The Quantum Negative Energy Problem Revisited

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1 Introduction

The aim of this note is to revisit the problem of the negative energy levels in the early Dirac electron theory. It is a problem that was not resolved satisfactorily in the context of Dirac's relativistic quantum theory, or in its extension to quantum electrodynamics, according to Dirac's own view. It will be my contention that this problem is automatically resolved in any continuous field theory of microscopic matter, as in the author's demonstration of quantum mechanics as a linear, asymptotic limit of a continuous, nonlinear field theory of the inertia of matter, rooted in the theory of general relativity, [1].

2 Review of the Quantum Electron Problem

In Dirac's generalization of Schrodinger's nonrelativistic wave mechanics, to a spinor form in special relativity, he was left with the problem of the existence of negative energy levels E_-, accompanying the positive energy levels E_+ of the electron. It was a problem not encountered in classical field theory or in nonrelativistic wave mechanics. According to Dirac [2],

"In the quantum theory, since in general a perturbation will cause transitions from states with E positive to states with E negative, such transitions would appear experimentally as the electron (whose charge) suddenly changes from –e to +e, a phenomenon which has not been observed".

In time, it was recognized that +e (the positron) in the Dirac theory, is an elementary particle that is independent of the elementary particle –e (electron). But the problem remained that in the context of the quantum theory, the electron with positive energy, $E_+ = +c[m^2c^2 + p^2]^{1/2}$ must, with a definite non-zero probability, drop to a negative energy state with energy $E_- = -c[m^2c^2 + p^2]^{1/2}$. The minimum gap between the positive and the negative energies of the electron, $(E_+ - E_-)_{min} = 2mc^2$, corresponding to zero electron momentum, p = 0, in each domain. In dropping to lower energy states, the electron would then continually lose energy until reaching the state at negative infinity - implying that matter could not be stable!

Dirac then postulated that the problem would be overcome by assuming at the outset that a separate electron already occupies each of the negative energy levels. Then, according to the *Pauli exclusion principle*, a positive energy electron could not drop into any of the negative energy states since other electrons occupy them all.

This model then left open the possibility that a negative energy electron near the top of this domain (i.e. near $E = -mc^2$) could be excited to an unoccupied positive energy state, thereby leaving a 'hole' in the negative energy state. Other negative energy electrons at lower energy levels could then fill the latter. An external electric field would then reveal what is equivalent to 'hole conduction' – the conduction of positive electrons in this domain.

Thus, Dirac's resolution of the negative energy problem required that whenever one postulates the existence of single positive energy electron, it must be accompanied by an infinite number of negative energy electrons. The situation was not satisfactory to Dirac as a permanent resolution to the problem!

Of course, one may postulate *ad hoc* that the minimum energy of the electron is $+mc^2$, and that the ground state (zero) energy would be the vacuum state. But this could not be done in the context of the Dirac theory, because it would reduce the completeness of the latter in its applications to physical problems. For example, the derivation of the (empirically correct) Klein-Nishina formula for Compton (electron-photon) scattering [3], requires the complete set of negative and positive energy states.

Dirac next considered the extension of quantum mechanics to quantum electrodynamics to resolve the problem. In the latter theory, the 'perfect vacuum state' is *postulated* to be at zero energy, where there are no electrons, positrons or photons. But he showed that this was also not acceptable because the 'perfect vacuum' is not stationary[4]. At the end of Dirac's book, he says [5]:

"It would seem that we have followed as far as possible the path of logical development of the idea of quantum mechanics as they are at present understood. The difficulties being of a profound character can be removed only by some drastic change in the foundations of the theory, probably as drastic as the passage from Bohr's orbit theory to the present quantum mechanics".

3 A Resolution – Back to a Continuous Field Theory

It has been my contention that the "change in the foundations of the theory" that Dirac alludes to would be a returning to the continuous field theory of matter, rooted in the theory of general relativity [1]. For in this view, the energy of a "particle of matter" (which, in the continuous field theory is a mode of a single continuum) is defined according to the Lagrangian formalism, according to *Noether's theorem*, as:[1]

$$E = \int \Sigma_{i=1}^{n} [\partial L/\partial(\partial_0 \Lambda_\alpha^{(i)})]\partial_0 \Lambda_\alpha^{(i)} - L)d^3x \qquad (1)$$

where L is the Lagrangian density for the continuum material system, ∂_0 is the time derivative and $\Lambda_\alpha^{(i)}$ are the α components of the n-dependent fields of the system of matter. The latter, in the Dirac electron theory, would be his bispinor solutions ψ, their hermitian conjugate solutions multiplied by the Dirac matrix, $\psi^+\gamma^0$, and their respective derivatives. The dependent fields, Λ_α, are generally all of the solutions of the field laws of nature. One arrives at the definition (1) for the conserved energy from the assumption that the corresponding Lagrangian density L is invariant with respect to arbitrary, continuous variations of the time parameter δt.

Thus, *energy is defined in the field theory in terms of continuous change only*. Since one value of energy E projects to another *continuously connected* value of energy, $E + \delta E$, for, say, an electron matter field ψ, one may not proceed from a positive value energy to a negative value energy as this would be a discontinuous change. This is the reason, as Dirac asserts[2], that one may automatically reject the negative energy values in a classical field theory.

At this juncture in the history of quantum mechanics, in 1928, the route taken was to quantum electrodynamics. But, as Dirac explained, this did not work! In addition to the problem that the vacuum state in quantum electrodynamics (*qed)* is not stationary, another of the critical reasons for its failure was that extension to *qed* automatically generated infinities in the formalism, thereby destroying the self-consistency of the theory. It was for this reason that Dirac referred to *qed* as an "ugly theory" [6]. I have previously also discussed some of these difficulties with *qed* [7].

I have demonstrated this route in resolving the problem of quantum mechanics in my research program[1]. In this view, quantum mechanics is a linear (asymptotic) approximation, at low energy, for a generally nonlinear field theory of the inertia of matter in general relativity. These results indeed confirm Dirac's assertion [8] :

"Some day, a new quantum mechanics, a relativistic one, will be discovered, in which we will not have these infinities occurring at all. It might very well be that the new quantum mechanics will have determinism in the way that Einstein wanted … I think it is quite likely, or at any rate quite possible, that in the long run Einstein will turn out to be correct."

References

[1] The development of this theory is given, most recently, in M. Sachs: *Quantum Mechanics and Gravity* (Springer, 2004). Earlier books that derive other explicit details of the theory are in M. Sachs: *General Relativity and Matter* (Reidel, 1982) and M. Sachs: *Quantum Mechanics from General Relativity* (Reidel, 1986).

[2] P. A. M. Dirac: "The Quantum Theory of the Electron", *Proceedings of the Royal Society* (London) **A118**, 351 (1928).

[3] T. J. Bjorken and S. D. Drell: *Relativistic Quantum Mechanics* (McGraw-Hill, 1964), p. 134.

[4] P. A. M. Dirac: *The Principles of Quantum Mechanics*, 4th edition (Oxford, 1958), p. 306.

[5] Dirac, *ibid*.

[6] H. S. Kragh: *Dirac: A Scientific Biography* (Cambridge, 1990), Chapter 8.

[7] M. Sachs: "Is Quantization Really Necessary?", *British Journal for the Philosophy of Science* **21**, 359 (1970).

[8] P. A. M. Dirac: "The Early Years of Relativity", in G. Holton and Y. Elkana (eds.) *Albert Einstein: Historical and Cultural Perspectives* (Princeton, 1982), p. 79.

Note added in Proof:

Later research that addresses Dirac's problem of a non-stationary vacuum is on the theory of field operator algebra, as discussed in G. G. Emch,

Algebraic Methods in Statistical Mechanics and Quantum Field Theory (Wiley Interscience, 1972), and O. Bratelli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics*, Vol I, II (Springer, 1979).

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