Quantum origin of transverse electromagnetic fields

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ABSTRACT. In the potential description of transverse electromagnetic fields in order to reduce degrees of freedom from four to two the constraint like Coulomb gauge, or radiation gauge, is used. These conditions are Lorentz non-covariant and serve as a source of several well-known difficulties in the theory of electromagnetic fields. In this paper we suggest Lorentz-covariant double-potential representation for transverse electromagnetic fields. In this approach, **E** and **B** are expressed by the pair of Lorentz-scalar functions instead of four-vectors. Such representation manifests quantum origin of the transverse electromagnetic fields because the pair of potentials obey Klein-Gordon equation. Within the framework of this theory the four-vector potential and the Lorentz-gauge equation can be interpreted as a current density and an continuity equation, respectively.

1 Introduction

The Maxwell's equations in the vacuum are equations for the electric field strength **E** and the magnetic flux density **B**. The four-potentials Φ, A_x, A_y, A_z were introduced into the Maxwell's electrodynamics in order to simplify the system of wave equations. In the sequel, largely thanks to the quantum mechanics essential conceptual role of the potentials for the theory has been manifested. Indeed, in the Schrödinger, Dirac and Klein-Gordon equations the field's strengths do not longer appear, but the four potentials instead. External e.m.[1] fields are introduced into quantum mechanics via four-potentials already at the level

[1] The abbreviation "e.m." we use instead of "electromagnetic"

of Hamilton-Jacobi equations. The principle of gauge invariance was born thanks to four-potential representation of e.m. fields. A form of the potential representation has a great significance in the quantum field theory, in the theory of superconductivity, in the theory of magnetic charge [1]. The puzzling role playing by the vector potential in the quantum mechanical motion of particles was strikingly illustrated by the Aharonov-Bohm effect [2].

In search of faithful potential representation for the electromagnetic fields several interesting ideas had been suggested (see, for instance, references [3], [4], [5], [6], [7], [8], [9] and references therein.)

Electromagnetic fields are characterized by two invariants $I_1 = (\mathbf{E}\cdot\mathbf{B})$ and $I_2 = \mathbf{E}^2 - \mathbf{B}^2$. Among them the field defined as a *pure radiation field*, or *null field*, for which the two invariants are identically zero has peculiar status. This field is propagating field with the well-known properties $\mathbf{E}\perp\mathbf{B}\perp\vec{n}$, where $\vec{n}$ is the direction of propagation. In other words we shall refer to the *transverse character* of the e.m. radiation field [10]. A potential representation of this field met difficulties with the Lorentz covariant description. The radiation fields posses two degrees of freedom whereas the theory describes them by four potentials corresponding to four degrees of freedom. Beside the theory uses subsidiary conditions like the radiation gauge, or the Coulomb gauge. These Lorentz non-covariant conditions together with Lorentz-gauge equation allow to reduce the degrees of freedom till two.

In this paper we suggest a new look on the nature of the transverse e.m. fields. According to this viewpoint, the propagating transverse e.m. fields, with $I_1 = 0,\ I_2 = 0$, have a nature distinct of the electric and magnetic fields characterized by two non-trivial invariants $I_1 \neq 0,\ I_2 \neq 0$. This viewpoint prompts an idea to use two-potential representation instead of four-potential one. Such representation implies transverse character of e.m. fields. The pair of potentials are Lorentz-scalars and obey Klein-Gordon equations. Hence, the transverse e.m. waves have a quantum origin. In the framework of this theory the four-potentials can be interpreted as a four-vector of current density for which a continuity equation is the Lorentz gauge equation.

2 Maxwell wave equations

In the solution of any electromagnetic problem the fundamental relations that must be satisfied are the four field equations– Maxwell equations:

$$[\nabla \times \mathbf{H}] = \frac{\partial}{\partial t}\mathbf{D} = \mathbf{J}, \tag{2.1a}$$

$$(\nabla \cdot \mathbf{D}) = \rho, \tag{2.1b}$$

$$[\nabla \times \mathbf{E}] = -\frac{\partial}{\partial t}\mathbf{B}, \tag{2.2a}$$

$$(\nabla \cdot \mathbf{B}) = 0. \tag{2.2b}$$

In addition there are three relations that concern the characteristics of the medium in which the fields exist. There are the constitutive relations

$$\mathbf{D} = \epsilon\mathbf{E},\ \ \mathbf{B} = \mu\mathbf{H}, \mathbf{J} = \sigma\mathbf{E}. \tag{2.2c}$$

Here ϵ, μ, σ are the permittivity, permeability, and conductivity of the medium, which is assumed to be homogeneous, isotropic, and source-free.

When relations (2.2c) are inserted in Eqs.(2.1a,b)-(2.2a,b), Maxwell's equations become differential equations relating the electric and magnetic field strengths $\mathbf{E}$, $\mathbf{H}$.

Consider the particular case of electromagnetic phenomena in free space, or more generally, in a perfect dielectric containing no charges and no conduction currents. For this case the field equations become

$$[\nabla \times \mathbf{B}] - \mu\epsilon\frac{\partial}{\partial t}\mathbf{E} = 0, \tag{2.3a}$$

$$(\nabla \cdot \mathbf{E}) = 0, \tag{2.3b}$$

$$[\nabla \times \mathbf{E}] + \frac{\partial}{\partial t}\mathbf{B} = 0, \tag{2.4a}$$

$$(\nabla \cdot \mathbf{B}) = 0. \tag{2.4b}$$

Since $c^2 = 1/\mu\epsilon$ where c is the speed of light in vacuum.

Equation (2.4b) is satisfied automatically if the *magnetic-flux density* is represented as a rotor of the vector-potential

$$\mathbf{B} = [\nabla \times \mathbf{A}], \tag{2.5}$$

because

$$(\nabla \cdot [\nabla \times \mathbf{A}]) = 0.$$

The expression for the electric field strength is defined by

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t}. \tag{2.6}$$

These formulae are such that the second group of Maxwell equations (2.4a,b), representing Faraday's law and the absence of magnetic charges, are satisfied automatically. The first group of equations (2.3a,b) are reduced into the following equations for the potentials

$$-\Delta\mathbf{A} + \nabla(\nabla \cdot \mathbf{A}) + \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla\Phi + \frac{\partial \mathbf{A}}{\partial t}) = 0, \tag{2.7a}$$

$$\nabla \cdot (\nabla\Phi + \frac{\partial}{\partial t}\mathbf{A}) = 0. \tag{2.7b}$$

Equations (2.7a,b) can be separated by choosing the Lorentz gauge equation

$$(\nabla \cdot \mathbf{A}) + \frac{1}{c^2}\frac{\partial}{\partial t}\Phi = 0. \tag{2.8}$$

Substitution of Eq.(2.8) into (2.7a,b) yields wave equations for the four-potentials

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A} = 0, \quad \Delta\Phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\Phi = 0. \tag{2.9}$$

Now let us re-write these formulae in a tensorial form. From the potentials Φ and $\mathbf{A}$ we pass to four-vector representation by

$$A^\mu := (\Phi, c\mathbf{A}), \quad A_\mu := (\Phi, -c\mathbf{A})$$

Further, introduce four-coordinates

$$x^\mu := (ct, \vec{r}), \quad x_\mu := (ct, -\vec{r}).$$

In these notations the vectors of electric field strength and magnetic-flux density cast into the form of screw-symmetric tensor

$$F_{\mu\nu} := (c\mathbf{B},\ \mathbf{E}).$$

Then, formulae (2.5) and (2.6) are joined in

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \text{with} \ \ \partial_\mu := \frac{\partial}{\partial x^\mu}. \tag{2.10}$$

The first group of Maxwell equations (2.3a,b) take the form

$$\partial_\mu F^{\mu\nu} = 0, \tag{2.11a}$$

whereas the second pair of Maxwell equations (2.4a,b) are written as

$$\partial_\rho F_{\mu\nu} + \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} = 0. \tag{2.11b}$$

The latter is a consequence of the potential representation (2.10).

In order to obtain wave equation for the four-potential, one has to use the Lorentz gauge

$$\partial_\mu A^\mu = 0. \tag{2.12}$$

Then the first pair of Maxwell equations (2.11a) is reduced to the wave equation for the four-potential:

$$\partial_\mu F^\mu{}_\nu = \partial_\mu \partial^\mu A_\nu = 0. \tag{2.13}$$

section 3 Representation of the electromagnetic fields by pair of Lorentz scalar potentials

The magnetic-flux density **B** defined in (2.5) as a rotor of the vector field **A** automatically satisfies Eq.(2.4b). This is not unique representation for **B**, however. In fact, in the following representation

$$\mathbf{B} = [\nabla\phi \times \nabla\psi] \tag{3.1}$$

the magnetic-flux density also satisfies Eq.(2.4b). This is because

$$(\nabla \cdot [\nabla\phi \times \nabla\psi]) = (\ [\nabla \times \nabla]\phi \cdot \nabla\psi) - (\nabla\phi \cdot [\nabla \times \nabla]\psi)\ ) = 0.$$

Correspondingly, the electric field strength is represented as follows

$$\mathbf{E} = \frac{\partial\phi}{\partial t}\nabla\psi - \frac{\partial\psi}{\partial t}\nabla\phi. \tag{3.2}$$

In tensorial notations these formulas are given by

$$F_{\mu\nu} = \partial_\mu\phi\partial_\nu\psi - \partial_\nu\phi\partial_\mu\psi. \tag{3.3}$$

Obviously, the functions ϕ, ψ (*two-potentials*) are invariant with respect to the Lorentz-transformation. Within these formulae, field Lorentz-transformation directly follows from the coordinate Lorentz-transformation.

In order to obtain expressions connecting four-potentials with two-potentials re-write (3.1) as follows

$$\mathbf{B} = \frac{1}{2}[\nabla \times (\phi\nabla\psi - \psi\nabla\phi)].$$

Hence, the tri-vector potential is related with the two-potentials by

$$\mathbf{A} = \frac{1}{2}(\phi\nabla\psi - \psi\nabla\phi).$$

Correspondingly, for the four-vector we obtain

$$A_\mu = \frac{1}{2}\left(\phi\partial_\mu\psi - \psi\partial_\mu\phi\right). \tag{3.4}$$

Two-potential representation corresponds to two degrees of freedom. The first main consequence of this approach is *transverse character* of e.m. fields. In fact, from formulae (3.1) and (3.2) it follows orthogonality of the e.m. fields,

$$I_1 = (\mathbf{B}\cdot\mathbf{E}) = 0.$$

As far as we reduced the degree of freedom from four to two, additional algebraic relations between fields and four-vector potentials arise. From (3.1), (3.2) by using (3.4) we obtain

$$[\mathbf{A}\times\mathbf{E}] - \Phi\mathbf{B} = 0, \quad (\mathbf{A}\cdot\mathbf{B}) = 0. \tag{3.5}$$

What kind interpretation can be done to these equations ? Seeking an answer we refer the reader to Refs.[11] and references therein. Stating briefly, the authors found a topological invariant and denominate this quantity by *"helicity"*[2]. This quantity is defined by the integral as

$$S = \int(\mathbf{A}\cdot\mathbf{B})dV.$$

For the static magnetic field the quantity S characterizes to what extent magnetic lines are coupled each other. For a single magnetic line this value estimates the screwness of the line. The relativistic generalization of "helicity" was introduced in Ref.[12]. In this case a density of the

[2]With this term the authors have introduced some confusion into the system of terminologies, because the term *helicity* in physics is understood as projection of the spin onto the direction of momentum.

"helicity" is defined as an integral over the 0-th component of the four-vector

$$\mathcal{J}^\mu = \widetilde{F}^{\mu\nu}A_\nu, \quad \widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}.$$

In the components

$$\vec{\mathcal{J}} = [\mathbf{A}\times\mathbf{E}] - \Phi\mathbf{B}, \quad \mathcal{J}_0 = (\mathbf{A}\cdot\mathbf{B}).$$

This four-vector satisfy the equation

$$\partial_\mu\mathcal{J}^\mu = -2(\mathbf{E}\cdot\mathbf{B})$$

From this equation the authors concluded that $\mathcal{J}^\mu$ is conserved only for transverse fields. However, within the framework of the present approach (see, Eqs.(3.5)), for the transverse electromagnetic fields the so called "helicity" is identically zero $\mathcal{J}^\mu = 0$, i.e. it is trivial.

section 4 Lorentz gauge equation as a continuity equation

Consider the Lorentz gauge equation for four-potentials defined via two-potentials. Substitute (3.4) into Eq.(2.12), we get

$$\partial_\mu(\phi\partial^\mu\psi - \psi\partial^\mu\phi) = 0,$$

which can be evaluated as follows

$$\phi\Box^2\psi - \psi\Box^2\phi = 0, \quad \text{with} \quad \Box^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2. \tag{4.1}$$

This equation is satisfied automatically by the following equations

$$\Box^2\psi = -\lambda^{-2}\psi, \quad \Box^2\phi = -\lambda^{-2}\phi. \tag{4.2}$$

We suggest interpret these equations as Klein -Gordon equations where $\lambda = \frac{\hbar}{mc}$ is Compton's length of the wave.

From two real Klein-Gordon equations with the same mass we can form a Klein-Gordon equation for the complex-valued function $\Psi = \phi + i\psi$. The expression for field tensor (3.3) now is defined as follows

$$F_{\mu\nu} = \frac{i}{2}\left(\partial_\mu\Psi\partial_\nu\bar{\Psi} - \partial_\nu\Psi\partial_\mu\bar{\Psi}\right) \tag{4.3}$$

Therefore we can define the current density by

$$j_\mu = \frac{i}{4}\left(\Psi\partial_\mu\bar{\Psi} - \bar{\Psi}\partial_\mu\Psi\right) =$$

$$= \frac{1}{2}\left(\phi\partial_\mu\psi - \psi\partial_\mu\phi\right).$$

Then from Eqs.(4.2) it follows the continuity equation for the current density

$$\partial_\mu j^\mu = 0,$$

which is the Lorentz gauge equation (2.12) because in these units $j^\mu = A^\mu$.

section 5 Monochromatic plane waves

In the previous section we have introduced two scalar functions instead of four-vector potential. These functions are defined in the field of real numbers. However, sometimes solutions of the equations more convenient to investigate by considering the four-vector potential as a complex- valued function [10]. In this case the pair of Lorentz-invariant potentials also are represented by complex-valued functions.

Consider the *monochromatic plane wave*

$$A_\mu = C_\mu \exp(-ikx), \tag{5.1}$$

with $k^2 = 0$ which is a solution of the wave equation

$$\Box^2 \; A_\mu = 0. \tag{5.2}$$

The subsidiary condition

$$\partial^\mu A_\mu = 0 \tag{5.3}$$

implies

$$k^\mu C_\mu = 0 \tag{5.4}$$

i.e. the amplitude C^μ is orthogonal to the wave vector k_μ.

Now, let us express this solution by the pair of complex valued potentials $\Psi,\ \Phi$. The free solutions of the Klein-Gordon equations are

$$\Psi = \rho_1 \exp(ip_\mu x^\mu), \quad \Phi = \rho_2 \exp(iq_\mu x^\mu), \tag{5.5}$$

where

$$q_\nu q^\nu = \lambda^{-2},\ p_\nu p^\nu = \lambda^{-2}. \tag{5.6}$$

From these functions we can construct four-potential according to formula (3.4). This formula now will be written as follows

$$A_\mu = \frac{1}{2i}(\Psi\partial_\mu\Phi - \Phi\partial_\mu\Psi). \tag{5.7}$$

Maxwell equations (2.11a) for the strength tensor

$$F_{\mu\nu} = \frac{1}{i}(\partial_\mu \Phi \partial_\nu \Psi - \partial_\nu \Phi \partial_\mu \Psi)$$

take the form

$$i\partial^\mu F_{\mu\nu} = \Box^2 \Phi \partial_\nu \Psi + \partial_\mu \Phi \partial^\mu \partial_\nu \Psi - \Box^2 \Psi \partial_\nu \Phi - \partial_\mu \Psi \partial^\mu \partial_\nu \Phi = 0.$$

By taking into account Eqs.(4.2), we obtain

$$i\partial^\mu F_{\mu\nu} = \lambda^{-2}(\Psi \partial_\nu \Phi - \Phi \partial_\nu \Psi) + \partial^\mu \partial_\nu \Psi \partial_\mu \Phi - \partial^\mu \partial_\nu \Phi \partial_\mu \Psi = 0.$$

From this equation it follows that the four-vectors p_ν, q_μ have to be related by the condition

$$(\lambda^{-2} + p^\nu q_\nu)(q_\nu - p_\nu) = 0 \tag{5.8}$$

This condition demands that at least one of the other two conditions have to be fulfilled:

1) $q_\nu - p_\nu = 0, \quad q_\nu = p_\nu$. We shall refuse this solution because in this case the strength tensor is trivial: $F_{\mu\nu} = 0$.

b) The other condition is

$$\lambda^{-2} + p^\nu q_\nu = 0. \tag{5.9}$$

By using Eqs.(5.6) we may transform Eq.(5.9) into

$$(q_\nu + p_\nu)(q^\nu + p^\nu) = 0. \tag{5.10}$$

According to formulae (5.5) and (5.7) we have

$$A_\mu = \rho_1 \rho_1 (q_\mu - p_\mu) \exp(i k_\nu x^\nu),$$

where $k_\nu = q_\nu + p_\nu$. Relationship (5.6)

$$q_\nu q^\nu = p_\nu p^\nu = \lambda^{-2},$$

we can write in the following form

$$(q_\nu + p_\nu)(q^\nu + p^\nu) = 0,$$

which provides the condition (5.4) corresponding to the Lorentz gauge. Notice, from (5.10) it follows

$$k_\nu k^\nu = 0,$$

which means that we deal with massless particle.

Apart of the condition $I_1 = 0$, the transverse e.m. fields must obey the other condition

$$I_2 = F_{\mu\nu}F^{\mu\nu} = 2(E^2 - B^2) = 0. \tag{5.11}$$

Let us show that the representation (5.7) provides this condition also. In fact,

$$I_2 = (q_\mu p_\nu - q_\nu p_\mu)(q^\mu p^\nu - q^\nu p^\mu)\rho_1\rho_1 \exp(ik_\nu x^\nu) =$$

$$(\lambda^{-2} + p^\nu q_\nu)(\lambda^{-2} - p^\nu q_\nu)\rho_1\rho_1 \exp(ik_\nu x^\nu) = 0, \tag{5.12}$$

due to (5.9).

section 6 Gauge Transformation. Interaction between electromagnetic fields

The four-potentials A_ν do not determine the electromagnetic fields uniquely because the *gauge transformations*

$$A'_\nu = A_\nu + \partial_\nu f, \tag{6.1}$$

where $f(x)$ is an arbitrary scalar function, does not effect the field tensor $F_{\mu\nu}$.

Let us consider the following transformations of the solutions of the Klein-Gordon equations

$$\psi' = \cos(g)\psi + \sin(g)\phi, \quad \phi' = \cos(g)\phi - \sin(g)\psi, \tag{6.2}$$

where g is a constant. This is gauge transformation of the first kind for the *matter* fields. The tensor $F_{\mu\nu}$ defined in (3.3) is invariant under this transformation. The situation becomes more clear when $F_{\mu\nu}$ is defined through the complex valued potential $\Psi = \phi + i\psi$ (see, (4.3)). Then formulae (6.2) are written as

$$\Psi' = \exp(ig)\Psi, \bar{\Psi}' = \exp(-ig)\bar{\Psi}. \tag{6.3}$$

If we require that $F_{\mu\nu}$ be invariant under (6.2) with $g = g(x,t)$ then we shall need a compensating gauge field transforming simultaneously according to law (6.1). Hence the partial differentiation $\partial_\nu\Psi$ has to be replaced as

$$\partial_\nu\Psi \to \partial_\nu\Psi - i\kappa\mathcal{A}_\nu\Psi,$$

where the four-potential $\mathcal{A}_\mu$ we refer to the external electromagnetic field, and κ is a parameter dimensionality of $[\frac{e}{\hbar}]$. The field tensor takes the form

$$F_{\mu\nu} = \frac{1}{2i}\left((\partial_\mu - i\kappa\mathcal{A}_\mu)\Psi(\partial_\nu + i\kappa\mathcal{A}_\nu)\bar{\Psi} - (\partial_\nu - i\kappa\mathcal{A}_\nu)\Psi(\partial_\mu + i\kappa\mathcal{A}_\mu)\bar{\Psi}\right) \tag{6.4}$$

By comparing (6.4) with (3.3), we find that the stress tensor consists the additional term,

$$F_{\mu\nu} \to F_{\mu\nu} + \frac{1}{2}\kappa[\mathcal{A}_\nu\partial_\mu|\Psi|^2 - \mathcal{A}_\mu\partial_\nu|\Psi|^2] \tag{6.5}$$

This additional term we treat as an interaction between transverse (or null) and ordinary (non-null) e.m. fields.

section 7 Representation of free Maxwell equations over solutions of Dirac equation

There were series of papers devoted to the investigation of the mapping between Dirac and Maxwell-like equations. In the section we will follow the Keller's review [13] of the papers Campolattaro [7], Daviau [8] and analysis of Rodrigues and Vaz [6].

Starting with the Dirac equation

$$(\gamma^\mu\partial_\mu + im)\Psi = 0, \tag{7.1}$$

Campolattaro derived Maxwell-like equations with e.m. tensors

$$F^{\mu\nu} = \bar{\Psi}S^{\mu\nu}\Psi, \quad S^{\mu\nu} = \frac{i}{2}\gamma^{\mu\nu}, \tag{7.2a}$$

and

$$\widetilde{F}^{\mu\nu} = \bar{\Psi}\gamma^5 S^{\mu\nu}\Psi, \tag{7.2b}$$

generated by two currents

$$j^\mu = g^{\mu\nu}Im(\bar{\Psi}\partial_\nu\Psi) + m(\bar{\Psi}\gamma^\mu\Psi), \quad \widetilde{j}^\mu = g^{\mu\nu}Im(\bar{\Psi}\gamma_\nu\gamma^5\Psi). \tag{7.3}$$

The former is ordinary in nature and the latter is magnetic monopolar. The both obey the Lorentz gauge equation because each of the spinor components satisfies Klein-Gordon equation and the current $m(\bar{\Psi}\gamma^\mu\Psi)$ is conserved.

Daviau, and later Rodrigues and Vaz, have worked within the framework of Hestenes formulation of the Dirac equation. They started with the Dirac-Hestenes solutionin free space given by

$$\Psi = \sqrt{\rho} exp(\frac{1}{2}\beta\gamma_5)R, \tag{7.4}$$

where ρ is a scalar density, β a duality rotation angle and R a spacetime rotation. There Ψ is in fact a multivector with four independent left ideals. They have shown that the multivector

$$F = exp(\beta\gamma_5)R\gamma_1\gamma_2 R^{\star}, \tag{7.5}$$

obey the free Maxwell equation

$$\Box F = 0. \tag{7.6}$$

Rodrigues and Vaz then look for solutions of Eq.(7.6) of the form (7.5); they concluded that Eq.(7.6) is valid only when F is non-null field $F^2 \neq 0$. *Thus, it is important to emphasize, this representation is applicable only for the non-null fields, on the contrary to our representation which is faithful only for the null fields.*

section 8 Concluding remarks

We postulated that the transverse radiation, or *null* electromagnetic fields with $I_1 = 0,\ I_2 = 0$ have a nature qualitatively different of a nature of the electromagnetic fields with $I_1 \neq 0,\ I_2 \neq 0$. In this context let us draw an analogue with the relativistic mechanics with the invariant constant of motion[14]

$$p_\mu p^\mu = (Mc)^2.$$

Within the scope of relativistic mechanics the particles are divided into two classes:
(a) $M^2 = m^2$, the class of massive particles;
(b) $M = 0$, the class of massless particles.
The massless particles have a nature qualitatively distinct of the nature of the massive particles. In the same way, the electromagnetic fields we should also divide into two classes:
(a) $I_1 \neq 0,\ I_2 \neq 0$ non- null fields;
(b) $I_1 = 0,\ I_2 = 0$ null fields.

The transverse e.m. fields have a quantum origin because the field's tensor is built of the matter fields obeying Klein-Gordon equations.

Within the framework of this theory the four-potential plays the role of the current density. Hence, the Lorentz gauge equation can be interpreted as an continuity equation for the current density.

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