Mass Protection and No Fundamental Charges Requires Extra Dimensions

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ABSTRACT. We call attention to that assuming no conserved charges in the fundamental theory, but rather only gravity and fermions with only a spin, the dimensions 4, 12, 20, ... are excluded under the requirement of mass protection. If it is required that we shall have several families the generic result is that even the other by 4 devisable dimensions are excluded and indeed only d=2 (mod 4) remains as acceptable.

1 Introduction

Looking at the Standard model of the electroweak and colour interaction, it is well known that it is strongly suggested that all the particles in this model - except for the Higgs particle itself - are a priori massless and only obtain masses different from zero by means of the interaction with the Higgs field (its vacuum expectation value). Especially the fermions - quarks and leptons - are in this sense mass protected, because their fields are composed from (a couple of) Weyl fields with such quantum numbers under the "weak" gauge transformations that a mass term is forbidden unless the interaction with the Higgs field vacuum expectation value causes the mass term, that means that the Higgs field "dresses" the right handed weak chargeless fermions with a weak charge.

The main point to which we call attention here is, however, that this mass protection *only works when there exists in the theory a* (for instance gauge) *symmetry ensuring the conservation of one or several charges distinguishing the right and the left handed Weyl-components.* Here, however, we want to look for, how it would go if we assume that there exists no such charge conservation a priori, or let us say at the fundamental level. Indeed if the charge conservation were broken, then there would be the allowance/possibility for a mass term being added to the Lagrange density. Even for the left handed neutrino, which in the Standard model does not have any right handed partner making it able to obtain a Dirac mass term, there is the possibility of a Majorana term provided the conservation laws are broken.

Really the possibility of Majorana terms is so general that we quite generally can claim that there is no way to get mass protection for any (Weyl) fermion without use of the charge conservation in the usual 3+1 dimensions. However, this statement is dimension-dependent, and the point is that we shall see that the dimensions which modulo 8 are equivalent to 4 are excluded under the assumption of there being no charge conservation imposed. Actually we shall see that it is even so that the potential Majorana mass terms for Weyl particles in dimensions divisible by 8 is only excluded even for no charge conservation because of the potential term having to be symmetric under the permutation of the (second quantized) fermion fields. Because of the Fermi statistics such a symmetry is not allowed and thus the mass protection at first looks to be possible, but this is only true as long as there is only one family/flavour. When there are more families of Weyl particles however even in by 8 divisible dimensions only one of these families will survive as massless. So if we insist on the number of families being mass protected should be bigger than one even these by 8 divisible dimensions are excluded under the assumption of no conservation of charges.

Since it is well known and rather trivial that there cannot be mass protection in an odd number of space-time dimensions[1, 2], since in odd dimensions Weyl spinors are at the same time Dirac spinors, we are at the end left with that under the assumption of there being no conserved charges we can only have mass protection of more than one family for the dimensions d = 2 (mod 4); i.e. for such dimension numbers as 2,6,10,14,...

One can of course complain against this discussion by saying that it is quite contrary to what we know to assume that there are no conserved charges: One of us have long worked on the speculation that the conservation laws which we conceive of as charges phenomenologically are at a deeper level to be identified with angular momenta in the extra dimensions[3, 4, 5, 6] (more precisely with spin degrees of freedom). Taken the generic point of view of not taking there to be a charge conservation without reason we see that one is first driven to some dimension having 2 as remainder when divided by four and then in order to obtain the phenomenologically known charges a scheme like the one mentioned of obtaining them as angular momenta (spin) would be highly suggested.

2 Massless and massive fermions

Let d be any integer number. We pay attention to only spinors (fermions). We see that:
i) In d even the operator of handedness Γ^d $((\Gamma^d)^2 = I,\ \Gamma^{d\dagger} = \Gamma^d)$ is a Clifford even operator, proportional to an even number of all the Clifford odd objects γ^a and can accordingly be expressed as a product of all the $d/2$ elements of the Cartan subalgebra set of the Lorentz generators S^{ab} for spinors. One finds

$$\{\Gamma^d, \gamma^a\}_+ = 0, \quad \{\Gamma^d, S^{ab}\}_- = 0, \quad \text{d even}. \tag{1}$$

ii) For d an odd number the handedness is a Clifford odd operator, proportional to a product of an odd number of γ^a, which means that it can be chosen to be proportional to the product of all the $(d-1)/2$ members of the Cartan subalgebra set of the commuting generators of the Lorentz group and one of γ^a, say γ^d. This means that Γ^d changes the Clifford character of a state, if being applied on a state of a definite Clifford character. One finds

$$\{\Gamma^d, \gamma^a\}_- = 0, \quad \{\Gamma^d, S^{ab}\}_- = 0, \quad \text{d odd}. \tag{2}$$

The mass term has in a second quantized formalism a form

$$-m\hat{\mathbf{\Psi}}^\dagger \gamma^0 \hat{\mathbf{\Psi}}, \tag{3}$$

where $\hat{\mathbf{\Psi}}$ is the operator annihilating a fermionic state

$$< 0|\hat{\mathbf{\Psi}}(x)|\Psi_k >= \Psi_k(x). \tag{4}$$

2.1 Spinors with no charge and not more than one family

We assume that a spinor carry nothing but a spin and that only a Weyl of one handedness and one family index exists.

Statement 1: There is no Dirac mass term for d even and it is always a Dirac mass term for d odd.

Proof: The proof is straightforward and well known. Namely, choosing only one handedness, say left, the mass term has a form $-m((1-\Gamma^d)\hat{\mathbf{\Psi}})^\dagger\gamma^0((1-\Gamma^d))\hat{\mathbf{\Psi}} = -m\hat{\mathbf{\Psi}}^\dagger(1-\Gamma^d)\gamma^0(1-\Gamma^d)\hat{\mathbf{\Psi}}$, which is zero (because of the factor $\gamma^0(1+\Gamma^d)(1-\Gamma^d)$) for d even and nonzero (the factor is now $\gamma^0(1-\Gamma^d)$) for d odd, due to Eqs. (1,2).

Statement 2: There is no Majorana mass term for $d = 2(2k+1)$, $k = 0, 1, 2, \cdots$ and is also no Majorana mass term for $d = 8k$, $k = 1, 2, \cdots$, for only one family of spinors. In all other dimensions there is always the Majorana mass term.

We shall prove this statement in several steps.

Let $\hat{\mathcal{C}}$ be a charge conjugation operator, operating on $\hat{\mathbf{\Psi}}$. Then the creation operator creating a Majorana state out of any left handed state, is defined as follows

$$\hat{\mathbf{\Psi}}_M = \frac{1}{\sqrt{2}}(\hat{\mathcal{C}}(1-\Gamma^d)\hat{\mathbf{\Psi}}\hat{\mathcal{C}}^{-1} \pm (1-\Gamma^d)\hat{\mathbf{\Psi}}). \tag{5}$$

Let us write $d = 4n + 2m$ for d even and $d = 4n + 2m - 1$ for d odd, with $n = 1, 2, \cdots$, and with $m = 0, 1$ only.

Statement 2. a: The following relation holds

$$\hat{\mathcal{C}}(1-\Gamma^d)\hat{\mathbf{\Psi}}\hat{\mathcal{C}}^{-1} = (-1)^{n-1+m} \prod_{Im\ \gamma^a} \gamma^a \ ((1-\Gamma^d)\hat{\mathbf{\Psi}})^{\dagger}. \tag{6}$$

(The index $Im\ \gamma^a$ under the product means that one takes the product over those γ^a-matrices which have only imaginary matrix elements.) This statement is proven in the subsect. 2.3.

We make a choice of γ^a so that the first two (γ^0, γ^1) are real, γ^2 is imaginary, γ^3 is real, (we skip index 4) γ^5 is imaginary and all the rest γ^a with a even are real and those with a odd are imaginary. We have $(\gamma^a)^{\dagger} = \eta^{aa}\gamma^a$ and $\eta^{aa} = diag(1, -1, -1, \cdots)$. (Accordingly γ^0 is a symmetric $2^{(d/2-1)} \times 2^{(d/2-1)}$ matrix and so are all the imaginary γ^a's, while the real γ^a matrices (except γ^0) are antisymmetric.) One finds that the number of imaginary γ^a in the product $\prod_{Im\gamma^a} \gamma^a$ is for d even equal to $\frac{d-2}{2}$ and for d odd $\frac{d-1}{2}$. To see in which dimensions the Majorana mass term

$$-m\hat{\mathbf{\Psi}}^{\dagger}_M \gamma^0 \hat{\mathbf{\Psi}}_M, \tag{7}$$

gives zero, if spinors of only one handedness and one family are assumed, we must evaluate the part

$$-(\pm)\, m\{(\hat{\mathcal{C}}(1-\Gamma^d)\hat{\mathbf{\Psi}}\hat{\mathcal{C}}^{-1})^\dagger\gamma^0\, ((1-\Gamma^d)\hat{\mathbf{\Psi}}) + ((1-\Gamma^d)\hat{\mathbf{\Psi}})^\dagger\, \gamma^0(\hat{\mathcal{C}}(1-\Gamma^d)\hat{\mathbf{\Psi}}\hat{\mathcal{C}}^{-1})\} =$$
$$= -(\pm)\, m\{\hat{\mathbf{\Psi}}(1-\Gamma^d)(\prod_{Im\gamma^a}\gamma^a)^\dagger\,\gamma^0\,(1-\Gamma^d)\hat{\mathbf{\Psi}}$$
$$+ (\hat{\mathbf{\Psi}})^\dagger(1-\Gamma^d)\gamma^0\,(\prod_{Im\gamma^a}\gamma^a)(1-\Gamma^d)\hat{\mathbf{\Psi}}^\dagger\}(-)^{(n-1+m)}. \tag{8}$$

It is easy to see that the term of Eq.(8) is zero for $d = 2(2k+1)$, for $k = 0, 1, 2, \cdots$, since $(1-\Gamma^d)\ (\prod_{Im\gamma^a}\gamma^a)^\dagger\gamma^0(1-\Gamma^d) = (\prod_{Im\gamma^a}\gamma^a)^\dagger\gamma^0(1+\Gamma^d)(1-\Gamma^d) = 0$ and similarly also the term $(1-\Gamma^d)\ \gamma^0(\prod_{Im\gamma^a}\gamma^a)(1-\Gamma^d) = 0$, namely in even dimensional spaces Γ^d anticommutes with a product of an odd number of γ^a's (and it commutes with a product of an even number of γ^a's). For $d = 8k$, however, we still get a zero contribution, due to the fact that the two terms: the term $(\prod_{Im\gamma^a}\gamma^a)^\dagger\gamma^0\ (1-\Gamma^d)$ and his Hermitean conjugate one $\gamma^0\prod_{Im\gamma^a}\gamma^a\ (1-\Gamma^d)$ cancel each other.

We can conclude: *In $d = 2(mod\ 4)$ and $d = 0(mod\ 8)$ dimensions, if we only have a Weyl of one handedness and one family and no charges, there is no Dirac mass term and also no Majorana mass term.*

In all the odd dimensions d the Dirac term by itself gives a nonzero contribution what ever the Majorana term is. But since in odd dimensional spaces Γ^d commutes with any product of γ^a, also Eq.(7) gives a nonzero contribution.

2.2 Spinors with no charge appearing in families

If we allow a family index without any special requirement about global or local symmetries of our Lagrangean with respect to the family index, it will in general happen that in $d = 0(mod\ 8)$ dimensions the two terms of Eq.(8) do not cancel each other, so that the mass term of the Majorana type of Eq.(5) will be non zero.

Really one should imagine that we have a Majorana like mass term given by a matrix in the space of families m^M_{fg} of the form

$$-m^M_{fg}\hat{\mathbf{\Psi}}^\dagger_{Mf}\gamma^0\hat{\mathbf{\Psi}}_{Mg}, \tag{9}$$

and it is seen that the cancellation takes place for the symmetric part of the family matrix m^M_{fg}. This means that if the number of families is *odd* there has to be a zero-eigenvalue for the antisymmetric part of this matrix and thus a single family will survive to have zero mass.

We can conclude: *It is only $d = 2(mod\ 4)$ dimensional spaces that Weyl spinors of one handedness, no charge and more than one family index are mass protected, having no Dirac mass term and also no Majorana mass term.*

2.3 The proof for the statement 2a

i. Let $X^p_> := \{\Phi_{k>}\}$ be a set of p occupied single particle states above the Dirac sea and let $X^p_< := \{\Phi_{l<}\}$ be a set of r holes in the Dirac sea.
ii. Let us make a choice of any phase for $\Phi_{k>}$, while we choose phases for $\Phi_{l<}$ so that

$$C\Phi_{l>} = \Phi_{l<}, \quad C = (\prod_{Im\gamma^a} \gamma^a)\ K, \tag{10}$$

with an odd number of γ^a's in $d = 0(mod\ 4)$ and $d = 0(mod\ 4) - 1$ and with an even number of γ^a's in $d = 2(mod\ 4)$ and $d = 0(mod\ 4) - 1$. K is an antilinear operator, transforming a complex number into its complex conjugate one. Then

$$\begin{aligned} &C^2\Phi_{k>}{=}C\Phi_{n<} = (-)^{n-1+m}\Phi_{n>}, \\ &\text{with } d{=}4n + 2m - 1, \text{ for } d \text{ odd}, \\ &\text{and } d{=}4n + 2m, \text{ for } d \text{ even}, \ m = 0, 1; n = 0, 1, 2, \cdots . \end{aligned} \tag{11}$$

We find $X^{p+1}_> = X^p_> U\{\Phi_{k>}\} = \Phi_{1>}, \Phi_{2>}, \cdots, \Phi_{k>}, \Phi_{p>}.(-)^{a_k X^p_>}$, with $a_k X^p_> = 0$, if $\Phi_{l>}$ jumps over an even number of $\Phi_{i>}$ and $a_k X^p_> = 1$, if $\Phi_{k>}$ jumps over an odd number of $\Phi_{i>}$. $X^p_> U\{\Phi_{k>}\} = 0$ for $\{\Phi_{k>}\} \in X^p_>$. It goes equivalently for $X^r_< U\{\Phi_{r<}\}$. We further find

$$\hat{\mathcal{C}}|X^p_>, X^r_< >= (-)^{(m+n-1)r}(-)^{pr}|X^r_<, X^p_> > . \tag{12}$$

We define the creation operator for a spinor (fermion) state $\Phi_{k>}$ above the Dirac sea as follows

$$\begin{aligned} &\hat{\mathbf{b}}^\dagger_{\Phi_{k>}}|X^p_>, X^r_< >=|X^p_> U\{\Phi_{k>}\}, X^r_< >= (-)^{a_k X^p_>}|X^{p+1}_>, X^r_< >, \text{ if } \Phi_{k>} \notin X^p_>, \\ &\hat{\mathbf{b}}^\dagger_{\Phi_{k>}}|X^p_>, X^r_< >=0, \text{ if } \Phi_{k>} \in X^p_>. \end{aligned} \tag{13}$$

We define the creation operator for a hole state, that is the annihilation operator which annihilates the state $\Phi_{l<}$ in the Dirac sea as follows

$$\begin{aligned} \hat{\mathbf{b}}_{\Phi_{l<}}|X^p_>, X^r_< >&=(-)^p|X^p_>, X^r_< U\{\Phi_{l<}\} >= \\ &=(-)^{a_l X^r_< + p}|X^p_>, X^{r+1}_< >, \text{ if } \Phi_{l<} \notin X^r_<, \\ \hat{\mathbf{b}}_{\Phi_{l<}}|X^p_>, X^r_< >&=0, \text{ if } \Phi_{l<} \in X^r_<. \end{aligned} \tag{14}$$

Equivalently we define the annihilation operator annihilating a spinor state $\Phi_{k>}$ above the Dirac sea as

$$\begin{aligned}&\hat{\mathbf{b}}_{\Phi_{k>}}|X^p_>, X^r_< >=(-)^{a_k X^p_>}|X^{p-1}_>, X^r_< >, \text{ if } \Phi_{k>}\in X^p_>,\\ &\hat{\mathbf{b}}_{\Phi_{k>}}|X^p_>, X^r_< >=0, \text{ if } \Phi_{k>}\notin X^p_>, \end{aligned}\tag{15}$$

while we define the annihilation operator annihilating a hole in the Dirac sea by

$$\begin{aligned}&\hat{\mathbf{b}}^{\dagger}_{\Phi_{l<}}|X^p_>, X^r_< >=(-)^{a_l X^r_< +p}|X^p_>, X^{r-1}_< >, \text{ if } \Phi_{k>}\in X^r_>,\\ &\hat{\mathbf{b}}^{\dagger}_{\Phi_{l<}}|X^p_>, X^r_< >=0, \text{ if } \Phi_{l<}\notin X^r_<. \end{aligned}\tag{16}$$

Statement 2. b. 1:

$$\hat{\mathcal{C}}\;\hat{\mathbf{b}}_{\Phi_{k>}}=\hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}}\;\hat{\mathcal{C}}.\tag{17}$$

Proof: Let $X^p_>$ does include the state $\Phi_{k>}$. One then finds that $\hat{\mathcal{C}}\;\hat{\mathbf{b}}_{\Phi_{k>}}|X^p_>, X^r_< >= \hat{\mathcal{C}}\;(-)^{a_k X^p_>}\;|X^{p-1}_>, X^r_< >= (-)^{(n-1+m)r}\;(-)^{a_k X^p_>}$ $(-)^{r(p-1)}|X^r_<, X^{p-1}_> > = (-)^{(n-1+m)r}(-)^{a_k X^p_>}\;(-)^{r(p-1)}(-)^{-a_k X^p_>}\;(-)^r$ $\hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}}\;|X^r_<, X^p_> >= \;\hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}}\;\hat{\mathcal{C}}\;|X^p_>, X^r_< >$, which completes the proof, since if $X^p_>$ does not include the state $\Phi_{k>}$, the very left hand side before the first equality sign is zero and so is evidently zero on the right hand side one equality sign before the last.

Statement 2. b. 2:

$$\hat{\mathcal{C}}\;\hat{\mathbf{b}}_{\Phi_{k<}}=(-)^{(n-1+m)}\hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}}\;\hat{\mathcal{C}}.\tag{18}$$

Proof: Let $X^r_<$ does not include the state $\Phi_{k<}$. One then finds that $\hat{\mathcal{C}}\;\hat{\mathbf{b}}_{\Phi_{k<}}|X^p_>, X^r_< >=\hat{\mathcal{C}}\;(-)^{a_k X^r_< +p}\;|X^p_>, X^{r+1}_< >= (-)^{(n-1+m)(r+1)}\;(-)^{a_k X^r_< +p}$ $(-)^{(r+1)p}|X^{r+1}_<, X^p_> > = (-)^{(n-1+m)(r+1)}\;(-)^{a_k X^r_< +p}\;(-)^{(r+1)p}\;(-)^{-a_k X^r_<}$ $\hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}}\;|X^r_<, X^p_> >= (-)^{n-1+m}\;\hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}}\;\hat{\mathcal{C}}\;|X^p_>, X^r_< >$, which completes the proof.

Statement 2. b. 3:

$$\hat{\mathcal{C}}\;\hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}}=\hat{\mathbf{b}}_{\Phi_{k<}}\;\hat{\mathcal{C}}.\tag{19}$$

Proof: The proof goes equivalently as in the case of Statements 2. b. 1 and 2. b. 2.

Statement 2. b. 4:

$$\hat{\mathcal{C}}\;\hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}}=(-)^{(n-1+m)}\hat{\mathbf{b}}_{\Phi_{k>}}\;\hat{\mathcal{C}}.\tag{20}$$

Proof: The proof goes equivalently as in the case of Statements 2. b. 1 and 2. b. 2.

One finds by repeating the operation with $\hat{\mathcal{C}}$ two times that

$$\hat{\mathcal{C}}^2\ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k>,<}}=(-)^{(n-1+m)}\hat{\mathbf{b}}^{\dagger}_{\Phi_{k>,<}}\ \hat{\mathcal{C}}^2. \tag{21}$$

And we have for the vacuum state

$$\hat{\mathcal{C}}^2\ |X^{p=0}_{>}, X^{r=0}_{<}>=|X^{p=0}_{>}, X^{r=0}_{<}>. \tag{22}$$

We can now write for the operator $\hat{\mathbf{\Psi}}(x)$ the relation

$$\begin{aligned}\hat{\mathbf{\Psi}}(x) &= \sum_{k_<} \Psi_{k<}(x)\ \hat{\mathbf{b}}_{\Phi_{k<}} + \sum_{k_>} \Psi_{k>}(x)\ \hat{\mathbf{b}}_{\Phi_{k>}},\\ \hat{\mathbf{\Psi}}^{\dagger}(x) &= \sum_{k_<} \Psi^{\dagger}_{k<}(x)\ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}} + \sum_{k_>} \Psi^{\dagger}_{k>}(x)\ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}},\end{aligned} \tag{23}$$

with $\Psi^{\dagger}_{k<,k>}(x) = \Psi^{*}_{k<,k>}(x) = K\Psi_{k<,k>}(x)$, that is complex conjugation.

We further find that (using the relation $C\Psi_{k>} = \Psi_{k<}$ and $C\Psi_{k<} = (-)^{n-1+m}\ \ \Psi_{k>}$ (Eq.(11))) $\hat{\mathcal{C}}\ \ \hat{\mathbf{\Psi}}(x) = \sum_{k<}\ \Psi_{k<}(x)\ \ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}}\ \ (-)^{n-1+m}$ $\hat{\mathcal{C}} + \sum_{k_>}\ \ \Psi_{k>}(x)\ \ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}}\ \ \hat{\mathcal{C}}\ \ = (-)^{n-1+m}\ \ [\sum_{k<}\ \ (C\Psi_{k>}(x))\ \ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k>}}$ $+\sum_{k_>}\ (C\Psi_{k<}(x))\ \ \hat{\mathbf{b}}^{\dagger}_{\Phi_{k<}}]\ \hat{\mathcal{C}}$ so that it follows

$$\hat{\mathcal{C}}\ \hat{\mathbf{\Psi}}(x) = (-)^{n-1+m}\sum_{\text{all } k}(C\Psi_k(x))\ \hat{\mathbf{b}}^{\dagger}_{\Phi_k}\ \hat{\mathcal{C}}, \tag{24}$$

Since (Eq.(10,11)) $C\Psi(x) = \prod_{Im\gamma^a}\ \gamma^a\ \ K\Psi(x)$, we have $K\Psi_{k<,k>}(x)$ $= (\prod_{Im\gamma^a}\ \gamma^a)^{-1}\ \ C\Psi_{k<,k>}(x)$ and accordingly

$$\hat{\mathcal{C}}\ \hat{\mathbf{\Psi}}(x) = (-)^{n-1+m}\ (\prod_{Im\gamma^a}\gamma^a)\ \hat{\mathbf{\Psi}}^{\dagger}(x)\ \hat{\mathcal{C}}, \tag{25}$$

from where the relation

$$\hat{\mathcal{C}}\hat{\mathbf{\Psi}}\hat{\mathcal{C}}^{-1} = (-1)^{n-1+m}\ (\prod_{Im\gamma^a}\gamma^a)\ \hat{\mathbf{\Psi}}^{\dagger} \tag{26}$$

follows, proving the Statement 2a of Eq.(6), since the Eq.(26) holds also for $(1-\Gamma^d)\hat{\mathbf{\Psi}}$, accordingly relating it with $((1-\Gamma^d)\hat{\mathbf{\Psi}})^{\dagger}$.

3 Conclusions

We have pointed out in this paper that the mass protection mechanism - put into the Standard model of the electroweak and colour interaction "by hand" with the assumption that only the left handed spinors carry the weak charge while the right handed spinors are weak chargeless - *only works in d is* $(1+3)-$*dimensional space, when there exist in the theory a symmetry ensuring the conservation of one or several charges distinguishing the right and the left handed Weyl-components.* If a spinor of only one handedness carries no charge and no more than one family index, then the mass protection mechanism works only for $d = 2(2n+1)$ and $d = 8n$, for any n. If a spinor of only one handedness and no charge has more than one family then the mass protection mechanism works only in dimensions $d = 2(2n+1)$. Otherwise one gets in a generic case only one massless family and that even only in the odd number of families case.

To appreciate this observation as a significant one, one should think in the philosophy put forward through many years by one us[3, 4, 5, 6]: It would be a nice simplification of the theory if instead of many gauge fields put "by hand" into the model explicitly only the gravity would exist and spinors would carry nothing but the spin and the momentum.

If one now in this philosophy nevertheless asks for having mass protection so that one can get particles with a zero or low mass to be observed by the physicists that have compared to say the Planck scale only very low energy accelerators at their disposal, then he has to ask for how can one get mass protected fermions - let us say a Weyl spinor - without any conserved charge which assigns differently to right and left components. This was the question we really addressed above, and the result turned out to be that *one would need a different dimension from the experimental* $1+3$ *one, namely if one wants to end up with more than one family (as we need phenomenologically) a space time dimension* must be $d = 2(2n+1)$.

In other words: taking mass protection and no fundamental conserved charges roughly speaking as the guiding principles one is driven to fundamentally there being a number of dimensions which is equivalent to 2 modulo 4.

Of course it can very well be that in the rough sense needed here the gauge symmetries of the Standard model are fundamental - and not what is here really what not fundamental in this rough sense means, Kaluza-Klein, - since so it is e.g. in string theories or it could appear in many other ways. However, in a sense it complicates the model if we anyway have to have gravity in the theory and gravity could via the Kaluza Klein mechanism produce the gauge fields, to then put in explicit gauge fields.

Now it must however be admitted that Kaluza-Klein-like theories suffer severely from a "no-go" theorem by Witten[10] which essentially says that Kaluza-Klein-like theories cannot manifest mass protected fermions in the $1 + 3$ dimensions. We want to mention here, however, our work on a toy model, in which a spinor in $1 + 5$-dimensional space manifests its masslessness in $d = 1 + 3$ space and yet chirally couples through a Kaluza-Klein charge, which is indeed the spin in $d - 4$ space, to the corresponding Kaluza-Klein gauge field, if an appropriate boundary takes care of the properties of a spinor[9], overtaking accordingly the "no-go" theorem. But should these troubles be overcome we better use a model with - since it should of course have at least 4 dimensions - 6, 10, 14, 18, ... dimensions.

The works of one of us has since long had some success with 14 dimensions.

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