2002.

[10] B.G. Sidharth, Found.Phys.Lett., 15 (6), 2002, pp.577-583.

[11] P.A.M. Dirac, in "Monopoles in Quantum Field Theory", Eds. N.S. Craigie et al., World Scientific, Singapore, 1982.

[12] B.G. Sidharth, Il Nuovo Cimento, 118B (1), 2003, pp.35-40.

[13] C.H. Ohanian, and R. Ruffini, "Gravitation and Spacetime", New York, 1994, p.397.

[14] P.G.Bergmann, "Introduction to the Theory of Relativity", Prentice-Hall, New Delhi, 1969, 245ff.

[15] B.G. Sidharth, Gravitation and Cosmology, 4 (2) (14), 1998, p.158ff.
[16] B.G. Sidharth, Found.Phys.Lett., 16 (1), 2003, p.91-97.

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