Landau-Lifshitz equation of motion for a charged particle revisited

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ABSTRACT. It is shown that Landau-Lifshitz equation of motion for a charged particle presents similar behaviors to Lorentz-Dirac equation. Indeed the reaction force obtained for the uniform electric field vanishes when the motion is parallel to it. A discusson of this unphysical result is exposed.

RÉSUMÉ. On démontre que l'équation de Landau-Lifshitz pour une particule chargée presente des solutions similaires à celles de l'équation de Lorentz-Dirac. En effet, la force de réaction obtenue pour un champ électrique constant quand le mouvement est parallèle a lui, disparaît. Une discussion, à propos de ce résultat nonphysique, est exposée.

1 Introduction

Since Dirac [1] obtained in 1938, the so-named Lorentz-Dirac equation (LDE) of motion for a charged point particle, many discussions about its validity have appeared. Indeed, it is one of the most controversial equations in the history of physics [2]. The third order time derivative term leads to runaway and preaccelerated solutions. Asymptotic conditions or appropriate boundary conditions are imposed to the equation in order to neglecting the non-classical results [3] leaving then the corrections to the quantum domain. Moreover, the development of quantum electrodynamics by the middle of the last century, left aside this problem. Nevertheless, during the seventh decade of the last century, Shen [4, 5] showed that there is a region over an Energy vs Field diagram where quantum effects can be neglected and a classical equation of motion is required. Indeed, this region corresponds to the order of magnitude analyzed in Plasma Physics and Astrophysics. Moreover, in this same order of ideas, this region permits us to design an experiment to know which is the equation of motion of a charged particle [6].

New proposals have appeared in the last four decades but none with an appreciable impact, except the Sphon's one[16]. As an example, Mo and Papas equation [7] has been criticized by Shen [8], and Cook series representation [9] was rejected by Peter [10] and Ares de Parga [11]. Bonnor proposed a radiating mass [12] and the idea was criticized by himself in the same paper and discarded by Ares de Parga [13] later on. The list of such examples is uncountable, but the summary is that each time a promising idea appears, there is always a counterpart and the problem remains open. The failure of an alternative equation and the formal works realized by Synge [14] and Teitelboim [15] supporting the LDE, indicate that the solution consists on an adequate interpretation of it. Recently, Sphon [16] has presented a mathematical work where he proved that the old LDE must be restricted to its critical surface yielding the Landau-Lifshitz [17] equation (LLE). Indeed, Dirac's asymptotic condition forces the solution to be on the critical manifold. So even if Landau and Lifshitz deduced their equation as the first order iteration of the LDE, it has to be considerated that the solutions to this last equation are the exact solutions to the old problems of the LDE, within the Shen region[5]. It must be noticed that Herrera [18] obtained a particular equation which coincides with the LLE for fields with $\frac{\partial F\mu\nu}{\partial x^{\sigma}} = 0$. The Herrera equation has been solved for different cases [18, 19] giving apparently physical results. In the same order of ideas Rorlhich [20] asserts, about the LLE, "The result is an equation free of unphysical solutions. The deeper mathematical meaning of this approximation can be learned from Kunze and Spohn [21]". Finally, we can conclude that nowadays the LL, within the Shen [5] region, supported by the mathematical work done by Kunze and Sphon [16, 20], represents the solution for the description of the motion of a classical charge. Unlike LDE, an important result is that LLE eliminates the runaway solutions and the preaccelerations. Preaccelarations survive even if we consider asymptotic conditions for the LDE. So the solutions for the LLE or the LDE with asymptotic conditions, are different. In this order of ideas, although we know that the physical solutions will correspond to the LLE, it will be interesting to consider the differences between both equations for critical situations. One of the critical situation, where unphysical results may appear, is for the simple case of a constant electric field. Indeed, the LDE [22, 23] reaction force vanishes when a constant electric field is applied in the same direction of the initial motion. It has to be noticed, as Parrott mentioned [23], that the LDE and other equations present the same problem. It will be expected that for the LLE, the result will be repeated. We shall discuss why this is an unphysical result and propose that the problem may not consist of considering the solution on a critical manifold of the LDE, but to analyze the classical deduction of the LDE.

2 Landau-Lifshitz equation

The Lorentz-Dirac [1] equation of motion for a charged particle is:

$$m\,\frac{du^{\mu}}{ds} = e\,F^{\mu\nu}u_{\nu} + \frac{2}{3}e^{2}\left[\frac{d^{2}u^{\mu}}{ds^{2}} - u^{\mu}u^{\nu}\frac{d^{2}u_{\nu}}{ds^{2}}\right]\,. \tag{1}$$

Here u is the four-velocity of a charged particle of mass m and charge e, s denotes its proper time, F is the field tensor for an external electromagnetic field and the velocity of light is taken as unity. Solutions of this equation for some physical situation appear physically unreasonable. Many authors have proposed modifications which might result in physically reasonable solutions, among these are an equation proposed in the classical text of Landau and Lifshitz[16, 17, 19, 21]. As we mentioned above, although Landau and Lifshitz deduced the equation by means of an iteration, for Sphon the solutions of the equation, have to be considerated as the exact physical results. The Landau-Lifshitz equation for a charged particle is:

$$m\frac{du^{\mu}}{ds} = eF^{\mu\nu}u_{\nu} + g^{\mu}_{LL}\,, \tag{2}$$

where g^{μ}_{LL} represents the Landau-Lifshitz reaction force and it is expressed by:

$$g^{\mu}_{LL} = \frac{2}{3}\frac{e^{3}}{m}\frac{\partial F^{\mu\nu}}{\partial x^{\gamma}}u_{\nu}u^{\gamma} - \frac{2}{3}\frac{e^{4}}{m^{2}}F^{\mu\gamma}F_{\nu\gamma}u^{\nu} + \frac{2}{3}\frac{e^{4}}{m^{2}}\left(F_{\nu\gamma}u^{\gamma}\right)\left(F^{\nu\alpha}u_{\alpha}\right)u^{\mu}\,. \tag{3}$$

For a constant electric field $\left(\frac{\partial F^{\mu\nu}}{\partial x^{\gamma}} = 0\right)$, the last expression reduces to Herrera [18] reaction force,

$$g^{\mu}_{LL} = g^{\mu}_{H} = \frac{2}{3}\frac{e^{4}}{m^{2}}\left(-F^{\mu\gamma}F_{\nu\gamma}u^{\nu} + \left(F_{\nu\gamma}u^{\gamma}\right)\left(F^{\nu\alpha}u_{\alpha}\right)u^{\mu}\right)\,. \tag{4}$$

Since $F^{\mu\nu}$ is antisymmetric, for any vector u^ν,

$$F^{\mu\nu} u_\nu u_\mu = F_{\nu\mu} u^\nu u^\mu = 0 \tag{5}$$

Thus the first tensor of Eq. (2), $eF^{\mu\nu}u_\nu$, is orthogonal to u. The left side of Eq. (2), $m\,\frac{du^\mu}{ds}$ is also orthogonal to u. Hence for consistency g^μ_{LL} must also be orthogonal to u. If we consider a charged particle moving in the direction of a constant electric field, ($\frac{\partial F^{\mu\nu}}{\partial x^\gamma} = 0$), for purposes of calculating the motion, Minkowskian space is effectively two dimensional. From Eq. (5), it follows that the first term of the Landau-Lifshitz reaction force, Eq. (4), namely

$$\frac{2}{3}\frac{e^4}{mc^3} F^{\mu\gamma} F_{\nu\gamma}\, u^\nu \tag{6}$$

must be in the direction of u, that is, a multiple of u. This is because of

$$\omega := F^{\alpha\beta} u_\beta$$

is orthogonal to u by Eq. (5), so that

$$F^{\mu\gamma} F_{\nu\gamma} u^\nu$$

is orthogonal to ω. In a two-dimensional space with nondegenerate inner product, as it is the case, if ω is orthogonal to u and v is orthogonal to ω, then v must be a multiple of u. Since the second term of the Landau-Lifshitz reaction force is in the direction of u, we can conclude that g^μ_{LL} is multiple of u which is also orthogonal to u. Hence it must vanish. In others words, the reaction force vanishes in this special case. As we mentioned above, the same result is obtained for the LDE and other equations. In the next section we will explain why we consider this result as an unphysical behavior.

3 Unphysical result

If a classical charged particle is accelerated, a momentum is transferred to the field, thus from momentum balance, a reaction force must act on the charged particle. Indeed, a reaction force will be needed to describe a Bremsstrahlung effect ("braking radiation") which is physically observed when charged particles are decelerated by a force in the direction of their motion (e.g. when a beam of charged particles hits a target). Even if the electric field is not constant, it can be consideratedas constant for a small time or simply to expose the beam of particles in a uniform electric field.

4 Conclusion

After more than a century that Abraham, Lorentz, Planck and later on Dirac claimed for a third order derivative equation of motion, it is time to think that drastic changes must be done to deal with the problem. Indeed, the hyperacceleration is the responsible of all this problematic. Although the mathematical work realized by Sphon is undeniable, it doesn't mean that the result is physically acceptable since the point of departure maybe wrong. Indeed, the reasoning for obtaining the LDE is founded in the use of the Maxwell stress tensor. This last one is defined from electric and magnetic fields which are meaningful by the use of an equation of motion. This equation of motion is Lorentz equation and not the LDE. So we depart from Lorentz equation of motion for a charged particle and after a mathematical process, we obtain another equation of motion for the charged particle. Something is misunderstood. In this order of idea, it is convenient to mention Galeriu's comment[24]: "The physical origin of this 4-force, which gives the accelaration energy, is not clear, and the mechanism by which a charged particle acquires rest mass from the field needs more investigation".

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