A New Way Beyond the Standard Model of the Electroweak and Colour Interactions

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ABSTRACT. The Standard model of the electroweak and colour interactions, which fits well all the experimental data with around 30 parameters and assumptions, leaves many questions unanswered, like: Where do families of quarks and leptons come from?, Why do only the left handed quarks and leptons carry the weak charge while the right handed ones are weak chargeless?, Where do Yukawa couplings come from?, Why do we have so many different internal degrees of freedom?, Why has our space-time $3+1$ dimensions?, and many others. I am trying to answer to these and other questions by proposing the approach [1, 2, 3, 4, 5, 6, 7, 8], which unifies spins and charges (manifesting in $d(=1+3)$-dimensional space) to only a spin in $d>4$, in particular in $d=1+13$. The approach assumes that fermions in d-dimensional space carrying nothing but a spin, interact with spin connection and vielbein fields - the gravitational gauge fields. Then a fermion with only a spin manifests in $d=1+3$ the spin and all the known charges, required by the Standard model, while the part of the starting Lagrangean in $d=1+13$ ($E\psi^{\dagger}\gamma^0\gamma^a p_{0a}\psi$, $a=0,1,2,3,5,\cdots,14$), transforming left handed weak charged quarks and leptons into the corresponding right handed weak chargeless quarks and leptons, manifests in $d=1+3$ the Yukawa couplings. No Higgs are needed to "dress" the weak chargeless fermions with the weak charge. I am also proposing a (new) way of generating families: by introducing a second kind of the Clifford algebra objects, with the corresponding gauge field included.

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1 Introduction

The very successful Standard model of the electroweak and colour interactions leaves unanswered many open questions, among which are also the questions about the origin of the families, of the Yukawa couplings of quarks and leptons, of the corresponding Higgs mechanism and of the weak scale. Understanding the mechanism for generating families, their masses and mixing matrices might be one of the most promising ways to physics beyond the Standard model. The Standard model also leaves open the question: Why does the Nature manifest 4 dimensions with the Minkowsky signature, the $SU(3), SU(2)$ and $U(1)$ charges and the spin ($SO(1,3)$). The approach unifying spins and charges[1, 2, 3, 4, 5, 6, 7, 8] into only a spin is proposing a way for answering the above questions by assuming that in $d \geq (1+13)$ spinors carry nothing but a spin and interact with the Poincarè gauge fields (vielbeins and spin connections), while families appear due to the second kind of the Clifford algebra objects and the corresponding gauge fields. One Weyl spinor of $SO(1,13)$ then manifests (by itself) in $d = 1+3$ the ordinary spin, the known charges and all the properties required (postulated) by the Standard model. It also manifests (an even number of) families.

A left handed $SO(1,13)$ Weyl spinor multiplet namely includes, if the representation is analyzed in terms of the subgroups $SO(1,3)$, $SU(2)$, $SU(3)$ and the sum of the two $U(1)$'s, all the spinors of the Standard model - that is the left handed $SU(2)$ doublets and the right handed $SU(2)$ singlets of (with the group $SU(3)$ charged) quarks and (chargeless) leptons - giving a possible answer to the question where does originate the connection between the weak charge and the handedness (which in $d = 1+3$ concerns only the spin).

The approach connects the existence of more than one family with the fact that there are two kinds of the Clifford algebra objects: the ordinary Dirac γ^a operators and the operators, called $\tilde{\gamma}^a$, which anticommute with γ^a ($\{\gamma^a, \tilde{\gamma}^b\}_+ = 0$). The first kind belongs to the generators of the Poincarè group for spinors and is responsible for the interaction of spinors with a spin connection field (while the momentum p^a connects spinors with a vielbein), the second kind is responsible for the generation of families and contribute - together with the first one - to Yukawa couplings. Every spinor carries accordingly two indices - the spinor index and the family index. A state signed by a spinor index changes due to S^{ab} ($S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$) into other members of the same Weyl representation, while the family index stays unchanged. A state belonging to a family (signed by the second index) changes under the application of $\tilde{S}^{ab}$ ($\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$) into a member of another family with unchanged properties with respect to S^{ab}.

The approach is a kind of the Kaluza-Klein-like theories, which suffer a very serious disadvantage, namely that there might not exist any massless, mass protected spinors, which are, after the break of symmetries, chirally coupled to the desired (Kaluza-Klein) gauge fields[9]. This would mean that there are no observable (massless or almost massless in comparison with let us say the Planck scale) spinors at observable energies.

We are trying hard[10, 11] to solve the problem and indeed it seems that we have found a possible way out of this problem by choosing appropriate boundary conditions. Namely, on an example of a spinor in $d = 1 + 5$, with boundaries allowing only one handedness, we demonstrate that a spinor does manifest masslessness and the Kaluza-Klein charge in $d = 1 + 3$, giving a hope also for more general cases.

We have search also for a possible answer to the question: How did Nature make a decision for one time and three space coordinates[19, 20]?, as well as for a possible answer to the question: How did breaks of symmetries from $SO(1, 13)$ down to the observed symmetries in $1 + 3$ occur[13]?, finding some possible explanations for both of these questions. But all these problems need further studies.

It is, of course, by itself questionable whether more than $1 + 3$ dimensions exist in Nature at all. The elegance of existing only one internal degree of freedom, namely spin, speaks for more than $1 + 3$ dimensions. Also the fact that in all the dimensions except in those of $d = 2(2n + 1), n = 0, 1, 2, \ldots$, there is no massless particles (since either both kinds of masses - namely the Dirac one and the Majorana one - exist, or at least the Majorana mass exists even if we start with only one Weyl and no families[12]) speaks for more than $d = 1+3$. Very probably d is any and then one should show how Nature has arrived from any d via $d = 1 + 13$ to observable $1 + 3$-dimensional space-time with the properties of spinors and vectors and tensors as observed today.

Together with collaborators we step by step build the approach towards a theory which could answer the open questions of the Standard model. In this contribution I shortly comment on the progress made up to now.

2 Two kinds of the Clifford algebra objects

We assume two kinds of the Clifford algebra objects defining two kinds of the generators of the Lorentz algebra[1, 2, 14, 15]. One kind are the ordinary Dirac γ^a operators defining the generators of the Poincaré algebra for spinors S^{ab} ($S^{ab} = \frac{i}{4}(\gamma^a\gamma^b - \gamma^b\gamma^a)$). The second kind[1] of the Clifford objects $\tilde{\gamma}^a$ anticommutes with γ^a ($\{\tilde{\gamma}^a, \gamma^b\}_+ = 0$) and defines accordingly $\tilde{S}^{ab}$ ($\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)$), with $\{\tilde{S}^{ab}, S^{cd}\}_- = 0$. They are responsible for the generation of families.
We define a basis of spinor representations as eigen states of the chosen Cartan subalgebra of the Lorentz algebra $SO(1,13)$, with the operators S^{ab} and $\tilde{S}^{ab}$ in the two Cartan subalgebra sets, with the same indices in both cases. When introducing the notation[14, 15, 7]

$$\begin{aligned} \overset{ab}{(\pm i)} &:= \frac{1}{2}(\gamma^a \mp \gamma^b), \quad \overset{ab}{[\pm i]} := \frac{1}{2}(1 \pm \gamma^a\gamma^b), \text{ for } \eta^{aa}\eta^{bb} = -1, \\ \overset{ab}{(\pm)} &:= \frac{1}{2}(\gamma^a \pm i\gamma^b), \quad \overset{ab}{[\pm]} := \frac{1}{2}(1 \pm i\gamma^a\gamma^b), \text{ for } \eta^{aa}\eta^{bb} = 1, \end{aligned} \tag{1}$$

it can be shown that the above binomials are all "eigen vectors" of the generators S^{ab}, as well as of $\tilde{S}^{ab}$

$$\begin{aligned} S^{ab} \overset{ab}{(k)} &= \frac{k}{2} \overset{ab}{(k)}, \quad S^{ab} \overset{ab}{[k]} = \frac{k}{2} \overset{ab}{[k]}, \\ \tilde{S}^{ab} \overset{ab}{(k)} &= \frac{k}{2} \overset{ab}{(k)}, \quad \tilde{S}^{ab} \overset{ab}{[k]} = -\frac{k}{2} \overset{ab}{[k]}\,. \end{aligned} \tag{2}$$

Defining $\overset{ab}{(\tilde{k})} = \frac{1}{2}(\tilde{\gamma}^a + \frac{\eta^{aa}}{ik}\tilde{\gamma}^b)$ we find the relations

$$\begin{aligned} \gamma^a \overset{ab}{(k)} &= \eta^{aa} \overset{ab}{[-k]}, \quad \gamma^b \overset{ab}{(k)} = -ik \overset{ab}{[-k]}, \\ \gamma^a \overset{ab}{[k]} &= \overset{ab}{(-k)}, \quad \gamma^b \overset{ab}{[k]} = -ik\eta^{aa} \overset{ab}{(-k)}, \\ \tilde{\gamma}^a \overset{ab}{(k)} &= -i\eta^{aa} \overset{ab}{[k]}, \quad \tilde{\gamma}^b \overset{ab}{(k)} = -k \overset{ab}{[k]}, \\ \tilde{\gamma}^a \overset{ab}{[k]} &= i \overset{ab}{(k)}, \quad \tilde{\gamma}^b \overset{ab}{[k]} = -k\eta^{aa} \overset{ab}{(k)}, \end{aligned}$$

[1] The operators $\tilde{\gamma}^a$ are introduced formally as operating on any Clifford algebra object B from the left hand side, but they also can be expressed in terms of the ordinary γ^a as operating from the right hand side as follows $\tilde{\gamma}^a B := i(-)^{n_B} B\gamma^a$, with $(-)^{n_B} = +1$ or -1, when the object B has a Clifford even or odd character, respectively.

$$
\begin{array}{llll}
\overset{ab}{(k)}\overset{ab}{(k)}=0, & \overset{ab}{(k)}\overset{ab}{(-k)}= \eta^{aa}\overset{ab}{[k]}, & \overset{ab}{[k]}\overset{ab}{[k]}=\overset{ab}{[k]}, & \overset{ab}{[k]}\overset{ab}{[-k]}= 0, \\
\overset{ab}{(k)}\overset{ab}{[k]}=0, & \overset{ab}{[k]}\overset{ab}{(k)}=\overset{ab}{(k)}, & \overset{ab}{(k)}\overset{ab}{[-k]}=\overset{ab}{(k)}, & \overset{ab}{[k]}\overset{ab}{(-k)}= 0, \\
\overset{ab}{\tilde{(k)}}\overset{ab}{(k)}=0, & \overset{ab}{\tilde{(-k)}}\overset{ab}{(k)}= -i\eta^{aa}\overset{ab}{[k]}, & \overset{ab}{\tilde{(k)}}\overset{ab}{[k]}= i\,\overset{ab}{(k)}, & \overset{ab}{\tilde{(k)}}\overset{ab}{[-k]}= 0.
\end{array} \quad (3)
$$

3 One Weyl spinor in $d = (1 + 13)$ manifesting at "physical energies" families of quarks and leptons

We assume a left handed Weyl spinor in $(1+13)$-dimensional space. We make a choice of $d/2 = 7$ Cartan subalgebra members in both sectors as follows:
$S^{03}, S^{12}, S^{56}, S^{78}, S^{9\;10}, S^{11\;12}, S^{13\;14}$ and
$\tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \tilde{S}^{78}, \tilde{S}^{9\;10}, \tilde{S}^{11\;12}, \tilde{S}^{13\;14}$.

The breaks of symmetries should bring one Weyl spinor from $SO(1,13)$ down to $SO(1,3)$ demonstrating then all the charges. Although some steps in studying possible ways of breaking symmetries have been done[13], this problem is very hard one and the study is in progress.

There are several ways of breaking the group $SO(1,13)$ down to subgroups of the Standard model. (One of) the most probable breaks, suggested by the approach unifying spins and charges, is the following one

$$
\begin{array}{c}
\mathrm{SO}(1,13) \\
\downarrow \\
\underbrace{\mathrm{SO}(1,7) \otimes \mathrm{SU}(3) \otimes U(1)} \\
\downarrow \\
\mathrm{SO}(1,3) \otimes \mathrm{U}(1) \otimes \mathrm{SU}(3)
\end{array}
$$

When analyzing one Weyl spinor in $d = 1 + 13$ with $2^{d/2-1}$ basic states in terms of the subgroups $SO(1,3)$, two $U(1)$, $SU(2)$ and $SU(3)$ we end up with quarks and leptons postulated by the Standard model, with right handed weak chargeless neutrino included. I am pointing out that the quarks of different colours and the colourless leptons are identical with respect to the subgroup $SO(1,7)$ of the group $SO(1,13)$.

They only differ in the part, which determines the colour and the one of the two $U(1)$ groups, which is a subgroup of the $SO(6)$.

We present in Table I all the quarks of one particular colour (the right handed weak chargeless u_R, d_R and the left handed weak charged u_L, d_L, with the colour $(1/2, 1/(2\sqrt{3}))$ in the Standard model notation). They all are members of one $SO(1,7)$ multiplet, which in the approach unifying spins and charges concerns the generators S^{ab} with the indices a and b from 0 to 8, while the colour degrees of freedom as well as one $U(1)$ charge degree of freedom concern S^{ab} with a, b from 9 to 14.

i		$\vert^a\psi_i>$	$\Gamma^{(1,3)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{21}	τ^{33}	τ^{38}	τ^{41}	Y	Y'
		Octet, $\Gamma^{(1,7)}=1,\ \Gamma^{(6)}=-1$, of quarks										
1	u_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	1/2	1	0	1/2	1/2	$1/(2\sqrt{3})$	1/6	2/3	-1/3
2	u_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	-1/2	1	0	1/2	1/2	$1/(2\sqrt{3})$	1/6	2/3	-1/3
3	d_R^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	1/2	1	0	-1/2	1/2	$1/(2\sqrt{3})$	1/6	-1/3	2/3
4	d_R^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	-1/2	1	0	-1/2	1/2	$1/(2\sqrt{3})$	1/6	-1/3	2/3
5	d_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	1/2	-1	-1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
6	d_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	-1/2	-1	-1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
7	u_L^{c1}	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	1/2	-1	1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6
8	u_L^{c1}	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\Vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	-1/2	-1	1/2	0	1/2	$1/(2\sqrt{3})$	1/6	1/6	1/6

Table 1: The 8-plet of quarks - the members of $SO(1,7)$ subgroup, belonging to one Weyl left handed ($\Gamma^{(1,13)} = -1 = \Gamma^{(1,7)} \times \Gamma^{(6)}$) spinor representation of $SO(1,13)$. It contains the left handed weak charged quarks and the right handed weak chargeless quarks of a particular colour ($(1/2, 1/(2\sqrt{3}))$). Here $\Gamma^{(1,3)}$ defines the handedness in $(1+3)$ space, S^{12} defines the ordinary spin (which can also be read directly from the basic vector, since, in particular, $S^{12}\stackrel{12}{(+)}=\frac{1}{2}\stackrel{12}{(+)}$), τ^{13} defines the third component of the weak charge, τ^{21} defines the $U(1)$ charge, τ^{33} and τ^{38} define the colour charge and τ^{41} another $U(1)$ charge, which together with the first $U(1)$ charge defines $Y = \tau^{21} + \tau^{41}$ and $Y' = -\tau^{21} + \tau^{41}$. Leptons differ from quarks only with respect to the part which concerns the indices $9, \cdots, 14$. A singlet in the colour, which belongs to the same Weyl, must be of the kind $\cdots|\cdots||\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$ (one finds that ν_R looks like $\stackrel{03}{(+i)}\stackrel{12}{(+)}|\stackrel{56}{(+)}\stackrel{78}{(+)}||\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$) The reader can find the whole Weyl representation in the ref.[6].

Looking at the first row of Table I, for example, and using Eq.(3)

(saying that $\gamma^0 \stackrel{03}{(+i)} = \stackrel{03}{[-i]}$, $\stackrel{78}{(-)}\stackrel{78}{(+)} = -\stackrel{78}{[-]}$) one sees that $\gamma^0 \stackrel{78}{(-)}$ transforms a right handed weak chargeless u_R quark of a particular colour into a left handed weak charged u_L quark of the same colour and the spin, presented in the seventh row. One also can notice that the generator $\tilde{S}^{07} = \frac{i}{2}\tilde{\gamma}^0\tilde{\gamma}^7$ transforms the expression of the first row on Table I into with respect to S^{ab} an equivalent state (namely the sate $\stackrel{03}{[+i]}\stackrel{12}{(+)} \mid \stackrel{56}{(+)}\stackrel{78}{[+]} \mid\mid \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}$, which is again a right handed weak chargeless u_R quark of the same colour and spin), which differs in properties from the u_R of the first row only with respect to $\tilde{S}^{ab}$.

Assuming that a kind of breaking symmetries[10] makes a starting Weyl spinor in $d = 1 + 13$ to manifest after a break into $SO(1,7) \times SU(3) \times U(1)$ as massless spinors - one $SU(3)$ triplet and one $SU(3)$ singlet - each of them a member of an $SO(1,7)$ octet[2], there must be accordingly also eight families since one can easily notice that each member of the octet on Table I carries indeed two indices: the index of the row and the family index. Namely, any $\tilde{S}^{ab}; a, b \in 0, 1.., 8$, not belonging to the Cartan subalgebra, transforms any member of the starting family, represented on Table I, into another family, with the same spin and charges.

4 The action for a Weyl spinor in $d = (1 + 13)$

A spinor carries only the spin (no charges) and interacts accordingly with only the gauge gravitational fields - with vielbeins and two kinds of spin connection fields - the gauge fields of p^a, S^{ab} and $\tilde{S}^{ab}$, respectively[1, 2, 4, 5, 6, 7]. One kind is the ordinary gauge field (gauging the Poincaré symmetry in $d = 1 + 13$). This field takes in $d = 1 + 3$ care of ordinary gravity and all the known gauge fields. The contribution of this field to the mass matrices manifests in only the diagonal terms - connecting the right handed weak chargeless quarks or leptons to the left handed weak charged partners within one family of spinors, as we shall see. The second kind of gauge fields is in our approach responsible for the appearance of families of spinors and for couplings among families of spinors - contributing to diagonal matrix elements as well.

[2] Antiquarks and antileptons appear in the second quantized procedure as their charged conjugate partners.

We write the action[7] for a Weyl (massless) spinor in $d(=1+13)$ - dimensional space as follows[3]

$$
\begin{aligned}
S=&\int d^dx\ \mathcal{L}\\
\mathcal{L}=&\frac{1}{2}(E\bar{\psi}\gamma^a p_{0a}\psi)+h.c.=\frac{1}{2}(E\bar{\psi}\gamma^a f^\alpha{}_a p_{0\alpha}\psi)+h.c.\\
=&E\{\bar{\psi}\gamma^m(p_m-\sum_{A,i} g^A\tau^{Ai}A^{Ai}_m)\psi+\sum_{s=7,8}\bar{\psi}\gamma^s p_{0s}\ \psi\}+\ \text{the rest},\\
p_{0\alpha}=&p_\alpha-\frac{1}{2}S^{ab}\omega_{ab\alpha}-\frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}.
\end{aligned}
\tag{4}
$$

Here $f^\alpha{}_a$ are vielbeins (inverted to the gauge field of the generators of translations $e^a{}_\alpha$, $e^a{}_\alpha f^\alpha{}_b=\delta^a_b$, $e^a{}_\alpha f^\beta{}_a=\delta_\alpha{}^\beta$), with $E=\det(e^a{}_\alpha)$, while $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ are the two kinds of the spin connection fields, the gauge fields of S^{ab} and $\tilde{S}^{ab}$, respectively. Index A determines the charge groups ($SU(3), SU(2)$ and the two $U(1)$'s), index i determines the generators within one charge group. τ^{Ai} denote the generators of the charge groups $\tau^{Ai}=\sum_{s,t}\ c^{Ai}{}_{st}\ S^{st}$, $\{\tau^{Ai},\tau^{Bj}\}_-=i\delta^{AB}f^{Aijk}\tau^{Ak}$, with $s,t\in 5,6,..,14$, while $A^{Ai}_m, m=0,1,2,3$, denote the corresponding gauge fields (expressible in terms of ω_{stm}). (We assume that no terms like $\sum_{A,i}\ \tilde{g}^A\tilde{\tau}^{Ai}\tilde{A}^{Ai}_m$ manifest at "low energy region", and the assumption needs a justification, which I leave for further studies.)

The subgroups and accordingly the coefficients $c^{Ai}{}_{st}$ are chosen so that the gauge fields in the "physical" region agree with the known gauge fields, while "the rest" in Eq.(4) is assumed to be small (which again would need the justification and therefore additional studies, which are in progress).

It is not difficult to see that the term $\sum_{s=7,8}\ \bar{\psi}\gamma^s p_{0s}\ \psi$ in Eq.(4) manifests as the Yukawa couplings of the Standard model. To manifest this fact let us write it as $\mathcal{L}_Y$ in the following way

$$\mathcal{L}_Y=E\psi^\dagger\gamma^0\gamma^s p_{0s}\psi$$

[3] Latin indices $a,b,..,m,n,..,s,t,..$ denote a tangent space (a flat index), while Greek indices $\alpha,\beta,..,\mu,\nu,..\sigma,\tau..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a,b,c,..$ and $\alpha,\beta,\gamma,..$), from the middle of both the alphabets the observed dimensions $0,1,2,3$ ($m,n,..$ and $\mu,\nu,..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s,t,..$ and $\sigma,\tau,..$). We assume the signature $\eta^{ab}=diag\{1,-1,-1,\cdots,-1\}$.

$$
\begin{aligned}
=&E\{\psi^\dagger\gamma^0\ \{\stackrel{78}{(+)}\ (\sum_{y=Y,Y'} yA^y_+ + \frac{-1}{2}\sum_{(ab)} \tilde{S}^{ab}\tilde{\omega}_{ab+})) + \\
&\stackrel{78}{(-)}\ (\sum_{y=Y,Y'} yA^y_- + \frac{-1}{2}\sum_{(ab)} \tilde{S}^{ab}\tilde{\omega}_{ab-}) + \\
&\stackrel{78}{(+)} \sum_{\{(ac)(bd)\},k,l} \stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\ \tilde{A}^{kl}_+((ac),(bd)) \ + \\
&\stackrel{78}{(-)} \sum_{\{(ac)(bd)\},k,l} \stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\ \tilde{A}^{kl}_-((ac),(bd))\}\psi\},
\end{aligned} \tag{5}
$$

with $k,l = \pm 1$, if $\eta^{aa}\eta^{bb} = 1$ and $\pm i$, if $\eta^{aa}\eta^{bb} = -1$, while Y and Y' are the two superpositions of the two $U(1)$ subgroups of the groups $SO(6)$ and $SO(1,7)$ as defined in refs.[7, 8]. The pairs (ab) in Eq.(6) run only over the Cartan subalgebra pairs of $SO(1,7)$ (namely over $(03),(12),(56),(78)$).

We namely rewrote $\sum_{(a,b)} -\frac{1}{2}\ \stackrel{78}{(\pm)}\ \tilde{S}^{cd}\tilde{\omega}_{cd\pm} = \sum_{(cd)}\ -\frac{1}{2}\ \stackrel{78}{(\pm)}\ \tilde{S}^{ab}\tilde{\omega}_{ab\pm} + \sum_{(ac),(bd),\, k,l} \stackrel{78}{(\pm)}\stackrel{ac}{(\tilde{k})}\stackrel{bd}{(\tilde{l})}\ \tilde{A}^{kl}_{\pm}((ac),(bd))$, where the pair (a,b) in the first sum runs over all the indices, with $a,b = 0,\ldots,8$, while the pairs in the second and the third sum $(cd),(ac),(bd)$ denote only the Cartan subalgebra pairs.

The Yukawa part of the starting Lagrangean (Eq.(4)) has the diagonal terms, that is the terms manifesting the Yukawa couplings within each family, and the off diagonal terms, determining the Yukawa couplings among families, as we shall demonstrate bellow.

One notices that in Eq.(6) only the part with the factor $\gamma^0\ \stackrel{78}{(+)}$ contributes to the mass matrix of the $d-$quarks and the electrons, while the part with the factor $\gamma^0\ \stackrel{78}{(-)}$ contributes to only the mass matrix of the $u-$quarks and the neutrinos. The first four sums of Eq.(6) contribute to only diagonal terms of either the $d-$quarks and the electrons or the $u-$quarks and the neutrinos. Terms with (Y,Y') distinguish among the u_R-quarks $(Y = 2/3, Y' = -1/3)$, the d_R-quarks $(Y = -1/3, Y' = 2/3)$, the left handed quarks $(Y = 1/6, Y' = 1/6)$, the ν_R $(Y = 0, Y' = -1)$, the e_R $(Y = -1, Y' = 0)$ and the left handed leptons $(Y = -1/2, Y' = -1/2)$. The terms with $\tilde{S}^{ab}$ do not distinguish among the members of one family, they only differ in values for different families. The last two sums in Eq.(6) contribute to the non diagonal matrix elements of either the $u-$quarks and the neutrinos (the term with the factor $\gamma^0 \overset{78}{(-)}$) or to the $d-$quarks and the electrons (the term with the factor $\gamma^0 \overset{78}{(+)}$).

5 Breaking symmetries lead to observable properties

We made the assumption that a break of symmetries leads from one Weyl representation in $d = 1+13$ to four massless octets (the representation of $SO(1,7)$) with the charges $SU(3), Y$ and Y' as presented above, leaving us with eight equivalent representations - that is with eight families. In order to be in agreement with what we observe, we must break further the symmetry of the octet in the charge sector. Namely, looking at Table I and recognizing[7] that $Q = \tau^{33} + Y = S^{56} + \tau^{41}$ must appear as a conserved quantity representing the electromagnetic charge, we assume that no terms of the types $S^{5a}\omega_{5a\pm}$ and $S^{6a}\omega_{6a\pm}$, with $a \neq 5, 6$ may appear in our $\mathcal{L}_Y$. Assuming that the break influences both sectors - S^{ab} and $\tilde{S}^{ab}$ - in a similar way, we let also all the terms $\tilde{S}^{5a}\tilde{\omega}_{5a\pm}$ and $\tilde{S}^{6a}\tilde{\omega}_{6a\pm}$, with $a \neq 5, 6$, contribute nothing, which means that we assume the break of $SO(1,7)$ into $SO(1,5) \times U(1)$, which further means that eight families decouple entirely into two times four families. Also this assumption needs the justification which we leave for further studies.

Let us look at the families. To manifest families we represent four equivalent representations of the first row on Table I, representing the u_R quark. All the other members of a particular family follow from the one for u_R by the application of $S^{ab}; a, b \in \{0, 8\}$ and equivalently for the quarks of other two colours and for the colourless leptons.

$$
\begin{aligned}
&I. \overset{03}{(+i)}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{(+)} \;||...\\
&II. \overset{03}{[+i]}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{(+)} \;||...\\
&III. \overset{03}{[+i]}\overset{12}{(+)} \mid \overset{56}{(+)}\overset{78}{[+]} \;||...\\
&IV. \overset{03}{(+i)}\overset{12}{[+]} \mid \overset{56}{(+)}\overset{78}{[+]} \;||... \,.
\end{aligned} \tag{6}
$$

Starting from one Weyl in d we only can have an even number of families. We have measured up to now three families and not four. How can we come from four families to three measurable ones up to now?

We may further break the symmetry, like $SO(1,5)$ into $SU(2) \times SU(2) \times U(1)$, by letting, for example, all the terms $\tilde{S}^{7a}\ \tilde{\omega}_{7a\pm}$ and $\tilde{S}^{8a}\ \tilde{\omega}_{8a\pm}, a \neq 7,8$, contribute nothing. Doing that we end up with twice two completely decoupled families. If we break instead $SO(1,5)$ into $SU(3) \times U(1)$, one family decouples from the rest three.

The experimental data seems at least for quarks to be closer to an assumption of an approximate break of $SO(1,5)$ to $SU(2)\times SU(2)\times U(1)$, since the first two families are much lighter than the third and also the quark mixing matrix seems not to disagree with such an assumption.

Following this way and fitting the experimental data within the experimental accuracy to the Eq.(6) we find masses for four families of quarks and leptons as follows[7, 8]

$$\begin{aligned}
m_{u_i}/GeV =& (0.0034, 1.15, 176.5, 285.2),\\
m_{d_i}/GeV =& (0.0046, 0.11, 4.4, 224.0),\\
m_{\nu_i}/GeV =& (1\ 10^{-12}, 1\ 10^{-11}, 5\ 10^{-11}, 84.0),\\
m_{e_i}/GeV =& (0.0005, 0.106, 1.8, 169.2). \qquad (7)
\end{aligned}$$

The results are in agreement with what the refs.[16, 17, 18] find as still experimentally acceptable values for the fourth family.

6 Concluding remarks

I briefly reviewed the work done up to now on the approach of mine unifying spins and charges, pointing out a possible way beyond the Standard model of the electroweak and colour interactions which this approach is offering.

The approach can explain, if one starts with one massless left handed Weyl representation, the postulate of the Standard model that only the left handed quarks and leptons carry the weak charge, while the right handed ones are weak chargeless, since one Weyl representation in $d = 1+13$, if analyzed in terms of the subgroups $SO(1,3), SU(3), SU(2)$ and two $U(1)$ of the group $SO(1,13)$, contains just such quarks and leptons: left handed weak charged quarks and leptons and right handed weak chargeless quarks and leptons with the right handed neutrino (with $Y = 0$ and $Y' = -1$ included). The approach also explains how families can appear through the application of the two kinds of the Clifford algebra objects and the corresponding gauge fields, predicting the Yukawa couplings of the Standard model as well. None Higgs fields must be put "by hand" (like it is done in the Standard model) to equip right handed fermions with a weak charge and accordingly allowing the mass term. It is, namely, a part of the Lagrange density in $d = 1 + 13$, which manifests in $d = 1 + 3$ as a term $(\psi^\dagger \gamma^0 \gamma^s p_{0s} \psi, s = 7, 8)$, which transforms a right handed weak chargeless quark or lepton into a left handed weak charged one, taking care also of couplings among families. This term manifests accordingly as the Yukawa couplings (assumed "by hand" in the Standard model).

To come from $d = 1+13$ to $d = 1+3$, the break of staring symmetries must occur spontaneously in a way, which guarantees massless spinors up to the symmetry $SO(1,7) \times SU(3) \times U(1)$ and then further the measured masses with three up to now measured families and yet experiments on the new accelerators must confirm the fourth family. This part of the approach is not yet studied well enough, so that the predicted properties of the fourth family are only approximate ones.

We have proven on a toy model that a particular boundary conditions might assure massless spinors with their charge chirally coupled to the Kaluza-Klein charge after the break. It stays for further studies to prove the same for the group $SO(1, 13)$ after the break to $SO(1, 7)$ and further. It also stays for further studies how do after breaks perturbative and nonperturbative effects influence properties of spinors, dressing the charges and the Yukawa couplings. This is a very nontrivial study, in particular since all the gauge fields are indeed the gravitational fields.

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