Pseudo-Dirac Neutrinos

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ABSTRACT. "Pseudo-Dirac" neutrinos, which are almost Dirac particles with tiny Majorana masses, are argued to provide a unique clue to the Majorana nature of neutrinos. The manifestation of the Majorana nature is through rich neutrino oscillation phenomena of pseudo-Dirac neutrinos, including the neutrino $\leftrightarrow$ anti-neutrino oscillation due to the lepton-number violating Majorana masses, even inside each generation. The formulae for the various oscillations of pseudo-Dirac neutrinos are obtained. We find that the reported specific features of neutrino mixings in the data of neutrino oscillation experiments, such as almost maximal mixing in the atmospheric neutrino oscillation, is naturally explained in the pseudo-Dirac scenario with small generation mixings, just as in the quark sector. We, however, also point out that there remain some problems to be settled for this scenario to be viable. As another interesting application, it is argued that the tiny pseudo-Dirac mass splitting due to the Majorana mass can be revealed, looking at high energy astrophysical neutrinos by measuring the flavor ratios as the function of L/E.

1 Introduction

When a fermion has a mass, there should be the mixing of different chiralities, i.e. the mixing of left-handed and right-handed fermions. This is because the speed of a massive particle is less than c (the speed of light), and the chirality observed by an observer whose speed is faster than the particle is opposite to the chirality observed by an observer at rest. The transformation between these two observers is nothing but a Lorentz transformation, and as long as we work in a relativistic theory the mixture of chiralities is inevitable. Namely the fermion mass term causes the mixing between "chiral-partners". The chiral-partner of a left-handed fermion can be independent right-handed fermion. In that case, the massive fermion is called a Dirac fermion. We, however, theoretically have another possibility: the chiral partner of the left-handed fermion can be the anti-particle of the left-handed fermion itself. In this case the mass term causes the violation of fermion-number. This type of massive particle is called a Majorana fermion having a Majorana mass. Such theoretical possibility had no useful application for a long time. But, nowadays in particle physics the concept of Majorana fermion is quite popular and has important physical consequences. In particular, many people expect that the peculiarity of the leptonic sector, i.e. extremely small neutrino masses and unexpectedly large neutrino mixing angles reported by recent neutrino oscillation experiments, probably has its origin in the fact that only neutrinos (among quarks and leptons) may have Majorana masses. This is simply because Majorana mass term, not only breaks fermion-number, but also violates charge conservation for charged particles.

In this occasion to celebrate 100 years of E. Majorana, I would like to discuss so-called "pseudo-Dirac" neutrino, which has a unique clue to the Majorana nature of neutrinos. Let us note that on-going neutrino oscillation experiments cannot distinguish the oscillations of Dirac and (see-saw type) Majorana neutrinos. Only pseudo-Dirac neutrino has quite different predictions for the neutrino oscillations. The essential reason is that in the experiments, the neutrino oscillation with chirality flip is strongly suppressed, while the Majorana mass causes the chirality flip.

Once neutrinos are allowed to have Majorana masses, we may think of three typical cases for neutrino masses depending on the magnitude of the Majorana mass relative to the Dirac mass, which we will discuss successively below. First, let us note that in the base of weak-eigenstates, ψ_{wL}, where active states are put in the up-stairs and "sterile" states (without weak interaction) are put in the down-stairs,

$$\psi_{wL} = \begin{pmatrix} \nu_{\alpha L} \\ \overline{\nu}_{\alpha L} \end{pmatrix} \qquad (\alpha = e, \mu, \tau; \quad \overline{\nu}_{\alpha L} = (\nu_{\alpha R})^C), \tag{1}$$

the neutrino mass term in the 3 generation scheme is generally written as

$$L_{\rm mass} = \frac{1}{2}\psi^t_{wL} C M \psi_{wL}, \tag{2}$$

where C is the charge-conjugation matrix and the 6×6 mass matrix M takes a form of

$$M = \begin{pmatrix} M_L & M_D^t \\ M_D & M_R^* \end{pmatrix}, \tag{3}$$

with the 3×3 matrices M_D, M_L and M_R being those for Dirac masses, left- and right-handed Majorana masses, respectively.

(1) pure Dirac

Imposing lepton number conservation, or ignoring all Majorana masses ($M_L = M_R = 0$), we get pure Dirac neutrinos. We all know that there are only 3 mass-eigenvalues for 3 generations, although the mass matrix M should have 6 eigenvalues, in general. What's really happening is that there are 3 degenerate pairs of mass-eigenstates. To see the situation we consider the simplified 1 generation case

$$M = \begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}. \tag{4}$$

It is easy to see that M has eigenvalues $m_D, -m_D$ and the angle, θ, in the orthogonal matrix to diagonalize M is just $\pi/4$. If processes we are interested in have no chirality flip, as in the case of on-going neutrino oscillations experiments, the sign of mass is irrelevant and we have degenerate mass-squared. It should also be noted that in this case a maximal mixing, $\theta = \pi/4$, between an active state and a sterile state, ν_L and $\overline{\nu}_L$, has been realized. Unfortunately, this maximal mixing does not lead to any neutrino oscillation, just because the mass-squared are degenerated.

(2) "Pseudo-Dirac"

What happens if we allow small lepton number violation, i.e. if we switch on very small Majorana masses, $M_L, M_R \ll M_D$. Neutrinos are still almost Dirac particles and are called "pseudo-Dirac" neutrinos [1]. The small Majorana masses, however, slightly lift the degeneracy of mass-eigenvalues, and we get almost degenerate pairs of eigenstates with tiny mass differences. As far as the Majorana masses are small, the mixing angles should remain almost maximal, $\theta \simeq \pi/4$. To understand the situation, we again consider the 1 generation case. The mass matrix now reads as

$$M = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}, \tag{5}$$

with $m_L, m_R \ll m_D$. We now get $|\tan 2\theta| = |\frac{2m_D}{m_R - m_L}| \gg 1$, leading to almost maximal mixing $\theta \simeq \frac{\pi}{4}$. We have two mass eigenstates, which are almost symmetric and anti-symmetric combinations of active and sterile states, i.e., $\nu_S = \sin\theta\ \nu_L + \cos\theta\ \overline{\nu}_L \simeq \frac{1}{\sqrt{2}}(\nu_L + \overline{\nu}_L)$, and $\nu_A = (-i)(\cos\theta\ \nu_L - \sin\theta\ \overline{\nu}_L) \simeq \frac{1}{\sqrt{2}i}(\nu_L - \overline{\nu}_L)$. Their masses are almost degenerate but are slightly different; $m_S \sim m_A \sim m_D$, $\Delta m \equiv m_S - m_A \sim m_L + m_R \ll m_D$. Now the tiny mass difference and the almost maximal mixing will lead to a neutrino oscillation between an active state and a sterile state, $\nu_L \to \overline{\nu}_L$, surprisingly even if we have only 1 generation.

(3) See-saw
The last possibility is the famous see-saw scenario [2] in which SU(2) invariant Majorana masses, M_R, are supposed to be much larger than the Dirac masses: $M_R \gg M_D$, $M_L \simeq 0$. The sterile states $\overline{\nu}_L$ approximately become mass-eigenstates and are "decoupled" from low energy processes such as neutrino oscillation. Thus only lighter 3 mass-eigenstates ($\simeq \nu_{\alpha L}$) participate in neutrino oscillation phenomena. In the see-saw scenario, therefore, the mixings relevant for the neutrino oscillations are generation mixings and there seems to be no immediate reason to expect large mixing angles. It is also worth noting that as far as chirality preserving neutrino oscillations are concerned, there is no observable distinction between the cases of (1) and (3), since in both cases only 3 light neutrino states participate in the oscillations.

From the above discussion we learn that only in the pseudo-Dirac scenario 6 neutrino states fully participate in low energy processes, and rich neutrino oscillation phenomena, both inter-generational and active $\leftrightarrow$ sterile, are expected. In fact a trial to explain existing data on neutrino oscillations based on the pseudo-Dirac scenario has been made [3]. It should be also stressed that only in the pseudo-Dirac scenario, the Mayorana nature of neutrinos manifests itself, as only in this case lepton-number violating ($|\Delta L| = 2$) neutrino oscillation is observable.

The presence of neutrino oscillation, strongly suggested by the recent Superkamiokande results on atmospheric neutrinos [4], is almost unique experimental clue to the physics beyond the well-established standard model. More precisely, recent data on neutrino oscillations seem to have put forward the following challenging theoretical problems, which may lead to the physics beyond the standard model;
(a) The data from (Super-)Kamiokande on atmospheric neutrinos necessitate a large or almost maximal mixing angle [4]. Solar neutrino deficit may also be explained by the "large angle solution" [5]. How can such

large or maximal mixing be naturally derived theoretically ?
(b) If we further accept LSND result [6], in addition to the solar and atmospheric neutrino data, the scheme with only 3 light neutrino states clearly gets into trouble. What kind of theoretical framework or model is needed to accommodate all of these neutrino oscillations ? It has been argued that we should introduce at least one sterile state.
(c) There seems to be a large disparity among the magnitudes of mixing angles implied by these experiments; The data on atmospheric neutrino and possibly that on solar neutrino indicate the necessity of large or almost maximal mixing angles [4], [5], while the neutrino oscillation at LSND experiment is well described by a small mixing angle [6]. How can such disparity be naturally explained theoretically ?

At least, one can clearly say that these problems altogether suggest that flavour mixing or mass matrices in leptonic sector are quite different from those in quark sector. In particular the presence of large mixings and the necessity of extending the scheme with only 3 light neutrinos are specific new features in the leptonic sector, not shared by the quark sector, and may lead to a drastic modification of the Standard Model.

In this article we will briefly argue how the above problems, (a), (b) and (c), are naturally (without any fine-tuning) solved simultaneously in the scenario of pseudo-Dirac neutrinos. (For the detail of the discussion, refer to the already published paper [3]). It is worth noticing that small generation mixings, similar to those in the quark sector, are shown to be exactly what we need to solve the problems; the nice features of pseudo-Dirac scenario, i.e. the maximal mixings, are utilized in the atmospheric and solar neutrino oscillations, while LSND data is naturally explained by the small generation mixings. In such a sense, we may say that
"The recent data on neutrino oscillations may be natural consequences of the property that lepton number is only slightly violated and generation mixings are small."
We, however, will also point out some serious problem encountered by the atmospheric neutrino oscillation into a sterile state [7].

2 Mass eigenstates and mixings of pseudo-Dirac neutrinos

In this discussion we allow arbitrary generation mixings or arbitrary off-diagonal mass matrices, except for the assumption of pseudo-Dirac property $M_L, M_R \ll M_D$. Thus we may naively expect that diagonalization of 6×6 mass matrix is quite complicated and various formulae for the probabilities of neutrino oscillations are expressed by use of 6 mass-eigenvalues and 15 mixing angles (together with possible multiple CP violating phases). It, however, turns out that under the assumption of pseudo-Dirac nature (small Majorana masses), all probabilities of neutrino oscillations are neatly describable in terms of 6 mass eigenvalues and just one 3×3 unitary matrix U, "MNS matrix" [8], which has three mixing angles and one CP violating phase and just corresponds to the CKM matrix in the quark sector.

To confirm this, let us now discuss the diagonalization of the mass matrix M. We will skip all the details of the argument and present only the conclusion here. It is known that the neutrinos emitted by weak interactions (weak eigenstates) are expressed in terms of 6 mass eigenstates ν_{jS}, ν_{jA} $(j = 1 - 3)$ and a unitary matrix U (MNS matrix), as follows:

$$\nu_{\alpha L} = U_{\alpha j} \frac{\nu_{jS} + i\nu_{jA}}{\sqrt{2}}. \tag{6}$$

The mass-eigenvalues of the mass eigenstates are given as

$$m_{jS}^2 = m_j^2 + m_j|\epsilon_j|, \quad m_{jA}^2 = m_j^2 - m_j|\epsilon_j|, \quad (j = 1, 2, 3), \tag{7}$$

where ϵ_j are small masses originated from the tiny Majorana masses of pseudo-Dirac neutrinos.

The fact that there appears only single 3×3 unitary matrix U, even though we started from an arbitrary 6×6 mass matrix M, is one of the main results.

3 Formulae for neutrino oscillations

We are now ready to derive the formulae for the probabilities of various neutrino oscillations in terms of the differences of 6 mass-squared and single unitary matrix U. Though there is no reason to expect apriori some specific pattern of neutrino masses, we can still get some useful information on the pattern from the reported data on neutrino oscillations [4], [5], [6]. Namely, once we regard the mixing angles in U as small, as suggested by the CKM matrix in the quark sector, the mass-squared difference, responsible for each observed neutrino oscillation, is given as

$$\text{solar neutrino} : m_1|\epsilon_1| \sim 10^{-5} - 10^{-4}(eV^2),$$

$$\text{atmospheric neutrino} : m_2|\epsilon_2| \sim 10^{-3} - 10^{-2}(eV^2),$$

$$\text{LSND} : \Delta m_{12}^2 \sim 10^{-1} - 1(eV^2). \tag{8}$$

This knowledge suggests (with a little prejudice) a hierarchical structure of mass differences

$$m_1|\epsilon_1| \ll m_2|\epsilon_2| \ll m_3|\epsilon_3| \ll \Delta m^2_{12} \ll \Delta m^2_{13}, \tag{9}$$

where $\Delta m^2_{ij} \equiv m^2_j - m^2_i$. The hierarchical structure makes the formulae for neutrino oscillations simple and easy to be compared with the data.

3.1 A general formula for vacuum oscillation

We first note that except for the case of solar neutrino oscillation, both of atmospheric neutrino oscillation and the oscillation in LSND experiment are well described by vacuum oscillations.

In general, the probability of finding a state, born as an "active" state (with weak interaction) $\nu_{\alpha L}$ at time 0, in an active state $\nu_{\beta L}$ at time t is given by

$$P(\nu_\alpha \to \nu_\beta) = \frac{1}{4}|\sum_{j=1}^{3} U_{\beta j}\{\exp(i\frac{m^2_{jS}}{2E}t) + \exp(i\frac{m^2_{jA}}{2E}t)\}U^*_{\alpha j}|^2. \tag{10}$$

By use of this general formula, we derive below simple formulas for atmospheric and LSND neutrino oscillations.

3.2 Formulae for atmospheric neutrino oscillation

As the oscillation of atmospheric neutrino is sensitive to the mass difference $m_2|\epsilon_2| \sim 10^{-3}$ (eV^2) [4], under the mass hierarchy eq.(9) $m_1|\epsilon_1|$ may be ignored and ν_1 can be regarded as pure Dirac particle, i.e. $\nu_1 = \frac{1}{\sqrt{2}}\{(\nu_{1S} + c.c.) + i(\nu_{1A} + c.c.)\}$ with a unique mass $m_{1S} = m_{1A} = m_1$. The oscillation of atmospheric neutrino is due to the interference between ν_{2S} and ν_{2A}, and the matter waves of other states, ν_1, ν_{3S} and ν_{3A}, do not interfere with ν_{2S}, ν_{2A} or with each other, when time-average is taken for the high frequency modes in the oscillation probability. Thus the formula for the probability of atmospheric ν_μ to survive till time t, relevant for the zenith angle distribution, simply reads as

$$P(\nu_{\mu L} \to \nu_{\mu L})_{\text{atm}} = |U_{\mu 1}|^4 + |U_{\mu 2}|^4 \cos^2(\frac{m_2|\epsilon_2|}{2E}t) + \frac{1}{2}|U_{\mu 3}|^4. \tag{11}$$

Let us note that there are constant terms $|U_{\mu 1}|^4$, $|U_{\mu 3}|^4$ coming from the time-average of the high frequency modes, in sharp contrast to the conventional formula in a simplified 2 states system.

3.3 Formula for LSND neutrino oscillation

As the neutrino oscillation observed by LSND is sensitive to the mass difference $\Delta m_{12}^2 \sim 10^{-1}{-}1$ (eV^2) [6], under the hierarchy (9) all neutrino states can be regarded as pure Dirac particles, i.e. $m_i|\epsilon_i| = 0$ and $m_{iS}^2 = m_{iA}^2 = m_i^2$ $(i = 1, 2, 3)$. The LSND neutrino oscillation is, therefore, due to the interference between ν_1 and ν_2, and the matter waves of another state, ν_3, does not interfere with ν_1, ν_2, when time-average is taken. Thus the formula for the transition probability of LSND neutrino simply reads as

$$\begin{aligned} P(\nu_{\mu L} \to \nu_{eL})_{\mathrm{LSND}} &= |U_{\mu 1}|^2|U_{e1}|^2 + |U_{\mu 2}|^2|U_{e2}|^2 + |U_{\mu 3}|^2|U_{e3}|^2 \\ &\quad +2\mathrm{Re}(U_{\mu 1}U_{e1}^*U_{\mu 2}^*U_{e2})\cos(\frac{\Delta m_{21}^2}{2E}t) \\ &\quad -2\mathrm{Im}(U_{\mu 1}U_{e1}^*U_{\mu 2}^*U_{e2})\sin(\frac{\Delta m_{21}^2}{2E}t) \qquad (12) \\ &= 4\{|U_{\mu 2}U_{e2}|^2 + \mathrm{Re}(U_{\mu 2}^*U_{e2}U_{\mu 3}U_{e3}^*)\}\sin^2(\frac{\Delta m_{21}^2}{4E}t) \\ &\quad +2\mathrm{Im}(U_{\mu 2}^*U_{e2}U_{\mu 3}U_{e3}^*)\sin(\frac{\Delta m_{21}^2}{2E}t) + 2|U_{\mu 3}U_{e3}|^2. \end{aligned}$$

4 Pseudo-Dirac scenario confronted by the data on neutrino oscillations

4.1 Comparing the formulae with the data

As the application of the formulae we have derived for the neutrino oscillations, we are now going to compare them with the corresponding experimental data. (The comparison with the date of solar neutrino oscillation is not made here.) As we have already advertised we will see that small generation mixing angles in the unitary matrix U is just what we need to explain the pattern of mixing angles observed in neutrino oscillations of our interest.

Thus we first consider the case where generation mixings are small, though the formulae we have derived above are applicable for arbitrary generation mixings. Retaining only the leading contributions for small generation mixing angles (say θ_1, θ_2, θ_3 just as the angles in CKM matrix), we get the following formulae, relevant for each neutrino oscillation

$$\text{atmospheric neutrino}: \quad 1 - P(\nu_{\mu L} \to \nu_{\mu L})_{\mathrm{atm}} \simeq \sin^2(\frac{m_2|\epsilon_2|}{2E}t),$$

$$P(\nu_{eL} \to \nu_{eL})_{\rm atm} \simeq 1,$$
$$P(\nu_{\mu L} \to \nu_{eL})_{\rm atm} \simeq 0, \quad (13)$$
$$\text{LSND} : \quad P(\nu_{\mu L} \to \nu_{eL})_{\rm LSND} \simeq 4|U_{e2}|^2 \sin^2(\frac{\Delta m^2_{21}}{4E}t). \quad (14)$$

In the formula for LSND $|U_{e3}U_{\mu 3}|$ has been neglected compared with $|U_{e2}U_{\mu 2}|$, as is suggested by the hierarchical mixing angles, $\theta_1 \gg \theta_2 \gg \theta_3$, in quark sector.

In the case of atmospheric neutrino oscillation, the factor in front of $\sin^2(\frac{m_2|\epsilon_2|}{2E}t)$ is $1 = \sin^2(2 \times \frac{\pi}{4})$, which just corresponds to a vacuum oscillation with maximal mixing, strongly suggested by the data on atmospheric neutrinos [4]. In a contrary, in the case of LSND our formula gives that of ordinary 2 generation scheme with small generation mixing, if we identify $4|U_{e2}|^2$ with $\sin^2 2\theta_1$. This is just consistent with the experimental data [6], which says $\sin^2 2\theta_1 \leq 0.04$, when combined with the data from BUGEY experiment.

4.2 Problems to be settled

We have seen that the pseudo-Dirac scenario just provides the favored solutions to the atmospheric neutrino problems with (almost) maximal mixings, suggested by the recent data, invoking the oscillations mainly into sterile states, while LSND data is naturally explained by ordinary generation mixing between active states with a small mixing. However, the recent data seem to regard the maximal mixing solution of neutrino oscillation into a sterile state with disfavour [7]. Namely, though we have neglected the matter effect in the atmospheric neutrino oscillation, the matter effect of the Earth becomes non-negligible for higher neutrino energies. It has been pointed out that ν_μ oscillations into ν_τ and a sterile state have different zenith-angle dependence, as only in the case of the oscillation into the sterile state the matter effect affects the time-evolution of the neutrino states. Comparing with the data, combining the analysis of neutral current enriched events, the Super-Kamiokande collaboration claims that the oscillation into the sterile state with maximal mixing is regarded with disfavour [7]. One possible way to evade this problem, we can think of, is the following. When the oscillation to the sterile state is analyzed, simplified 2 states system of (ν_μ, ν_S) (ν_S denoting a sterile state) is assumed. In the scenario of pseudo-Dirac, however, we have 6 neutrino states to participate in the oscillation, and the formula for the oscillation, as seen in Eq.(11), is different from that in the simple 2 states system, typically having additional constant terms (for non-vanishing generation mixings). Including the matter effect of the Earth, our formula is modified into

$$\overline{P}(\nu_{\mu L} \to \nu_{\mu L})_{\rm atm} = |U_{\mu 2}|^4 \; \overline{P}(\nu_{2L} \to \nu_{2L})_{\rm eff} + \; |U_{\mu 1}|^4 + \frac{1}{2} \; |U_{\mu 3}|^4, \quad (15)$$

where the survival probability $\overline{P}(\nu_{2L} \to \nu_{2L})_{\rm eff}$ in the effective 2 states system $(\nu_{2L}, \; \overline{\nu}_{2L})$ is that for the mass-squared difference 2 $m_2 \epsilon_2$ with the matter effect being included. Thus both the depletion rate of atmospheric ν_μ and the zenith angle dependence should be reanalyzed by use of this formula before some definite conclusion is derived.

5 Another application

As another application of the pseudo-Dirac neutrinos, the authors of Ref. [9] have discusses the possibility to observe the pseudo-Dirac nature of neutrinos in the neutrino telescopes.

They assume that pseudo-Dirac mass splittings $\delta m_i^2 = m_i|\epsilon_i|$ are too small to play any roles for solar and atmospheric oscillations, and they also do not attempt to account for the LSND data. Instead, they argue that the very small δm_i^2 up to 10^{-18} (eV^2) can be searched for, looking at high energy astrophysical neutrinos by measuring their flavor ratios as the function of L/E (L is the distance from the source of the high energy neutrino to the detector and E is the neutrino energy).

Without pseudo-Dirac mass splitting, neutrinos produced through π decay at the source have the following flavor ratios (with $\theta_{atm} \simeq \frac{\pi}{4}$ and $\theta_{sun} \simeq \frac{\pi}{6}$)

$$\nu_e : \nu_\mu : \nu_\tau \simeq 1 : 1 : 1. \quad (16)$$

They argue that with very tiny pseudo-Dirac mass splitting , the ratio

$$\nu_e : \nu_\mu = \nu_\tau \quad (17)$$

will change as the function of L/E, depending on δm_i^2, with the ratios being calculable by use of the general formula we obtained above. If the modification of the flavor ratios is observed, it clearly signals the presence of Majorana mass, or the lepton number violation, which is otherwise quite difficult to check (except for the neutrino-less double beta decay search).

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