Special Relativity: Einstein's Spherical Waves versus Poincaré's Ellipsoidal Waves

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ABSTRACT. We show that the image by the Lorentz Transformation of a spherical light wavefront, emitted by a moving source, is not a spherical light wavefront but an ellipsoidal light wavefront. Poincaré's elongated ellipse is the direct geometrical representation of Poincaré's relativity of simultaneity. Einstein's circles are the direct geometrical representation of Einstein's convention of synchronisation. Poincaré's ellipse supposes another convention for the definition of space-time units involving that the Lorentz Transformation (LT) of an unit of length is **directly proportional** to the LT of an unit of time: This is Poincaré's definition of isotropic elongated distance (with dilated travel time). The historical (polemical) problem of priorities is therefore scientifically solved because Einstein's explicit kinematics and Poincaré's implicit kinematics are not the same.

It is generally admitted that the observed expansion of space (Hubble) can only be considered in the framework of General Relativity (Robertson-Walker's metric) and not in the framework on Special Relativity (Minkowski's metric). We show that Poincaré's relativistic kinematics, with the invariance of the quadratic form by LT, predicts both a dilation of time and an isotropic **expansion of space** as well.

1 Introduction: Einstein's Spherical Wavefront & Poincaré's Ellipsoidal Wavefront

Einstein writes in 1905, in the third paragraph of his famous paper:

> At the time $t = \tau = 0$, when the origin of the two coordinates (K and k) is common to the two systems, let a **spherical wave** be emitted therefrom, and be propagated

> -with the velocity c in system K. If x, y, z be a point just attained by this wave, then
>
> $$x^2 + y^2 + z^2 = c^2t^2 \tag{1}$$
>
> Transforming this equation with our equations of transformation (see Einstein's LT, 29), we obtain after a simple calculation
>
> $$\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2 \tag{2}$$
>
> The wave under consideration is therefore no less a **spherical wave** with velocity of propagation c when viewed in the moving system k. [Einstein A.1905]

Poincaré writes in 1908 in his second paper on "La dynamique de l'électron" with the subtitle "Le principe de relativité":

> Imagine an observer and a source involved together in the transposition. The wave surfaces emanating for the source will be **spheres**, having as centre the successive positions of the source. The distance of this centre from the present position of the source will be proportional to the time elapsed since the emission - that is to say, to the radius of the sphere. But for our observer, on account of the contraction, all these spheres will appear as **elongated ellipsoids**. The compensation is now exact, and this is explained by **Michelson's experiments**. [Poincaré H. (1908)]

We can further find in Poincaré's text the equation (two dimensions) of an elongated ellipse. The observer at rest (let us call him: O) is situated at the centre C and the source S in moving (together with "our observer", "Michelson", let us call him O') is situated at the focus F of the ellipse.

The contrast between both great relativists, Einstein et Poincaré, is very clear: according to Einstein, the image of a spherical wave is a *spherical* wave whilst according to Poincaré the image of a spherical wave is an *ellipsoidal* wave. Does the latter not know the invariance of the quadratic form? Not at all because he does demonstrate, with the structure of group, in his first paper on ,"La dynamique de l'électron"

[Poincaré H. 1905], that the Lorentz Transformation (**LT**) "doesn't modify the quadratic form $x^2 + y^2 + z^2 - c^2t^2$". We must point out that Poincaré's lengthened ellipse has been completely ignored for a whole century by the scientific community[1]. We note also that Poincaré doesn't use LT in the previous quotation and directly deduces the ellipsoidal shape of the light wavefront (in the system of the source !) from the principle of contraction of (the unit) of length (see conclusion).

So who is right: Einstein or Poincaré? The best thing that we can do, to solve this dilemma, is to apply a LT to a spherical wavefront.

2 Image by LT of the Object "Circular Wave"

What is the image, by LT, in the system K of a spherical wave emitted in $t' = t = 0$ by a source S at rest in the origin O' of the system K' ? The LT defined by Poincaré (K' is in uniform translation with respect to K) is:

$$x' = k(x - \varepsilon t) \qquad y' = y \qquad t' = k(t - \varepsilon x) \tag{3}$$

We keep Poincaré's notations where ε, k correspond to Einstein-Planck's notations β, γ because, according to Poincaré in his 1905 work about the theory of relativity, *"I shall choose the units of length and of time in such a way that the velocity of light is equal to unity"* [Poincaré H. 1905]. The deep meaning of Poincaré's choice of space-time units *with* $c = 1$ will be specified in §6. In order to have *one only* wavefront, we have to define a time t' as unit of time $1_{t'}$. The equation of the circular wave front in the system of the source K' (the geometrical locus of the object-points in K') in $t' = 1$ is :

$$x'^2 + y'^2 = t'^2 = 1_{t'} \tag{4}$$

The unprimed coordinates of the image-points are given by the inverse LT:

$$x = k(x' + \varepsilon t') \qquad y = y' \qquad t = k(t' + \varepsilon x') \tag{5}$$

[1] Poincaré's ellipsoidal wavefront was mentioned in his course of 1906 "Les limites de la loi de Newton" [Poincaré H. (1906)]. We also find them in "La Mécanique Nouvelle" [Poincaré H. (1909)] and in "La dynamique de l'électron" [Poincaré H. (1912)]. In fact it was in 1904, at a talk in Saint Louis, that Poincaré first introduced the elongated ellipsoidal wave as an *alternative* (non relativistic alternative, see note 12) and not as a *consequence* of the contraction of the unit of length [Poincaré H. (1904)]. Poincaré's non-relativistic 1904 ellipsoid is, very curiously, developped by Guillaume[Poincaré H. (1909)], Leroux [Le Roux J.] and Dive[Dive P.].

The coordinates $(0,0,1)$ in K of the source in $t' = 1$ are $(k\varepsilon, 0, k)$ and $(k\varepsilon t', 0, kt')$ in $t' \neq 1$. As LT is a *punctual* transformation (a transformation of *events*), let us determine the images (x, y, t) in K of different events or object-points in K' with $t' = 1$: $(1,0,1)$, $(-1,0,1), (0,1,1), (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1)$ *etc.* (see **figure 1**).

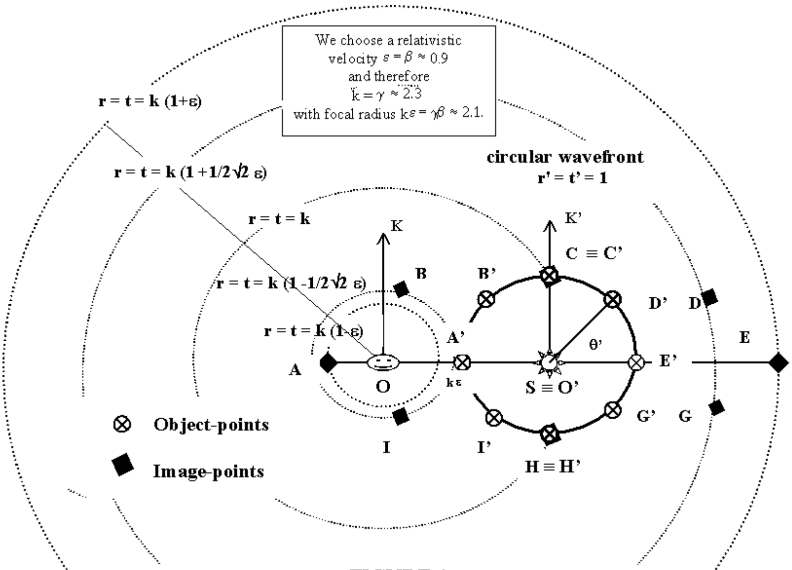

FIGURE 1
Image-Points (A, B, C) in K of Object-Points (A', B', C') of a spherical wavefront emitted by a source in K'.
The events D' and I' are simultaneous in K' (images D and I) are not simultaneous in K.

The image-point E, $k(1+\varepsilon)$, 0, $k(1+\varepsilon)$, is on the large dotted circle $x^2 + y^2 = t^2$ with radius $r = t = k(1+\varepsilon)$.

The image-point A, $k(\varepsilon - 1)$, 0, $k(1-\varepsilon)$, is on the small dotted circle $x^2 + y^2 = t^2$ with radius $r = t = k(1-\varepsilon)$.

The image-point C, $(k\varepsilon, 1, k)$, is on the dotted circle $x^2 + y^2 = t^2$ with the radius k. The image-point D, $k(\frac{\sqrt{2}}{2} + \varepsilon)$, $\frac{\sqrt{2}}{2}, k(1 + \frac{\sqrt{2}}{2}\varepsilon)$, is on the dotted circle $x^2 + y^2 = t^2$ with radius $r = t = k(1 + \frac{\sqrt{2}}{2}\varepsilon)$ *etc...*

The images in K, by LT, (5) of the simultaneous events in K', contrary to what one might expect, are not situated on one circular wavefront but, *given the invariance of the quadratic form* (the dotted circles $x^2+y^2=t^2$), in a circular ring between $k(1-\varepsilon) \leq r \leq k(1+\varepsilon)$, **figure1**).

By introducing , in the system K', the angle θ' determined by both the radius vector **r'** and the Ox' axes, we have $x' = r' \cos\theta'$ et $y' = r' \sin\theta'$. So with $r' = t' \neq 1$ we have:

$$t = kt'(1 + \varepsilon cos\theta') \quad (6)$$

which is the temporal LT (5), $t = k(t' + \varepsilon x')$, with $x' = r' cos\theta' = t' cos\theta'$. We can also write $(r = t)$ the locus of the images-points:

$$r = kr'(1 + \varepsilon cos\theta') \quad (7)$$

If $r' = t' = 1$ (**figure1,** normalization in K'), we then have:

$$t = r = k(1 + \varepsilon cos\theta') \quad (8)$$

We will show now that this "temporal equation" (6) or "radial equation"(7) is the equation of an elongated ellipse in polar coordinates if we define the polar angle θ (see **figure 2**) as the relativistic transformation of the angle θ'(§3.1).

3 Poincaré's Elongated Ellipse deduced from LT

3.1 Poincaré's ellipse (I) and the relativity of simultaneity

Let us first determine Poincaré's elongated ellipse in Cartesian coordinates. We are seeking for the shape of the wavefront $t' = 1$ in K, given that the quadratic form (4) is invariant:

$$x^2 + y^2 = t^2 \quad (9)$$

If the time t were *fixed* (see paragraph 4 on Einstein's synchronisation), we would obviously have a circular wavefront; but t depends by LT on x'. If t is written in function of x', we would not have the image of the wave in K. We must write *t in function of x*. By using the first and the third (x and t) LT (5), we have respectively if $r' = t' \neq 1$ *or* $r' = t' = 1$:

$$t = k^{-1}t' + \varepsilon x \qquad or \qquad t = k^{-1} + \varepsilon x \quad (10)$$

We immediately obtain the Cartesian equation of Poincaré's elongated ellipse respectively if $r' = t' \neq 1$ *or* $r' = t' = 1$ (**figure2**, normalization in K'):

$$x^2 + y^2 = (k^{-1}t' + \varepsilon x)^2 \qquad or \qquad x^2 + y^2 = (k^{-1} + \varepsilon x)^2 \tag{11}$$

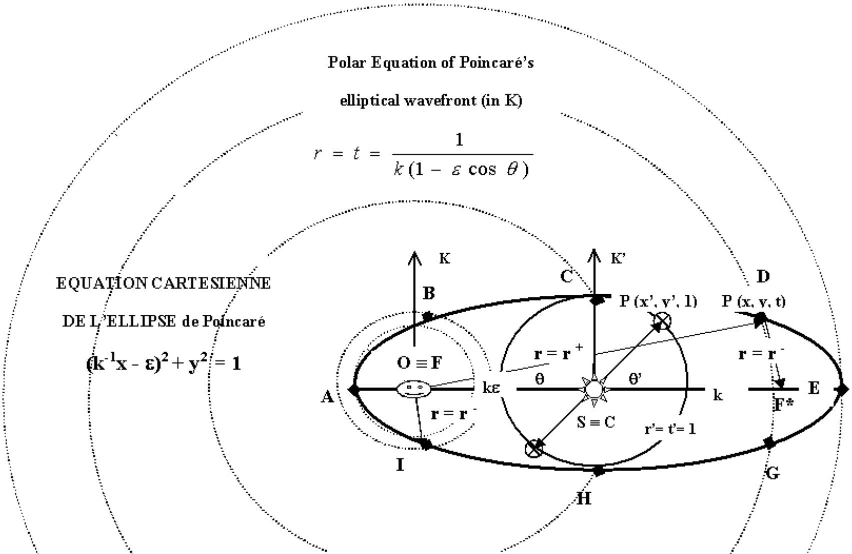

FIGURE 2
The relativity of simultanéity and Poincaré's elongated wavefront.

The circular set of simultaneous events in K' is transformed by LT into a set of non-simultaneous events in K (except for events of the same absissa)

The image in coordinates of K (x, y, **and t**) of the circle in K' is *elliptical* because t depends by LT on x. To one circular wavefront with constant radius t' in K' **does not** correspond, by LT, one circular wavefront with constant radius t in K (see paragraph 4): **a set of simultaneous events in K' is not transformed, by LT, into a set of simultaneous events in K**.

Poincaré's ellipse $t(x, y)$ can be also written $(y' = y)$ by replacing x' in function of x (5):

$$x' = k^{-1}x - \varepsilon t' \qquad or \qquad x' = k^{-1}x - \varepsilon t \tag{12}$$

in $x'^2 + y'^2 = t'^2 = 1_{t'}$ (4) , respectively if $r' = t' = t'_0$ or $r' = t' = 1$ (normalization in K'):

$$(k^{-1}x - \varepsilon t'_0)^2 + y^2 = t'^2_0 \qquad or \qquad (k^{-1}x - \varepsilon)^2 + y^2 = 1 \tag{13}$$

At once we check that (11) $\equiv$ (13). Given that LT is a punctual transformation, the front wave in K is determined by the set of the images of the simultaneous events in K': **this is an elongated ellipse with Observer O at the focus F and Source S at the centre C**[2]. To one circular wavefront in K' ***does*** correspond, by LT, one elliptical wavefront in K (**figure 2,** normalized Poincaré's ellipse).

The eccentricity of the ellipse is $\varepsilon = \frac{k\varepsilon}{k}$ where k is the length of the large axis (we choose, in figure 2, the small axis of the ellipse $r' = t' = 1$). The equation of Poincaré's *normalized* ellipse can be written in polar coordinates with pole O, focus F and the polar angle θ defined *in K* (with both standard parameters of the ellipse e, p and with the small axe of the ellipse $b = r' = 1$):

$$r = \frac{p}{1 - e\cos\theta} \tag{14}$$

with

$$p = a(1 - \varepsilon^2) = ak^{-2} = kk^{-2} = k^{-1}$$

we immediately deduce the polar equation of Poincaré's ellipse:

$$r = \frac{\sqrt{1 - \varepsilon^2}}{1 - \varepsilon\cos\theta} = \frac{1}{k(1 - \varepsilon\cos\theta)} \tag{15}$$

with eccentricity $e = \frac{f}{a} = \frac{k\varepsilon}{k} = \varepsilon$ and with the two standard parameters of the special relativity ε, k :

$$a^2 - f^2 = b^2 \qquad\qquad k^2 - \varepsilon^2 k^2 = 1$$

[2]This is **not** historical elongated Poincaré's ellipse (source at the focus) because the respective roles of the source and the observer are inverted.

If $r' = r'_0 \neq 1$, we have the equation of the ellipse:

$$r = \frac{r'_0}{k(1 - \varepsilon \cos\theta)} \tag{16}$$

with $r'^2_0(k^2 - \varepsilon^2 k^2) = r'^2_0$.

It should be reminded that the "radial equation" (7) of the ellipse is

$$r = kr'_0(1 + \varepsilon cos\theta') \tag{17}$$

Thus we obtain from (16 and 7) the **formula of relativistic transformation of angle**:

$$\cos\theta = \frac{\cos\theta' + \varepsilon}{1 + \varepsilon\cos\theta'} \tag{18}$$

So it is now utterly demonstrated that Poincaré is right and that *the geometrical image by LT of a circular wavefront is an elongated ellipse* its polar equation being (16) and its Cartesian equation being (13). Poincaré's ellipse gives the other formulae of aberration, in particular:

$$\sin\theta = \frac{\sqrt{1 - \varepsilon^2}}{1 + \varepsilon\cos\theta'}\sin\theta' \tag{19}$$

What is now the **physical interpretation** of Poincaré's elongated ellipse? We underline, at this stage, two points (for the relativistic Doppler effect, see conclusion):

1) Poincaré's elongated ellipse, is the concrete representation of the **relativity of simultaneity**: the **set** of simultaneous events in K' of the spherical wavefront in time $t' = 1_{t'}$ is *not* a **set** of simultaneous events in K, in time t.

2) Poincaré's elongated ellipse is the general representation of the "**headlight effect**" (18, 19): the isotropic emission of a moving source is not isotropic seen from the rest system (relativistic transformation of angles θ'into θ). In three dimensions of space the reduction of the angle of aperture of the cone of emission of a moving source is physically (synchrotron radiation, bremsstrahlung...) represented, *on the whole* (from any angle), by the ellipsoidal shape of the wavefront. We note here that the relativistic formulae transformation of angles is in the hart of Poincaré's implicit kinematics while these formulae are in the second part (application of his explicit kinematics) of Einstein's famous 1905 work. These two fundamental points put the emphasis on the fact that Poincaré's ellipse is not only a geometrical *image* but also a physical space-time *shape* of the wavefront.

3.2 Poincaré's ellipse (II) and Michelson's Experiment

It is now essential to interpret the historical case (see introduction and footnote 2) considered by Poincaré (in connection with Michelson's experiment where the *source is on the Earth, with "our observer Michelson*). We have indeed defined until now the case (ellipse I) where the observer O is in moving relative to the source S.

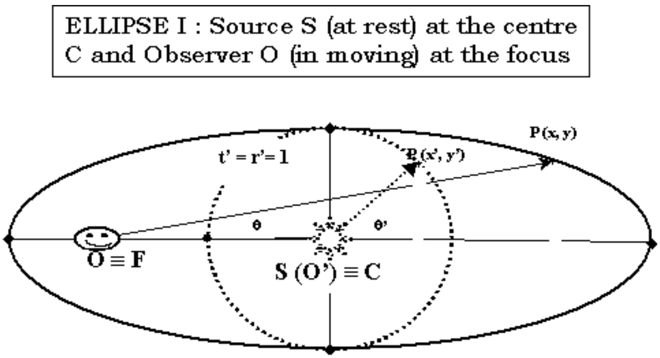

FIGURE 3
"Ellipse of observation"

Let us show now that the (normalized) Ellipse II can be immediately deduced from (normalized) ellipse I by inversion. We have respectively the radial, polar and Cartesian equation of ellipse I:

$$r = k(1{+}\varepsilon cos\theta') \qquad r = \frac{1}{k(1-\varepsilon\cos\theta)} \qquad (k^{-1}x{-}\varepsilon)^2{+}y^2 = 1_{t'} \tag{20}$$

We invert primed and unprimed and change ε in -ε and we obtain:

$$r' = k(1{-}\varepsilon\cos\theta) \qquad r' = \frac{1}{k(1+\varepsilon\cos\theta')} \qquad (k^{-1}x'{+}\varepsilon)^2{+}y'^2 = 1_t \tag{21}$$

with

$$\cos\theta = \frac{\cos\theta' + \varepsilon}{1+\varepsilon\cos\theta'} \qquad \cos\theta' = \frac{\cos\theta - \varepsilon}{1-\varepsilon\cos\theta} \tag{22}$$

(21) are the equations (respectively radial, polar and Cartesian) of Ellipse II in K'

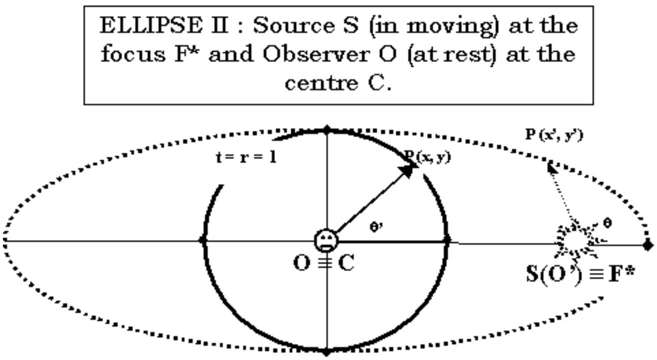

FIGURE 4
"Source Ellipse"
(Poincaré's historical ellipse)

The respective roles of the source S and the observer O are reversed. The circular light wavefront are now developed around O: $r = t = r_0 = t_0 = 1$ (normalization in K). The image of the circular locus of the points (determined now by θ) seen from O' (where the source is at rest in K', "system of the Earth") is Poincaré's ellipse II.

Poincaré's elongated ellipse is the direct translation of Poincaré's completely **relativistic ether**: *put the ether at rest in one (K) or in the other system (K')* is *mathematically* equivalent to *define the ellipse with the direct LT or the inverse LT*. This relativistic role of the ether is very clear in the Doppler effect: the two non-relativistic different situations ("source in moving" or "observer in moving" relative to the ether) are now completely equivalent[3]. In Poincaré's own words: if t' is the true time ("circular" time), t then is the local time ("elliptical" time) and inversely (by LT) if t is the true time ("circular" time), t' then is the local time ("elliptical" time). That is completely relativistic and Poincaré's elongated ellipse II is physically "the immediate interpretation of Michelson experimental result"(c=1, see §5).

CRITERION: Objectively we have two possibilities to choose the criterion of the *relativistic state of rest of a system*: **the source of light or the medium of light**. In Einstein-Minkowski's relativistic *kinematics*, the criterion is clearly the **source** (*the proper system, see paragraph 4*). In Poincaré's relativistic *kinematics*, the criterion is clearly the **ether ("circular waves")**. Poincaré's ether is relativistic[4] but *not deleted* (as Einstein's one) because it remains *the relativistic definition of state of rest*: when we choose by definition ether at rest in one system (spheres or *true time*), it is *not at rest* in the other system (ellipsoids or *local time*).

The "fine structure" of special relativity is very clear in the following quotations respectively of Einstein and Poincaré:

> The introduction of a "lumineferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space provided with special properties *nor assign a velocity-vector to a point*

[3] We show that Poincaré's relativistic Doppler formula is not the same as Einstein's relativistic Doppler formula [Pierseaux Y. (Doppler)].

[4] For example the "background" in the cosmological background radiation (CBR) is a relativistic ether in Poincaré's meaning. We can measure by Doppler effet, relative to this "background", a relative velocity (about 300km/s).

> *of the empty space in which electromagnetic processes take place.*"[Einstein A.1905]
>
> Quoi qu'il en soit, il est impossible d'échapper à cette impression que le principe de relativité est une loi générale de la Nature, qu'on ne pourra jamais par aucun moyen imaginable, mettre en évidence que des VITESSES RELATIVES, et j'entends par là non pas seulement des vitesses des corps PAR RAPPORT A L'ETHER mais les vitesses des corps les uns par rapport aux autres [Poincaré H. (1908)]

According to Einstein we cannot assign any velocity to the ether ("at rest within the two systems" or it is *deleted*) whilst according to Poincaré we can assign a RELATIVE velocity to the ether (it is at rest in ONE of the two systems, it is *relativistic*).

4 Einstein's Kinematics: identical Spheres, identical rigid Rods and Convention of Synchronisation

If according to Einstein, the object (1) and the image (2), are both spherical and *concentric* within the two systems, then two simultaneous events in K, for example (1, 0, 1) and (-1, 0, 1), must be also simultaneous in k.

That seems in contradiction not only with Poincaré's ellipse but also with Einstein's well known definition of relativity of simultaneity. Therefore **the image by LT** in k of a spherical wave in K **cannot be a spherical wave** (see paragraph 3). So it could appear at this stage that *Poincaré is right* and *Einstein is wrong.*

However the question is: "What in Einstein's reasoning is true?". Let us return to Einstein's 1905 quotation (paragraph 1). The two quadratic forms $x^2 + y^2 + z^2 = c^2t^2$ and $\xi^2 + \eta^2 + \zeta^2 = c^2\tau^2$ are the *geometrical* equations of two spheres (two circles in two dimensions or two equidistant points respectively from O and O' in one dimension) *if and only if τ is fixed.* Let us note that Einstein doesn't specify, in the previous quotation, in *which system* the source is at rest. So if we consider now that we have *two identical sources,* in O' and O, emitting a signal of light *simultaneously* at the time,

$$\tau = t = 0 \tag{23}$$

the physical situation is perfectly identical in each system (Einstein's deletion of ether[5]). So we must have two identical spherical wavefronts, around O and around O', simultaneously at the time:

$$\tau = t = 1_t = 1_\tau = T_0 \tag{24}$$

It immediately follows from the latter choice of two *identical* units of time that we have two identical units of length $L_0 = 1_x = 1_\xi = c1_t = c1_\tau$.

$$\tau = t = \frac{L_0}{c} = T_0 \tag{25}$$

"Einstein's definition of identical units" of time or "Einstein's double normalization" (in K and in K') is therefore completely coherent with Einstein's **identical rigid rods** *(*but incompatible with Poincaré's definition of units, see §6). Einstein writes in this sense in 1905:

> Let there be given a stationary rigid rod; and let its length be L_0 as measured by a measuring-rod which is also stationary. In accordance with the principle of relativity the "length L_0 of the rod in the moving system" - **must be equal** to the length L_0 of the stationary rod . [Einstein A.1905]

In this respect, M. Born is perhaps the only physicist (see also [Weisskopf V. (1)]) who underlined that Einstein introduces in fact a tacit assumption (1921):

> A fixed rod that is at rest in the system K and is of length 1 cm, will, of course, also have the length 1 cm, when it is at rest in the system k. We may call this **tacit assumption** of Einstein's theory the *principle of the physical identity of the units of measure.* [Born M.]

Let us point out that the travel time of the circular wavefront either towards the right or towards the left (on the x, ξ axis) are identical within the two systems. **Einstein's rigid rod (2L_0) is defined by two simultaneous events within each system (K and k)**. Einstein's two spherical waves are not in contradiction with "Einstein's relativity of simultaneity (after LT)" because it is "Einstein's convention of

[5] Einstein's deletion of ether is completely inseparable of Einstein's photon (1905) [Pierseaux Y. (Doppler)].

simultaneity (before LT)" or in other words "Einstein's **convention of synchronisation** *of identical clocks in A and B* with the exchange of a signal of light in K (before LT)". Let us now demonstrate that point by rigorously distinguishing the two stages of Einstein's deduction: before LT and after LT.

4.1 Before LT (the proper Systems)

According to Einstein, "It is essential to have time defined by means of *stationary* clocks in *stationary* system". Einstein's famous repetition of the concept "stationary" is essential because he notices about his second system k (ξ, η, ζ, τ):

> To do this [deduce LT] we have to express in equations that τ is nothing else than the set of data of clocks at rest in system k , which have been synchronized according to the rule given in paragraph 1 [for the system K] [Einstein A.1905].

It is clear that Einstein's convention synchronisation does not take into account the relative velocity v between K and k[6].

We have for *stationary* clocks in *stationary* system K with a source at rest in K(paragraph 1):

$$t_B - t_A = t_A^* - t_B \tag{26}$$

The forth travel time is equal to the back travel time in K (in Poincaré's notations $t^+ = t^-$, §6).

We have for *stationary* clocks in *stationary* system k with a source at rest in k:

$$\tau_{B'} - \tau_{A'} = \tau_{A'}^* - \tau_{B'} \tag{27}$$

with $A'B' = AB$. The forth travel time is equal to the back travel time in K (in Poincaré's notations $t'^+ = t'^-$, §6).

Einstein's interpretation of the invariant quadratic form as **two physical spherical wavefronts** (1 & 2) is therefore exactly the same concept as Einstein's 1905 **convention of synchronisation**

[6]The ether is at rest (or stationary) within the two systems and it is therefore superfluous. The deletion of the medium of light involves that the relativistic state of rest is defined with respect to the source of light.

within the two systems. This is Einstein's synchronisation with spherical wavefronts and *without contraction* (For Poincaré's convention of synchronisation with ellipsoidal wave and *with contraction* see [Pierseaux Y. (identity)] and [Reignier J.]).

Einstein defines first the simultaneity of two events at the same place ($A = B$). Secondly he defines the simultaneity of two events at different places ($A \neq B$) [Einstein A.1905, paragraph 1]. But the departure (from A), the arrival (in B) and the return (in A) of the light signal are 3 *successive* events. What are finally Einstein's two simultaneous events in A and B? These two events are in k: $(\xi_A, \frac{1}{2}T_0)$ and $(\xi_B, \frac{1}{2}T_0)$. These are the two ends of the rigid rod at the same time. So we have Einstein's *relativistic* definition of rigidity:

EINSTEIN'S DEFINITION of the **proper length,**

$$L_0 = \xi_B - \xi_A \tag{28}$$

is defined **by two events at the same time** τ

$$\tau = \frac{1}{2}T_0 \tag{28bis}$$

in the proper system k

4.2 After LT (the improper Systems)

Einstein's construction of invariant quadratic form as physical spherical wavefront means that Einstein (and Minkowski) defines the units before taking in account the relative velocity v : Einstein's units are completely *independent*[7] on the relative velocity v (and also β and γ). Therefore there is no contradiction with LT, because the definition of space-time units is a preparation of the two systems not only prior to the use of LT and even, more fundamentally, prior to the *deduction of LT* (see note

[7]Einstein's spheres are not necessarily identical. Fock V. underlined about the scale factor: " *We have* $c^2\tau^2 - \xi^2 - \eta^2 - \zeta^2 = \varphi^2(x, y, z, t)\ (c^2t^2 - x^2 - y^2 - z^2\ )$. *The factor* φ^2, *or rather* φ, *evidently characterises the ratio of the scales of measurement in the primed and unprimed frames. Further, it follows that this factor cannot depend on the relative velocity. It is usually said, following Einstein, that the scale factor can "evidently" depend on nothing but the relative velocity, and it is subsequently proved that, in fact, it does not have any dependence but is equal to 1:* $\varphi(x, y, z, t) = 1$." [Pierseaux Y. (identity), principe d'identit]. In Poincaré's relativistic kinematics the dependance of the scale factor on the velocity, $l(\varepsilon)$, is paramount. The group property of LT proves that the velocity is a relative velocity.

4). Einstein's 1905 deduction of LT is very complicated but Einstein's immediate 1907 deduction of LT from the invariance of Einstein's two spheres is now a classic [Einstein A.1907]:

$$\xi = \gamma(x - vt) \qquad \eta = y \qquad \tau = \gamma(t - \frac{v}{c^2}x) \tag{29}$$

If Einstein's definition of space-time units is not in contradiction with LT, it requires on the other hand a specific use of LT. In Einstein's own words there is two operations *"operation (a)"* that consists of defining identical units and *"operation (b)"* that consists of discovering *"the length L of the (moving) rod in the stationary system" [Einstein A.1905, paragraph2].* In current words, the length to be discover by LT is the improper length L, and also the improper time T, respectively relatively to the proper length L_0 or the proper time T_0. The role of the LT consists fundamentally of introducing the velocity v or defining the *improper moving* system (k relative to proper K or inversely). In all standard books we can find Einstein's deduction, with the use of LT, of the dilation of proper time $T = \gamma T_0$ and the **contraction** of proper length $\gamma^{-1}L_0$. Therefore the *improper time and the improper length (in the moving system) are **inversely*** proportional. Let us examine Einstein's use of LT (the standard deduction) in details.

4.2.1 Dilation of proper time

The proper time, $T_0 = \tau_2 - \tau_1$, is the duration between this two events *at the same place* ($\xi_1 = \xi_2 = \xi$) in k. We find the duration T in K by the *second LT:*

$$t_1 = \gamma(\tau_1 - \frac{v}{c^2}\xi) \qquad t_2 = \gamma(\tau_2 - \frac{v}{c^2}\xi)$$

The duration, $T = t_2 - t_1$, in the moving system K is

$$T = \gamma T_0 \tag{30}$$

With the first LT we remark that the two considered events are not at the same place in K:

$$x_1 = \gamma(\xi - v\tau_1) \qquad x_2 = \gamma(\xi - v\tau_2) \tag{31}$$

This is a very well known result: Einstein's dilation is the consequence of the fact that we must use *two clocks in different places*

$$\Delta x = \gamma v T_0 \tag{31bis}$$

of the moving system.

4.2.2 Contraction of proper length

According to Einstein (as in all standard books on SR), the proper length $L_0 = \xi_2 - \xi_1$ is the length at rest in k (ξ_2, ξ_1 are the coordinates of the ends of the rod in k). The length of the moving rod is then defined as the distance between the two ends of the rod *at the same time* ($t = t_1 = t_2$) in K. We immediately find this length, $L = x_2 - x_1$, by the inverse first LT:

$$\xi_1 = \gamma(x_1 + vt) \qquad\qquad \xi_2 = \gamma(x_2 + vt)$$

and therefore we obtain Einstein's famous contraction:

$$L = \gamma^{-1} L_0 \tag{32}$$

Both Einstein's deductions, dilation of time and contraction of length, are presented in all standards books as perfectly symmetric: two *events* at the same place (in k) for the dilation of duration and two *events* at the same time (in K) for the contraction of length. Nevertheless: what are the complete coordinates of the two events (ends of the rods) in the proper system k? In order to have the complete symmetry, we must consider the other LT not only in the case of dilation of duration (31) but also in the case of contraction of length. The second LT is:

$$\tau_1 = \gamma(t + \frac{v}{c^2} x_1) \qquad\qquad \tau_2 = \gamma(t + \frac{v}{c^2} x_2) \tag{33}$$

This is a completely ignored result: The second LT (33) determines obviously the times, τ_1 *and* τ_2, of the ends of the rods $\xi_1$and ξ_2 in the proper system k and thus the complete coordinates of the two events (ξ_1, τ_1) and (ξ_2, τ_2): the simultaneous events in k are, obviously **by LT**, not simultaneous events in k, the proper system. So by symmetry with 31bis, we must have:

$$\Delta\tau = \gamma \frac{v}{c^2} L_0 \tag{33bis}$$

This is in contradiction with Einstein's definition of identical RIGID rods (see above **DEFINITION 28-28bis**) that implies that the proper length must be defined by simultaneous events in the proper system (the ends of the rigid rods are defined at the same time τ). So **Einstein's contraction** is **not** deduced directly from LT but from only one LT with *a supplementary hypotheses*:

$$(\Delta\tau)_{\mathrm{Ei}\,nstein} = 0 \tag{33ter}$$

The proper length L_0 is defined by two simultaneous events $(\xi_{1,}\ \tau)$ and $(\xi_2,\ \tau)$ in k **and** Einstein's improper length is also defined by two simultaneous events $(x_{1,}\ t)$ and $(x_2,\ t)$ in K. But these events (33ter) are *not* the images by LT one another.

We will show now that Einstein's inverse (γ^{-1}) contraction (32) or "Einstein's breaking of symmetry" is in opposition with Poincaré's direct proportionality of the transformation of time and length.

5 Cosmological definition of the distance in Poincaré's Relativistic Kinematics.

Poincaré writes in 1911 in "L'espace et le temps" on the special theory of relativity:

> Today some physicists want to adopt a new convention. This is not that they have to do it; they consider that this convention is easier, that's all; and those who have another opinion may legitimately keep the old assumption in order not to disturb their old habits.[Poincaré H. (1911)]

"Some physicists" is a clear allusion to Einstein's convention of synchronisation or Einstein-Minkowski's definition of space-time units. What is the difference, according to Poincaré, between the "old convention" and the "new convention"? We note that both conventions are relativistic because we have the invariance of the velocity of light (or the quadratic form) in both cases. Indeed we have with the ellipse we have by construction:

$$\frac{r_0}{t_0} = \frac{r'}{t'} = c = 1 \tag{34}$$

The essential difference between Einstein's kinematics and Poincaré's kinematics is situated in the definition of the DISTANCE . At the end of the deduction of his elongated ellipse Poincaré writes (in italics in the text):

> This hypothesis of Lorentz and FitzGerald will appear most extraordinary at first sight. All that can be said in its favour for the moment is that it is merely the immediate interpretation of Michelson experimental result, **if we *define* distances by the time taken by light to traverse them**.[Poincaré H. (1908)]

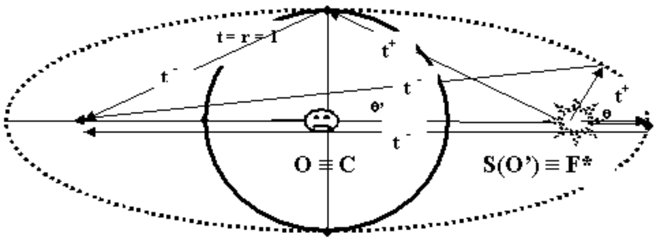

FIGURE 5
Poincaré's isotropic (c = 1) ellipse
and Michelson's experiment

So with Poincaré's ellipse II (the ellipse I inverted, see paragraph 2, **figure 2 and 4**) we see that the time of the *round trip* t_M (35) of the light is the same in all directions (and therefore for the two Michelson's perpendicular directions). So Poincaré's historical ellipse is the immediate interpretation of Michelson's null result[8] without postulating that the source in the system of the Earth emits spherical waves (new convention).

According to Poincaré we can indeed deduce, from the *geometrical* properties of an ellipse, two expressions with the forth travel time t^+ and the back travel time t^- (respectively the *sum* t_M, from the foci, and the *difference* Δt, from the directrices, [Poincaré H. (1906)] & [Poincaré H. (1912)]). First Poincaré's expression is:

$$t_M = \frac{t^+ + t^-}{2} = kt' \qquad \text{and by (34)} \qquad r_M = \frac{r^+ + r^-}{2} = kr' \qquad (35)$$

So the (isotropic) dilation of time is directly proportional to the (isotropic[9]) dilation of space. We note that this definition of distance $r_M = \frac{r^+ + r^-}{2} = kr'$ is physically very concrete because it is only based on the mean travel time of the light (forth and back). Second Poincaré's expression is:

$$\Delta t = \frac{t^+ - t^-}{2} = k\varepsilon x' \qquad (36)$$

So the difference between the forth travel time t^+ and back travel time t^- is not by definition "zero" (new convention[10]) as in Einstein's synchronisation (26, 27) or in Einstein's contraction (33ter). The subtle interpretation of this second expression (35), that depends only on the difference of abscissas, will be developed in another paper (with Poincaré's interpretation of the contraction).

[8]And also an immediate explanation of Sagnac *non null* result (1913). The main problem of rotating platform with Einstein's kinematics is precisely Einstein's invariance of one way speed of light, $t^+ = t^-$ or $\Delta t = 0$ in the proper system[Selleri F.]. This is the reason why Langevin solves the problem in the framework of General Relativity. In Poincaré's relativistic kinematics we can have in the system of the source $t^+ \neq t^-$ (see figure 5). With Poincaré's elongated ellipse [Pierseaux Y. (Ph.D, ULB)] and Poincaré's group with rotations [Reignier J.], we predict immediately (at the second order k) the experimentally measured difference of time $\frac{t^+ - t^-}{2} = k\varepsilon L$ ($L = 2\pi R$, R being the radius of the platform).

[9]This is not the case of Poincaré's 1904 ellipsoide (see note 1) that is an *non-relativistic* alternative to Lorentz contraction (the velocity of light depends on the direction).

[10]Let us note that Einstein's new convention with spherical waves (1905) is historically formulated *before* Poincaré's old convention with ellipsoidal wave (1908): l'ancien et le nouveau sont donc "relatifs" (en français dans le texte).

We will now show that Poincaré's elliptical deduction of *dilated space-time (35)*, is based, and *only* based, on the application of LT (the "old convention"). He doesn't need like Einstein a supplementary hypotheses (see §4). In order to delete Einstein's space-time asymmetry in the use of LT, we have to define rigorously the events firstly for the time and secondly for the space.

5.1 Dilation of proper time

For the time we consider two isotopes (at the same place) but non-simultaneous events $(0,0)$ et $(0,T_0)$

The immediate calculus with both LT gives $(0,0)$ et $(\gamma\beta T_0, \gamma T_0)$. So we have:

$$T = \gamma T_0 \tag{37}$$

POINCARE'S **DEFINITION 1 :** *the improper duration in K' is the dilated duration between two non-isotopes (not at the same place) events (in harmony with Einstein's dilation, 30).*

5.2 Dilation of proper length

For the space, we consider two simultaneous but non-isotopes events (not at the same place) $(0,0)$ et $(L_0,0)$

The immediate calculus with both LT gives $(0,0)$ et $(\gamma L_0, \gamma\beta L_0)$. So we have:

$$L = \gamma L_0 \tag{38}$$

POINCARE'S **DEFINITION 2:** *the improper distance in K' is the dilated distance between two non-simultaneous events.*

DEFINITION 2 is completely symmetric with DEFINITION 1. Poincaré's definition of the distance induced by elongated ellipse (old convention, see 35) is not compatible with Einstein's contraction (new convention, 32) but it is in perfect harmony with LT (both LT). Let us note that it is the distance in the well known meaning of the cosmologists. Lachièze-Rey writes[Lachièze-Rey]:

> Une difficulté fondamentale imprègne la cosmologie et il est capital de la garder à l'esprit. Toute l'information sur telle ou telle partie de l'univers nous parvient par l'intermédiaire du rayonnement électromagnétique qui se propage à une

> vitesse finie, celle de la lumière c. Aucun des objets que nous observons ne nous est donc contemporain mais se révèle tel qu'il était à une époque vieille de L/c (...) L'ensemble des objets qui nous sont accessibles par l'observation n'est donc pas situé dans l'espace pris à un instant donné. Au contraire, on peut dire que chacun se situe dans une tranche d'espace-temps *dont l'éloignement dans le temps est proportionnel à l'eloignement dans l'espace*; la relation entre les intervalles de temps T et d'espace L nous séparant de lui est donnée par la loi de propagation de la lumière c=L/T.

Lachièze-Rey focuses the attention on the cosmological definition of distance:(p72-73)

> Une difficulté supplémentaire provient d'ailleurs du fait que L n'est pas une *VRAIE* distance puisque c'est un intervalle de longueur entre la positionoccupée par nous 'l'observateur) aujourd'hui et la galaxie observée à un instant différent.
>
> Rappelons-nous néanmoins que la distance ainsi considérée n'est pas une distance au sens propre du mot puisqu'elle mesure un intervalle spatial entre deux événements- l'émission et la réception d'un signal lumineux- non contemporains.

About the first and the second note, we point out that Poincaré's relativistic definition of the distance (35-38) is clearly the distance of the cosmologists that becomes a true relativistic distance by LT: It is an improper elongated distance (38) in Poincaré's meaning (between two non-simultaneous events). We focus the attention on the fact that Einstein's contraction (32) is incompatible with Poincaré's definition of expansion of space (35). We will show in a next paper that only Poincaré's interpretation of longitudinal contraction (*"The ellipse is elongated* ***because*** *the units of length are contracted")* is compatible with **both** LT.

6 Conclusion: Poincaré's expansion of space

Poincaré's relativistic kinematics is based on a fundamental isotropic *space-time proportionality* (a dilation by a factor k) in perfect harmony with the invariance of the speed of light:

$$\frac{r_M}{t_M} = \frac{kr'}{kt'} = c = 1 \tag{39}$$

Poincaré's **space-time symmetry (35 &37-38)** (very strange in Einstein's kinematics **30 & 32**) characterizes fundamental Poincaré's choice of space-time units in relativistic kinematics: "*I shall choose the units of length and of time in such a way that the velocity of light is equal to unity*". This is the reason why we kept Poincaré's notations ε (β) and k (γ): behind Poincaré's notations, there is not only Poincaré's perfectly space-time symmetrical representation of LT (5) but also Poincaré's relativistic convention about the **metric** (in the sense of space-time units of measure) underlying the invariance of quadratic form in SR. We have $c = 1$ in the second (moving) system *if we define distances, as the cosmologists, by the time taken by light to traverse them.*

The existence of a "fine structure" of SR (two very close but not merged theories) is therefore rigorously demonstrated [Pierseaux Y. (Ph.D, ULB)]. Given that the velocity is isotropic(the same in all directions both in Einstein's two systems and Poincaré's two systems), there is two different ways of defining the metrics of space-time: Einstein's theory of distance (**32**) and Poincaré's theory of distance **(35 or 37-38).**

According to Minkowski (1908), Einstein's SR was not a local theory but a theory of "the world" or " the Universe" (Minkowski's "worldline", "worldpoint", "worldinterval" and even "world principle", which is Minkowski's name for the principle of special relativity [Minkowski H.]).

The problem is that Minkowski's metric is incompatible with an expansion of the Universe (Hubble 1929). It is then generally admitted that the observed cosmological expansion of space (Hubble) can only be considered in the framework of General Relativity (Robertson-Walker's metric) and not in the framework on Special Relativity (Minkowski's metric). Finally we point out that *Poincaré's metric* involves not only a dilation of time but also an **expansion** of space. We will show in another paper that Poincaré's *completely relativistic* expansion (with cosmological definition of distance) is directly connected with the deduction of the relativistic Doppler formulae (for remote objects) from Poincaré's successive elongated ellipsoidal wavefronts.

7 Annex: Penrose's and Poincaré's elongated ellipsoid (invisibility versus visibility of Lorentz contraction)

The question under discussion is directly connected to another question: Penrose-Terrel's analysis on "The Apparent Shape of a Relativistic Moving Sphere" (1959) or "The Invisibility of Lorentz Contraction" (1959) in Einstein's SR. If we search the apparent shape for *one* observer of a moving material sphere, according to Penrose [Penrose R.], we have to send a signal of light that is reflected on the surface of the sphere and that finally returns to the observer. Penrose shows that we have to take into account Einstein's 1905 relativistic formulae of aberration and Doppler effect. Terrel writes thus:

> The factor M is the magnification, the ratio between subtended angles as seen by the observers O' and O, or the ratio of apparent distances of the objects from the two observers. It is interesting that M is precisely the Doppler shift factor becoming $\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$ for $\theta = 0 = \theta'$. [Terrel J.]

In one dimension we avoid the question of aberration (for $\theta = 0 = \theta'$), which is the main problem of Penrose-Terrel and not under discussion in the present paper. If we try to measure a moving contracted rod $L = \gamma^{-1} L_0$ with the mean time travel of the signal of light (forth $+$ and back $-$travel), Lampa [Lampa A.], before Penrose and Terrel, shows that, the longitudinal Doppler effect is respectively $\nu^+ = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$ and $\nu^- = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$. We have the mean travel time[11] $t^+ = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} + t^- = \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} = \gamma T$. The mean "apparent distance" from O is, according to Lampa, $L_{app} = \gamma L$. Penrose explains how his elongated ellipsoid disappears:

> The length of the image of the sphere in the direction of motion is thus greater than might otherwise be expected so that if it were not for the flattening the sphere would appear to be elongated. [Penrose R.]

And also Rindler:

[11] In the deduction of Poincaré's ellipse we have immediately by LT: $t^+ = k(1+\varepsilon)$ and $t^- = k(1-\varepsilon)$.

> This shows that a moving sphere presents a circular outline to all observers in spite of length contraction (or rather: because of length contraction; for without length contraction the outline would be distorted). [Rindler W.]

So according to Lampa-Penrose-Terrel the image of the rigid rod is, by compensation with Einstein's contraction (32), a rigid rod (with aberration we have a rotation). This enigmatic compensation, $L_{app} = \gamma\gamma^{-1}L_0 = L_0$, might be true (the image of a sphere, not by LT but "by Doppler and aberration", would be a sphere but only for "sufficiently small subtended solid angle" [Terrel J., Weisskopf V. (2)]). However it is clear that Einstein's convention is different to Poincaré's one: in Poincaré's SR the elongated *light* ellipsoid *appears* because of the contraction of unit of length (visibility of Lorentz contraction in "expansion of space" connected with Doppler effect, see conclusion) whilst in Einstein-Minkowski-Penrose's SR the elongated *material* ellipsoid *disappears* because of Einstein's contraction (invisibility of Lorentz contraction). Let us remark that in this scientific tradition (the relativistic shape of a sphere of matter), which begins in 1924 with Lampa, nobody made the slightest reference to Poincaré's 1906 elongated ellipse (the relativistic shape of a sphere of light).

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