

# Majorana Neutrino and New Space-Time Geometry

GUENNADI G. VOLKOV

On leave from PNPI, Russia. E-mail:Guennadi.volkov@cern.ch

Ettore Majorana (05.08.1906- )  
was one of the last genius of  
the romantic geometric period  
starting at the XIX century  
and ending at the first half  
of the XX century

**ABSTRACT.** The existence of the Majorana fermion matter [1] in nature can help us in understanding the Lorentz symmetry and the matter-antimatter symmetry. It can provide a new tool to study a geometrical origin of the gauge symmetries of the Standard Model. The Majorana nature of neutrino can be related with a new ternary symmetry which can also help to solve the 3-quark-lepton family problems. Based on the primordial Majorana fermion matter we discuss a possibility to solve the baryo-genesis problem through the Majorana-Diraco genesis in which we have a chance to understand a creation of  $Q_{em}$  charge and its conservation in our  $D = 1 + 3$  universe after Big Bang. In Majorana-Diraco genesis approach, there is the possibility of the proton and electron non-stability on the very low energy scale. In cosmology the Majorana-Diraco genesis could be useful to study new space-time symmetry which can give a completely new insight on the the dark matter and dark energy problems. The new ambient geometry can be related to a new space-time symmetry leading at high energies to generalization of the special theory of relativity.

## 1 Photon, electron/positron in discovery D=3+1 space-time

At the end of the XIX century the long efforts of many civilizations the study of the electromagnetism and light has been finished by Maxwell [2] who wrote his fundamental equations. Then Lorentz found that these equations are satisfied to the symmetry, which has got his name. Based on this symmetry Minkowsky suggested the geometry for our world, Minkowsky space-time, which became the fundamental corner for the quantum physics. Thus, shortly one can say that the photon extended our geometrical representations about ambient space-time structure of our world. More precisely, it was conjectured by Minkowsky that our world is 4-dimensional Riemannian homogeneous space-time geometrical object with the structure described by the metrics  $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ . Thus, shortly one can say that the study of the electromagnetism + light and its Grand Unification in the Maxwell equations extended our representations about ambient geometrical structure of our world from dimension  $D = 3$  up to dimension  $D = 4 = 3 + 1$ .

Later theory of electron/anti-electron just confirmed this conjecture [3]. The spin structure of electron can be described by spinor wave function described by  $SU(2)$  group symmetry which is double covering group of the  $SO(3)$  group symmetry of the spatial part of our space [4]. The extension of the external  $SO(3)$  group up the Lorentz group  $SO(1, 3)$  with its double covering group  $SL(2, C)$  gave two additional degree of freedoms in the spinor wave function which has been interpreted by Dirac as the states of the anti-particles [3] what was nicely confirmed by experiments. The theory of light and electron/anti-electron has been unified into the quantum electrodynamics, based on the space-time Poincare'/Lorentz symmetry and  $U(1)_{em}$  gauge symmetry acting in the Minkowsky D=1+3 space-time. One of the fundamental consequence of this quantum theory is the CPT theorem which tells us that the two conjugated "families", matter and anti-matter, must have some "degenerated" properties, *i.e.* the masses and times life of particle and antiparticle must coincide:

$$m = \bar{m}, \quad \tau = \bar{\tau}, \quad \text{BINARY RELATION} \quad (1)$$

## 2 Majorana neutrino - the way to see large extra dimensional world

We would like to discuss that the new possible space-time properties of neutrino, its Majorana properties [1], can also indicate about new geom-

etry of our world, more exactly, about some large (infinite) extra space-time dimensions [9]. Kaluza was the first who tried to understand geometrically the electromagnetism, its internal  $U(1)_{em}$  symmetry, through the extra 5-th small compact dimension of our space. The existence of Majorana neutrino give some new ideas about the possible composite structure of electromagnetism in our universe through the relation between Dirac and Majorana fermions and new large extra dimensions [9].

After prominent discoveries in Cosmology and Astrophysics and a lot of new experimental results in particle physics for the last 30 years, the situation for a theory in and beyond the Standard Model and for cosmology is still very intriguing and problematic. Theoretically, after the attractive proposal of Supersymmetry and Superstrings it seems clear that some basically new ingredient is missing: there could exist some new exotic symmetries at high energies. The Lie types symmetries in the framework of the  $SU(5)$ -,  $SO(10)$ -,  $E(6)$ -,... grand unified theories, simple or extended supersymmetries or Kac-Moody, Virasoro and some other symmetries related to the superstrings do not seem to be enough to get the complete description of the Standard Model. A plausible way, based on our long previous experience, to explain such phenomena is to generalize Lie symmetries [38, 36, 37, 9].

Therefore the first evidence of Majorana neutrino nature in HEIDELBERG-MOSCOW experiment of  $0\nu\beta\beta$  decay [5, 6, 7] has the very important significance for the modern physics since gives the real impact in the progress in cosmology, in gravity, in astrophysics, in special theory of relativity, in quantum field theory, in high energy physics inside and beyond the Standard Model, in neutrino physics and etc.

Firstly, one can say that the discovery of  $0\nu\beta\beta$  decay is the indication of a new fermion Majorana matter in our Universe what was expected not just in some extension of the Standard Model, like  $L - R$  models or Grand Unified Theory, but also in all supersymmetry-, supergravity-, superstring- approaches.

Secondly, the Majorana fermions can have a much more closed link with the geometrical properties of our space-time and, what is the most important, the Majorana neutrino can give the first evidence of existence of new large extra dimensions space with  $D > 1+3$  and, correspondingly, to observe the new space-time and new internal symmetries.

In extra-dimensional cosmology we can propose a new way to solve

the baryo-genesis problem which, correspondingly, can give a new universal mechanism of the proton and electron non-stability. By universality we mean that proton and electron decays are related with the same dynamics. And this new dynamics can be directly related with dark matter.

### 3 The Dirac fermion matter and global $U(1)$ -symmetries

Until now, thanks to electromagnetic interactions, we know very well the properties of Dirac particles, *i.e.* complex charged quarks and leptons, which form our visible Universe. The electro-weak  $SU(2) \times U(1)$  interactions gave us not just some additional information about the charged Dirac particles but also helped us to discover, in the examples of neutrinos, a new fermion matter which from the mathematical point of view could produce a real states (Majorana fermions). Note that neutrinos with respect to  $SU(2)_L$  symmetry together with the charged lepton form the isodoublet, but with respect to the space-time symmetry  $SO(1,3)^+$  have different properties, *i.e.* the charged lepton is Dirac fermion, but neutrino is Majorana?! This means that a real theory of neutrinos contain some puzzles which could have some prominent links with new symmetries in/beyond the SM and/or with some extra dimensional geometry.

The Lagrangian density for Dirac fermions can be written in the well-known form:

$$\begin{aligned} L_D &= i\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi \\ &= i\xi_1^\dagger\bar{\sigma}^\mu\partial_\mu\xi_1 + i\xi_2^\dagger\bar{\sigma}^\mu\partial_\mu\xi_2 + im(\xi_2^T\sigma^2\xi_1 - \xi_1^\dagger\sigma^2\xi_2^*), \end{aligned} \quad (2)$$

where the Dirac spinor in terms of Weyl spinors has been written as  $\psi = (\xi_1, i\sigma^2\xi_2^*)^T$ . If  $\xi_1 = \xi_2$ , then  $\psi$  would be a Majorana spinor, and above action can be reduced to twice copies the Majorana action. For the Dirac complex particles one can embed the  $U(1)_x$ -global symmetries,

$$\psi \rightarrow \exp(i\alpha N_x)\psi, \quad \psi^\dagger \rightarrow \exp(-i\alpha N_x)\psi^\dagger, \quad (3)$$

which can be related with some global invariance laws, for example,

- $x = L$       Lepton number conservation,  $\Delta L = 0$ ;
- $x = B$       Baryon number conservation,  $\Delta B = 0$ ;

- $x = Q_{em}$       Electromagnetic charge conservation  $\Delta Q_{em} = 0$ .

The first and the second laws of conservation can be unified into the particle-antiparticle notion and its invariance, for example,  $F = B - L$  with  $\Delta F = 0$ . Dirac [3] searched for a relativistic equation of the first order to replace the one particle Klein-Gordon equation since the last had some defects with negative energy solutions. The Dirac equation also has the negative energy solutions but he solved this problem relies on the fact that electrons have spin 1/2 and obeys to the Pauli's exclusion principle. Dirac supposed that the negative energy states are already filled and the Pauli principle forbids any more electrons being able to enter the sea of negative states. The Dirac vacuum sea is an infinite sea of negative energy electrons, protons, neutrinos, and all other a spin 1/2 particles. The holes in such Dirac sea are anti-particles. So, the Dirac equation describe both particle and antiparticle.

The idea of quantization has been used to overcome the problem of negative energy solutions of Klein-Gordon and Dirac equations. Dirac solved this problem by understanding that his equation involves also the anti-particles, which are holes in Dirac sea. But quantization solved this problem without interpretation through the Dirac sea. The role of the complex structures in the quantum fields can be understood much easier on the examples of the scalar particles, starting from quantization of the real scalar field theory going to the complex scalar fields where appear both states, particle and anti-particle. Majorana understood this phenomenon for fermions studying the Dirac equation. Majorana found the representation for gamma matrices in which the particle and anti-particle states could be related by single complex conjugation [1]. Labelling the  $\gamma$  - matrices in Majorana representation by subscript  $M$ , the main property of a Majorana representation is that all matrices are pure imaginary, *i.e.*:

$$\gamma_M^{\mu*} = -\gamma_M^\mu, \quad \mu = 0, 1, 2, 3 \quad (4)$$

In Majorana representation one can see manifest the particle-anti-particle symmetry. The Dirac operator in this representation will be pure real

$$(i\gamma_M^\mu \frac{\partial}{\partial x^\mu} - mc). \quad (5)$$

Hence if  $\psi_M$  is a solution of Dirac equation immediately follows that  $\psi_M^\dagger$  will also a solution of such equation.

A particular Majorana representation for  $\gamma_M^\mu$  is obtained from the Dirac-Pauli representation by unitary transformation  $U\gamma^\mu U^\dagger$ :

$$\begin{aligned}\gamma_M^0 &= \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, & \gamma_M^1 &= \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{pmatrix}, \\ \gamma_M^3 &= \begin{pmatrix} 0 & -\sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}, & \gamma_M^2 &= \begin{pmatrix} -i\sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix}, \\ \gamma_M^5 &= \begin{pmatrix} \sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix}.\end{aligned}\tag{6}$$

#### 4 Majorana neutrino and ternary symmetries

The existence of Majorana can help to understand much better the role of the  $D = 3 + 1$  Lorentz group and the discrete P-, T-, C-, ...-symmetries for quantum field theories [10], [11], [12], [13]. But  $D$ -dimensional Lorentz groups cannot allow to go into the  $D > 4$  world, *i.e.* to build the renormalizable theories for the large space-time geometry with dimension  $D > 4$  [10]. We suppose that to make one should apply the new symmetries beyond binary Lie algebras/superalgebras.

Moreover, the Standard Model can be itself related with the new symmetry beyond Cartan-Lie symmetries [38, 36, 37, 9], what can give the possibilities to solve the so long standing 3-family problem (see for historical review the list of the articles during  $\sim 20 - 25$  years [14]), to describe the fermion mass spectrum (see for example, [15, 16]), to explain the bootstrap between family-color-symmetries and space-time dimensions.

The main our argument is that if the binary Lie algebras/symmetries can describe all known now the interactions - Newton gravity, electromagnetism, weak-interactions, chromodynamics - which can be unified in the superstring approaches, the new ternary algebras/symmetries [21], [22], [23], [24], [25], [26], [27], [28], [29], [31], [32], [30], [39] could be related with membrane dynamics. It seems very plausible that using such ternary symmetries will appear a real possibility to overcome the problems with quantization of a membrane theory and what could be a further progress beyond the string/superstring theories. Also these new ternary algebras could be related with some new supersymmetry approaches. Getting the renormalizable quantum field theories in  $D > 4$

space-time we could find the point-like limits of the string and membrane theories for some new dimensions  $D > 4$ .

Note, that if the physics of the Standard Model and Standard Cosmology Model is related with new external and internal symmetries, then it can give us the direct physical predictions for Large Hadron Collider!

So, starting from the Majorana neutrino nature we can discuss the following remarkable phenomena for high energy physics and Cosmology:

- 1) the existence of the real Majorana particles among the set of the complex Dirac fermions in the framework of the Standard Model could put naturally the question about the mechanism of production of Dirac fermion matter through the Majorana fermion matter in our Universe after Big Bang. Moreover, this mechanism could also explain the origin of the electromagnetic charge  $Q_{em}$  and the  $U(1)_{em}$  symmetry. Such a mechanism can be naturally related with the baryo-genesis and, correspondingly, lead by an universal way for the proton and electron non-stability. By universality we mean that an energy scale,  $M_S$ , for proton and electron decays could be the same. To construct such a mechanism one could start from the extension of our space-time geometry, *i.e.* to propose the existence some new large extra-dimensions and, correspondingly, to embed some new space-time and internal symmetries which cannot be based anymore just on the ordinary Lie groups/algebras. We already know enough well all the drawbacks of using such symmetries in construction field theories in the space-time geometry with  $D > 4$ . We could propose an extension of the binary Lie symmetries up to the more universal symmetries adding there some new ternary symmetries. To link our world with extra dimensional space-time space apart from gravity we can also use the Majorana neutrino. Moreover, according our way we can propose that in the manifold cosmology [17] with large extra dimensions Majorana neutrino could live in the bulk [9]. It means that the relativistic equation for Majorana neutrino could correspond to a space-time symmetry with some large extra dimensions what can lead to the some high energy corrections for our  $D = 1+3$  world. This circumstance could lead to some new high energy properties of neutrino  $v > c$  which have been discussed in [20], [41], [40], [9]. Also in the bulk geometry with new space-time symmetry we should propose the existence a dark matter presented by the "real" sterile

fermions.

These new exotic symmetries can be related to the algebras which are based on the generalization of the Lie binary commutation relation,  $[A, B]_2 = AB - BA$ , by the ternary commutation relations:

$$[A, B, C]_3 = ABC + BCA + CAB - BAC - ACB - CBA, \quad (7)$$

which have been already discussed in physics [22, 24, 31, 32, 9], in algebra [21, 23, 25, 30, 28, 29, 39], in  $CY_3$  geometry [34, 36, 37], respectively. Correspondingly, we can propose that the charged quarks and leptons could weakly interact with real fermions in the bulk. The mass scale  $M_S \geq O(1 - 10) - TeV$  of such interactions can give an universal mechanism for proton and electron non-stability. The geometry of the bulk and, correspondingly, the space-time properties of exotic real fermions and the dynamics of such interactions in the bulk could be described by the new space-time symmetries. In principle, it can be the natural question how our world could be built in the bulk from the real exotic fermions of the bulk? Or the opposite question, we could study how the known geometrical and physical properties of the brane-world related with ambient geometry of the bulk and with the possible physical processes going in the bulk? We can add that the inclusion of a new ternary symmetry could also propose a new gravity beyond Newton? Physically, the addition of the ternary symmetries into the dynamics of the Standard Model or Standard Cosmology Model means that some physical phenomena can be related with the three body problem.

- 2) the Majorana neutrino is "real" just with respect to the usual external space-time Lorentz symmetry and internal  $U(1)_{em}$  gauge group, *i.e.* it has no antiparticle,  $\nu^c = \nu$ , and its electromagnetic charge is zero,  $Q_{em} = 0$ , respectively. But there could exist a new ternary Abelian group  $TU(1)$  which is related with ternary complex numbers [26, 27]. With respect to the new ternary Abelian group  $TU(1)$ , one can prescribe for Majorana neutrinos a new charge,  $Q_i^G$ ,  $i = 1, 2, 3$ , different for all three families, and with natural condition [9]:

$$Q_1^G + Q_2^G + Q_3^G = 0. \quad (8)$$



Roughly speaking, one can say that the Majorana neutrinos,  $\nu_1, \nu_2, \nu_3$ , are real with respect to the usual  $Z_2$  complex conjugation, but they can be complex with respect to the new ternary  $Z_3$  complex conjugation. In this case one can describe geometrically the all three neutrino species by one six-dimensional spinor. And the new space-time symmetry should be related with new ternary internal  $TU(1)$  symmetry. The invariance of the gauge  $TU(1)$  symmetry in the extra  $D = 6$ -dimensional space-time could play the similar role as the  $U(1)_{em}$  gauge symmetry is playing for  $D = 4$ -space-time. We expect that ternary symmetries could be enough until the  $D = 6$ .

- 3) in analog with the CPT theorem consequences for the particle and anti-particles the ternary  $D = 6$ -space-time and its discrete symmetries can lead to the similar mass relation

$$m_{\nu_1} = m_{\nu_2} = m_{\nu_3}, \quad TERNARY \quad RELATION \quad (9)$$

In this case the neutrino oscillation experiments indicates on the existence on a new gauge interaction what mixes the three neutrino states. Thus, in our interpretation the  $\nu_2$  and  $\nu_3$  states can be considered as "anti-particles" of the  $\nu_1$  state. The same could be valid for the other particles in 3-families, but with our proposal the charged Dirac particles are living in the  $D=3+1$  space-time (confinement).

- 4) this new ternary symmetry could shed light on the "dark" symmetry of the SM:

$$N_{Color} = N_{Family} = N_{dim.space} = 3. \quad (10)$$

In [39] it was found the ternary generalization of quaternions. Just as the unit quaternions discovered the  $SU(2)$  group, the unit ternary "quaternions" discover the new fundamental ternary group,  $TSU(3)$  [39], which can be considered as the ternary generalization of  $SU(3)$ . One can hope that this symmetry can be applied to the Standard Model symmetries, *i.e. this symmetry could be a generalization of the  $SU(3^C)$ , and apart from usual gluons there could exist exotic membrane gluons.* In [39] we found

the 3-rank ternary algebra, "tetrahedron", which we plan to use for understanding of the "dark" symmetry of the Standard Model. Note that the ternary symmetries naturally generalize the binary symmetries [39].

We can illustrate the  $3 \times 3$  matrix realization of ternary  $q$ -algebra [39]:

$$\begin{aligned}
 q_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad q_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & j \\ j^2 & 0 & 0 \end{pmatrix}, \quad q_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & j^2 \\ j & 0 & 0 \end{pmatrix} \\
 q_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad q_5 = \begin{pmatrix} 0 & 0 & j \\ 1 & 0 & 0 \\ 0 & j^2 & 0 \end{pmatrix}, \quad q_6 = \begin{pmatrix} 0 & 0 & j^2 \\ 1 & 0 & 0 \\ 0 & j & 0 \end{pmatrix} \\
 q_7 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & j & 0 \\ 0 & 0 & j^2 \end{pmatrix}, \quad q_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & j^2 & 0 \\ 0 & 0 & j \end{pmatrix}, \quad q_9 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
 \end{aligned} \tag{11}$$

which satisfy the ternary algebra:

$$[q_k, q_l, q_m]_{S_3} = f_{klm}^n q_n. \tag{12}$$

We can check that each triple commutator  $\{q_k, q_l, q_m\}$  is defined by triple numbers,  $\{klm\}$ , with  $k, l, m = 0, 1, 2, \dots, 8$ , that it gives just one matrix  $q_n$  with the corresponding coefficient  $f_{klm}^n$  given in Table 1.

## 5 The Dirac/Majorana fermion mass problems

The very important consequence of the ternary symmetries can be directly related with the solving of the fermion mass problem, in particular, this symmetry can lead to the intriguing equality of all three neutrino masses [9]. To make such a proposal more correctly one should take into account also the known experimental results on sun's and earth's neutrino oscillations, which indicate on the very small mass difference between three neutrino species comparing with the average HEIDELBERG-MOSCOW results?! So, a new gauge symmetry could help to understand the neutrino family mixing and an origin of the mechanism

Table 1: *The ternary commutation relations for  $Tsu(3)$ -algebra,  $j = \exp 2\pi i/3, [39]$ .*

$N$	$\{klm\} \rightarrow \{n\}$	$f_{klm}^n$	$N$	$\{klm\} \rightarrow \{n\}$	$f_{klm}^n$	$N$	$\{klm\} \rightarrow \{n\}$	$f_{klm}^n$
1	$\{123\} \rightarrow \{0\}$	$3(j^2 - j)$	2	$\{124\} \rightarrow \{2\}$	$j(1 - j)$	3	$\{125\} \rightarrow \{1\}$	$2(j^2 - j)$
4	$\{126\} \rightarrow \{3\}$	$j(1 - j)$	5	$\{127\} \rightarrow \{5\}$	$2(1 - j)$	6	$\{128\} \rightarrow \{4\}$	$2(j^2 - 1)$
7	$\{120\} \rightarrow \{6\}$	$(j^2 - j)$	8	$\{134\} \rightarrow \{3\}$	$(j^2 - j)$	9	$\{135\} \rightarrow \{2\}$	$2(j - j^2)$
10	$\{136\} \rightarrow \{1\}$	$(j^2 - j)$	11	$\{137\} \rightarrow \{4\}$	$2(j - 1)$	12	$\{138\} \rightarrow \{6\}$	$2(1 - j^2)$
13	$\{130\} \rightarrow \{5\}$	$(j - j^2)$	14	$\{145\} \rightarrow \{5\}$	$(j - j^2)$	15	$\{146\} \rightarrow \{6\}$	$(j^2 - j)$
16	$\{147\} \rightarrow \{7\}$	$(j^2 - j)$	17	$\{148\} \rightarrow \{8\}$	$(j - j^2)$	18	$\{140\} \rightarrow \{0\}$	0
19	$\{156\} \rightarrow \{4\}$	$2j(j - 1)$	20	$\{157\} \rightarrow \{0\}$	$3(1 - j)$	21	$\{158\} \rightarrow \{7\}$	$2(1 - j)$
22	$\{150\} \rightarrow \{8\}$	$(1 - j)$	23	$\{167\} \rightarrow \{8\}$	$2(1 - j^2)$	24	$\{168\} \rightarrow \{0\}$	$3(1 - j^2)$
25	$\{160\} \rightarrow \{7\}$	$(1 - j^2)$	26	$\{178\} \rightarrow \{1\}$	$(j - j^2)$	27	$\{170\} \rightarrow \{2\}$	$(j - 1)$
28	$\{180\} \rightarrow \{3\}$	$(j^2 - 1)$	29	$\{234\} \rightarrow \{1\}$	$2(j^2 - j)$	30	$\{235\} \rightarrow \{3\}$	$(j - j^2)$
31	$\{236\} \rightarrow \{2\}$	$(j - j^2)$	32	$\{237\} \rightarrow \{6\}$	$2(1 - j)$	33	$\{238\} \rightarrow \{5\}$	$2(j^2 - 1)$
34	$\{230\} \rightarrow \{4\}$	$(j^2 - j)$	35	$\{245\} \rightarrow \{4\}$	$(j - j^2)$	36	$\{246\} \rightarrow \{5\}$	$2(j - j^2)$
37	$\{247\} \rightarrow \{8\}$	$2(1 - j^2)$	38	$\{248\} \rightarrow \{0\}$	$3(1 - j^2)$	39	$\{240\} \rightarrow \{7\}$	$(1 - j^2)$
40	$\{256\} \rightarrow \{6\}$	$(j - j^2)$	41	$\{257\} \rightarrow \{7\}$	$(j^2 - j)$	42	$\{258\} \rightarrow \{8\}$	$(j - j^2)$
43	$\{250\} \rightarrow \{0\}$	0	44	$\{267\} \rightarrow \{0\}$	$3(1 - j)$	45	$\{268\} \rightarrow \{7\}$	$2(1 - j)$
46	$\{260\} \rightarrow \{8\}$	$(1 - j)$	47	$\{278\} \rightarrow \{2\}$	$(j - j^2)$	48	$\{270\} \rightarrow \{3\}$	$(j - 1)$
49	$\{280\} \rightarrow \{1\}$	$(j^2 - 1)$	50	$\{345\} \rightarrow \{6\}$	$2(j^2 - j)$	51	$\{346\} \rightarrow \{4\}$	$(j^2 - j)$
52	$\{347\} \rightarrow \{0\}$	$3(1 - j)$	53	$\{348\} \rightarrow \{7\}$	$2(1 - j)$	54	$\{340\} \rightarrow \{8\}$	$(1 - j)$
55	$\{356\} \rightarrow \{5\}$	$j - j^2$	56	$\{357\} \rightarrow \{8\}$	$2(1 - j^2)$	57	$\{358\} \rightarrow \{0\}$	$3(1 - j^2)$
58	$\{350\} \rightarrow \{7\}$	$(1 - j^2)$	59	$\{367\} \rightarrow \{7\}$	$(j^2 - j)$	60	$\{368\} \rightarrow \{8\}$	$(j - j^2)$
61	$\{360\} \rightarrow \{0\}$	0	62	$\{378\} \rightarrow \{3\}$	$(j - j^2)$	63	$\{370\} \rightarrow \{1\}$	$(j - 1)$
64	$\{380\} \rightarrow \{2\}$	$(j^2 - 1)$	65	$\{456\} \rightarrow \{0\}$	$3(j^2 - j)$	66	$\{457\} \rightarrow \{1\}$	$2(1 - j)$
67	$\{458\} \rightarrow \{2\}$	$2(j^2 - 1)$	68	$\{450\} \rightarrow \{3\}$	$(j^2 - j)$	69	$\{467\} \rightarrow \{1\}$	$2(j - 1)$
70	$\{468\} \rightarrow \{1\}$	$2(1 - j^2)$	71	$\{460\} \rightarrow \{2\}$	$(j - j^2)$	72	$\{478\} \rightarrow \{4\}$	$(j^2 - j)$
73	$\{470\} \rightarrow \{6\}$	$(1 - j)$	74	$\{480\} \rightarrow \{5\}$	$(1 - j^2)$	75	$\{567\} \rightarrow \{2\}$	$2(1 - j)$
76	$\{568\} \rightarrow \{3\}$	$2(j^2 - j)$	77	$\{560\} \rightarrow \{1\}$	$(j^2 - j)$	78	$\{578\} \rightarrow \{5\}$	$(j^2 - j)$
79	$\{570\} \rightarrow \{4\}$	$(1 - j)$	80	$\{580\} \rightarrow \{6\}$	$(1 - j^2)$	81	$\{678\} \rightarrow \{6\}$	$(j^2 - j)$
82	$\{670\} \rightarrow \{5\}$	$(1 - j)$	83	$\{680\} \rightarrow \{4\}$	$(1 - j^2)$	84	$\{780\} \rightarrow \{0\}$	0

of neutrino oscillations. One can compare such a mechanism to the well-known mechanism of the  $X_v^0 - \bar{X}_v^0$  mixing ( $X_v^0 = K^0, B_d^0, B_s^0, \dots$ ) where the start point for calculations is the CPT-invariance consequences:  $m(X_v^0) = m(\bar{X}_v^0)$  (see, for example, [14]).

Comparing such a ternary relation for three neutrino masses to the following phenomenological relations for the up- and down- quark masses [15], [16]:

$$\begin{aligned}
m_{i_k} &\approx (q^u)^{2k} m_0, & k = 0, 1, 2; & & i_0 = u, i_1 = c, i_2 = t, \\
m_{i_k} &\approx (q^d)^{2k} m_0, & k = 0, 1, 2; & & i_0 = d, i_1 = s, i_2 = b, \\
m_e : m_\mu : m_\tau &\approx 1 : 200 : 4000.
\end{aligned} \tag{13}$$

where  $q^u = (q^d)^2$ ,  $q^d \approx 4 - 5 \approx 1/\lambda$ ,  $\lambda = \sin \theta_C$ . From the first two expressions one can see an interesting universal law of the fermion mass growth starting from the first family until the third family depends on the up-quark charge to down-quark charge ratio. The most remarkable fact

of these mass quark relations that they can be defined by two parameters, Cabibbo angle and mass parameter, which could be related with the mass scale of the  $SU(2) \times U(1)$  gauge symmetry breaking. It means that the origin of the quark masses could be closely related with a geometrical mechanism of the electro-weak symmetry breaking. For charged leptons such a dependence is more complicated. But in general, one can say that for Dirac fermions living in the  $D = 3 + 1$  manifold universe there some  $D = 6$  discrete symmetries can be broken and, consequently, the ternary relation is no more valid. Also, we should take into account a possibility of existing broken non-Abelian ternary gauge symmetry related with 3-families.

At last, for Majorana neutrinos, as we already discussed, one could expect the ternary mass relation, related with ternary  $D = 6$ -space-time symmetry [9]:

$$m_{\nu_1} = m_{\nu_2} = m_{\nu_3}. \quad (14)$$

Thus, so big difference between Majorana and Dirac fermions we could explain by geometrical way: Dirac charged fermion matter is living in  $D = 3 + 1$  space-time manifold, but the Majorana neutrino can live in  $D = 6$ .

Finally, we supposed that our investigations in the discovery of new ternary algebras could be connected directly with the problem of finding of the real symmetry of the Standard Model [39], [9]. The big number of the parameters inside the SM and our non-understanding of many phenomena like families, Yukawa interactions, fermion mass spectrum, confinement, the nature of neutrino and its mass origin give us a proposal that the symmetry what we saw inside the SM is only a projection of more fundamental bigger symmetry based on the ternary extension of the binary Cartan-Lie symmmtries. There is also an analogy with the Dark matter problem and following this analogy we proposed an existence of some new ternary symmetries in SM.

## 6 The Dirac and Majorana matter in the visible and invisible worlds.

So we propose that our visible world which is described by the Poincare' external symmetry and  $SU(3) \times SU(2) \times U(1)$  gauge internal symmetry with all discrete symmetries is confined in the our manifold Universe. We can also propose that this symmetry based on Lie binary algebra is not

complete to describe our world and can contain a new type of symmetry, "dark" symmetry, which is beyond the Lie binary algebra/superalgebra. We apply this symmetry to understand the geometrical properties of Majorana neutrino and how the Majorana neutrino could help us to see invisible world through the possible solutions some cosmological problems, like as baryo-genesis, dark matter,...

The prediction the anti-matter by Dirac in his relativistic equation and the further experimental confirmation of his idea was one of the excited discovery in particle physics. But this discovery gave the very big puzzle for cosmology, which was not solved yet, after 70-80 years of discovery anti-matter. From the experiments we have got indications that the Universe is matter-antimatter asymmetric. For example, it was found that in cosmic rays antiproton  $\bar{p}$  to proton  $p$  ratio is equal to  $\bar{p}/p \approx 10^{-4}$ . If we starting with  $n_B = n_{\bar{B}}$  for the equilibrium abundance of nucleons and antinucleons can be get the following result

$$\eta = \frac{n_B}{\gamma} = \frac{n_{\bar{B}}}{\gamma} = 2 \times 10^{-18}, \quad (15)$$

what contradicts

$$\eta = (6 \pm 3) \times 10^{-10}, \quad \Omega_B = \frac{\rho_B}{\rho_c} = 0,045 \pm 0.001, \quad (16)$$

getting from Big Bang Nucleosynthesis and cosmic microwave background anisotropy. To solve the baryo-genesis problem three main Sacharov,s ideas were suggested:

1. the underlying model must have baryon number violation,  $\delta B \neq 0$ ;
2. Charge  $C$  and  $CP$  symmetries also should be violated;
3. The thermal equilibrium of  $X$ - particles or interactions mediating the  $\Delta B \neq 0$  processes should be moved in one from the two possible directions. In other case if the all processes with  $\Delta B \neq 0$  are in thermal equilibrium due  $CPT$  invariance the baryon asymmetry vanishes.

What can be the role of Majorana particles in baryo-genesis problem? These three ideas have been used in some baryo-genesis models based on

the binary Grand Unified Theory and baryo-genesis through the lepto-genesis in which could exist heavy particles like lepto-quark gauge bosons  $X_v$  or Majorana right-handed neutrinos  $N_M$ . In these two approaches the main source of the baryo-genesis is related with the C-and CP violating decays of such heavy non observable particles,  $X_v$  and  $N_M$ . Both these scenario proposed on the extension of the Standard Model symmetries and can predict at very high energy scale a new physics which could have some links with low energy, like proton decay in GUT baryo-genesis, or neutrino masses in lepto-genesis. Also, more naturally to describe the baryo-genesis through the lepto-genesis in the frames of GUT (SO(10), for example), and so both these directions could lead to the proton decays, which until now has not been founded on the very high level?! We would like to stress that in these scenarios (in electro-weak baryo-genesis also) it was used the common idea of extension just the internal symmetries:  $SM \rightarrow GUT$ . In all these approaches we have the question, is still survive the Lorentz group up so high energies.

To understand the ambient geometry of our world with some extra infinite dimensions one can suggest that our visible world (universe) is just a subspace of a space which “invisible” part one can call by bulk. The visibility of such bulk is determined by our understanding of the SM and our possibilities to predict what could happened beyond its. To find an explanation of the small mass of neutrinos in the sea-saw mechanism it was suggested that in this bulk could exist apart from gravitation fields some sterile particles, like heavy right-handed neutrinos which could interact with light left-handed neutrinos. The Majorana neutrino can travel in the bulk?! For this we should introduce a new space time-symmetry which generalizes the usual D-Lorentz symmetry.

The discovery of Majorana neutrino nature among the set of the all other Standard Model Dirac charged fermions prompts another dynamics of baryo-genesis, based on the composite dynamics of the fermion Dirac matter from the more simple “real” fermions like Majorana neutrino which can live in the extra dimensional world?! The dynamics based on the extra dimensional geometry could be beyond the Sacharov,s three conjectures. The fundamental conception of such idea is related with attempts to find a common mechanism of the creation of  $Q_{em}$  charge symmetry with baryo-genesis. We propose that such a mechanism must use a  $D = 3 + 1$ -duality between the  $Q_{em}$  conservation and  $CPT$  invariance. This is very important proposal because we suggest a correlation between the external and internal symmetries (Poincare’ duality).

The Standard Model which is described by the Lie algebra  $iso(1, 3) \times su(3)_c \times su(2)_L \times u(1)_Y$ . The Poincare algebra ( non-homogeneous Lorentz algebra),  $iso(1, 3)$ , is related to the space-time symmetries, having 10 parameters, 6=3-rotations+ 3-boosts+ 4-translations,  $x^{\mu\prime} = x^\mu + \xi^\mu$ . Also, the external symmetries include some discrete symmetries, like  $P, T, C$ , which unify in one of the principal symmetry of the SM,  $CPT$  - invariance. the Standard Model which is described by the Lie algebra  $iso(1, 3) \times su(3)_c \times su(2)_L \times u(1)_Y$ . The Poincare algebra ( non-homogeneous Lorentz algebra),  $iso(1, 3)$ , is related to the space-time symmetries, having 10 parameters, 6=3-rotations+ 3-boosts+ 4-translations,  $x^{\mu\prime} = x^\mu + \xi^\mu$ . Also, the external symmetries include some discrete symmetries, like  $P, T, C$ , which unify in one of the principal symmetry of the SM,  $CPT$  - invariance.

In this theoretical construction, external and internal symmetries have been included as a direct product,  $G_{ext} \times G_{int}$  without any relation between them, what can be reinterpreted as a generalization of the well-known Coleman-Mandula no-go theorem [38]. This may be one of the main obstacles to further progress in the understanding of a lot of open questions in the Standard Model and in Cosmology . However, the progress in supergravity and in superstring model compactifications which has been related to the introduction extra-dimensional new geometrical objects beyond Cartan-Lie geometry opened new paths. Especially, the significant progress was done with a geometric interpretation of the internal symmetries ( Klein-Du-Val singularities, Kaluza-Klein models, superstring compactifications on  $CY_n$  or  $G_2$  spaces, etc) ( see also [35, 34, 36, 37, 39]).

Thus, our proposal, a link between internal and external global conservation laws such as electric charge conservation and  $CPT$  invariance:

$$CPT - invariance \quad \leftrightarrow \quad (Q_{em}) \text{ charge conservation}, \quad (17)$$

gives us an important ingredient in our scheme. If such a duality exists, processes violating  $CPT$ -invariance should accompanied by electromagnetic charge violation too, *i.e.* the invariance of  $CPT$  in  $D = 4$  space-time means the conservation of  $Q_{em}$  charge there , and opposite, what can be checked on the experiments.

We suppose that primordially the Universe was neutral  $Q_{em} = 0$ , more over, there were no absolutely the charged particles, in particularly, Dirac complex fermions. The Dirac complex fermions appeared in  $D =$

1 + 3- manifold world from real high dimensional fermions as result of a dynamics having the geometrical origin. In this case the part of space-time where it was happened such process must have the total charge equal zero. Note, that the higher dimensional space-time world could have a symmetry which is more fundamental than the symmetry which we use in the Standard Model and Standard Cosmology Model based on the binary Lie algebras and Lie superalgebras. We already found some candidates for such symmetries. They can be related with the  $n$ -ary generalizations ( $n=3,4,\dots$ ) of Lie algebras and superalgebras. The geometry of such space-time and particle-states in this world can be determined by these new universal symmetries, for example, based on the extension of binary algebras up ternary. This could be a new way to construct a realistic Grand Unified Theory. Our proposal is that the Majorana neutrino could link with such neutral real fermion particles and therefore gives us useful information about this high-dimensional world with  $D > 4$ . For terminology we use the word “brane world” which means that the Standard Model matter is “living” in the three dimensional submanifold embedded into the higher dimensional space (a confinement for Dirac charged particles).

Mainly we will be interested just by the scenario when the extra dimensions are large or infinite. A link of the brane world with the bulk- the other world- depends on the confinement model. Based on the Majorana neutrino nature we accept a scenario in which the left-handed neutrino can trap in the bulk and some exotic real Majorana fermions from the bulk could interact weakly with Dirac fermions on the brane.

We can relate these real bulk fermions with a possibility of creation of Dirac fermions and the creation of the  $Q_{em}$  charge symmetry in  $D = 3+1$  world from them. Such a dynamics could lead to the production of baryon and leptons simultaneously after Big Bang by unique mechanism leading to the  $Q_p + Q_e = 0$ . In such scenario Majorana neutrino through its properties should have a “memory” about this process-production. The physical models with extra dimensions we can study through the new symmetries, which could help us to observe a duality between external and internal dynamical symmetries, *i.e.* to overcome the Coleman-Mandula no-go theorem [38]. One of the ways in this direction is related to the search for generalizations of the principle of the special theory of relativity, generalizations of the Klein-Gordon or Dirac-Majorana equations. Here one can find in [32] a very interesting example of non-relativistic equation in the frames of ternary Clifford algebra.



Thus for our purpose we can propose a correspondence between the  $Q_{em}$  charge conservation and  $CPT$ -invariance, which, by our opinion, could have a geometrical origin from Poincare' duality. This correspondence link in  $D=1+3$  space-time the matter-antimatter symmetry and  $Q_{em}$  charge conservation. This proposal can be interpreted as an attempt to find a correlation between external and internal symmetries overcoming by this the no-go Coleman-Mandula theorem. In our conjecture following to Poincare' duality the internal properties of the particle fields could have a correlation/link with their space-time symmetry properties. Of course, it should be valid a reverse sentence: one can study space-time through the study of the properties of the particles [38].

## 7 Large or/and small extra dimensions

In standard quantum physics the differential equations of the particles in space-time or the evolution of ambient spaces is based on some known global external symmetries of these spaces, such as the Lorentz  $SO(3,1)$  group. The usual description of the physical world made use of global geometry and symmetric homogeneous spaces, directly related to the Cartan classification of the simple Lie groups. The internal local physical gauge symmetries were embedded into quantum physics using the Yang-Mills gauge procedure and this, too, is formulated in the same Lie-Cartan symmetry framework. At the top of this Lie dynamics culminate the (super)string models in which one claims to unify all (so far) known gauge interactions, mediated through the graviton, W,Z-bosons, gluons and photon.

After discovery the electroweak interactions in the framework of the  $SU(2) \times U(1)$  gauge symmetry one of the main question was to understand the so big difference between two energy scales,  $M_{EW} \sim 250 GeV$  and Plank scale  $M_{Pl} \sim 2 \times 10^{19} GeV$ . This question transformed into the hierarchy problem of the Standard Model, which can be solved by different ideas, one of them can be the  $N=1$  supersymmetry extension of the Standard Model getting the name Minimal Supersymmetric Standard Model. The so big value of the  $M_{Pl}$  allowed to consider the Grand Unified Theory with the gauge symmetries  $G = SU(5), SU(5) \times U(1), SO(10), E(6), E(8)$  with possible their supersymmetry variants and with the unification scale  $M_U \sim 10^{16} GeV \leq M_{Pl}$ . All these models were interesting practically as they intended to give the baryon violation processes like proton decays, which can be used in cosmological scenario of the baryo-genesis. From another side these theories

based on the Cartan-Lie symmetries predicted a very big energy scale without any physics (desert) and had no enough dynamics to give exact predictions for low energy Standard Model physics, like confinement, family problem, fermion mass problem, Dirac or Majorana neutrino nature and others.

The alternative scenario to understand the problems of the Standard Model and physics beyond the Standard Model is based on the geometrical ideas. One of the main possibility of such approach is an extension of the dimension of our space-time, how it was discussed by Kaluza-Klein ( $D=5$ ), how it was happened in string approach ( $D=26$ ), in superstring approaches  $4 \leq D \leq 10$  and further its developments in M/F theories with  $D=11/12$ , respectively. But in all these approaches the extra dimensional compact spaces have the very small characteristic sizes related to the  $M_{Pl}$  energy scale.

In the beginning of the 90's due to [18] and later to [19] the interest of the physics community was excited by the suggestion of new large extra dimensions and their link to the high energy physics near Standard Model energy scale. The main attraction of this approach is the possibility to reduce the mass Planck scale from  $10^{19} GeV$  down to  $\sim 1 - 10$  TeV, an energy scale that can be explored experimentally on LHC. So, scenario with large or infinite extra dimensions can be more acceptable than the scenario based on a new physics near  $\sim 10^{19} GeV$ . So, there is a dilemma:

1. To continue to study possibilities at very large energies  $\sim 10^{14-19} GeV$ ;
2. To study new geometry/symmetries which can be related with new physics at energy scale  $\sim O(1 - 10) TeV$

One can suppose that on the second way we have much more possibilities to get the answers on the Standard Model problems by checking our ideas directly in the current experiments. On this way we could consider by applying of the two possibilities: small extra dimensions and large extra dimensions.

The first possibility can be studied by Kaluza-Klein method as it has been already intensively used in supergravity and superstring approaches. The second possibility needs to study the new geometrical spaces and its new symmetries.

The superstring/D-branes approaches cannot help us to solve these problems. Their point limit on the small compact manifolds cannot give us in  $D > 4$  renormalizable theories also. The main argument is that the string/superstring theories are based just on the Lie binary algebras and its infinite dimensional generalizations with central charge, *i.e.* Kac-Moody algebras. And in addition, the superstring compactifications cannot give us exact and unique predictions (degeneration of the superstring vacuum) <sup>1</sup>.

Both these problems could be solved by one way finding the new symmetries beyond Cartan-Lie algebras. Actually, to study the geometry of physical space-time with extra dimensions, one can proceed by investigating its symmetries. To build a quantum field theory in a space-time with extra dimensions ( $D > 4$ ), some of which have a different status from our usual 4 ones, it seems inappropriate to use the usual the Lie-Cartan symmetries, as compactification, mentioned above, has taught us. So there could be just a chance to uncover new symmetries beyond Lie-Cartan type. We propose first to realize such an aim by the generalizing binary Lie algebras/symmetries to  $n$ -ary algebras/symmetries (*That is, instead of binary operations like addition or product of elements, one starts with composition laws involving at least  $n$  elements of the considered algebra*).

The effective 4-dimensional  $M_{Pl}$  can be expressed via 4-dimensional  $M_D$  ( $D = 4 + n$ ), and radii of the compact volume  $R$  by the following [18], [19]:

$$M_{Pl} \sim M_D^{2+n} R^n. \quad (18)$$

The fundamental  $D$ -dimensional Newton's constant became:

$$G_{N_D} = \frac{1}{M_D^{n+2}}. \quad (19)$$

Following to [18], [19] one can consider  $M_D \sim 1TeV$ , demanding that  $R$  should reproduced the observed very big Plank scale,  $M_{Pl}$ :

$$R \sim \frac{1}{M_D} \left( \frac{M_{Pl}}{M_D} \right)^{2/n} \sim 10^{32/2} \times 10^{-17} cm. \quad (20)$$

---

<sup>1</sup>We would like to express our acknowledgements to I. Antoniadis and L. Lipatov for useful discussions on such interesting question

Thus this picture proposes that at the smaller distances the gravity could be stronger and stronger. But for the SM particles there could be some possibilities, for example, some of them can probe the extra dimensions. Especially, it can be happened at high energies, for example, at the energies more than some hundred GeV we have no stringent restrictions for this. So the picture ADD reproduces the idea of Kaluza-Klein, but at much lower mass scale. The extra dimensions could be found even at the energies in TeV region. Let us remind the main predictions of Kaluza-Klein from the [18],[19] point of view. In (3+1) world one can see KK modes which can be interpreted as a separate particle with mass  $m_n = n/R$  (infinite tower of the particles), where  $R \leq 10^{-17}cm$ . At low energies one can see only massless modes, which are usual particles and all K-K modes should be near or heavier the TeV range. In principle, according this picture only graviton and KK-graviton could penetrate into extra dimensions, i.e. into a bulk. To explain this picture more correctly one can show they all known particles and gauge bosons of SM are localized on a brane.

We should go further. We propose that the “life” in the bulk is much richer and can be responsible for creation the Dirac matter and symmetries in our D3 brane.. For this we should suggest a new symmetries beyond the binary Lie symmetries, which were useful until now. But just with them we cannot get a progress in the geometry of the bulk and physical processes are going there.

This symmetry can help us to go beyond the  $D > 4$  dimensional geometry and to get an information about possible generalization of D-Lorentz symmetry in extra dimensional geometry of our universe. We suppose that the D-Lorentz symmetry will be not enough to build a realistic theory. A real symmetry of such world should be more universal, which could be responsible for dynamics of creation of complex Dirac particles from real more simple fermions.

Of course, these unusual properties of neutrinos should be related with a new progress in the study of the new symmetries which can be related with extra dimensional geometry of our Universe.

A mechanism of the geometrical origin of electromagnetic charge  $Q_{em}$  and the Dirac complex matter, baryon and lepton matter and baryon-antibaryon (lepton-antilepton) asymmetry, proton-electron non-stability and etc can be called by Majorana-Diraco genesis and could be considered as a further step of development of baryo-genesis and lepto-genesis. Note that the baryo-genesis-theory or lepto-genesis-theory to explain

baryon or lepton asymmetry of Universe demand some exotic particles, lepto-quarks or right-handed neutrino, correspondingly. The masses of such particles are very big:  $M_{lq} \sim 10^{18} GeV$  and  $M_{RH} \sim 10^{12} - 10^{14} GeV$ , respectively. The Majorana-Diraco genesis is based on the existence of a new extra dimensional geometry and can predict the proton/electron decays at the TeV mass scale which can be checked on the modern experiment (LHC). Here one can find a similarity with the situation with  $M_{Plank} \sim 10^{19} GeV$  in  $D = 4$  and a possibility to diminish this parameter using some extra dimensions [18], [19].

We already know that the superstring Grand Unified Theory's did not bring us an expected success for explanation or understanding as mentioned above many problems of the Standard Model. The main progress in superstrings (strings) was related with understanding that we should go to the extra dimensional geometry with  $D > 4$ . Also the superstrings turned us again to study the geometrical approach, which has brought in XIX century the big progress in physics. This geometrical objects, Calabi-Yau spaces with  $SU(3)$  holonomy, appeared in the process of the compactification of the heterotic  $E(8) \times E(8)$  10-dimensional superstring on  $M_4 \otimes K_6$  space or study the duality between 5 superstring/M/F theories. Mathematics [33] discovered such objects using the holonomy principle. To get  $K_6 = CY_3$ , the main constraint on the low energy physics was to conserve a very important property of the internal symmetry, i.e., to build a grand unified theory with  $N = 1$  supersymmetry. It has been got the very important result that the infinite series of the compact complex  $CY_n$  spaces with  $SU(n)$  holonomy can be described by algebraic way [34]. This series starts from the torus with complex dimension  $d = 1$  and  $K3$  spaces with complex dimension  $d = 2$ , with  $SU(1)$  and  $SU(2)$  holonomy groups, respectively. We would like to stress that consideration of the extended string theories leads us to a new geometrical objects, with more interesting properties than the well-known symmetric homogeneous spaces using in the Standard Cosmology Model. For example, the  $K3 = CY_2$ -singularities are responsible for producing Cartan-Lie ADE-series matter using in the Standard Model. The singularities of  $CY_n$  spaces with  $n \geq 3$  should be responsible with producing of new algebras and symmetries beyond Cartan-Lie and which can help us to solve the questions of the Standard Model and Standard Cosmology Model [36, 37, 9, 39]. This geometrical direction is related with Felix Klein's old ideas in his Erlangen program which promotes the very closed link between geometrical objects and symmetries linked with

them.

For example, embedding the Majorana neutrino into the higher dimensional space-time we need to find a generalization of relativistic Dirac-Majorana equation which should not contradict to low energy experiments in which the properties of neutrino are known very well! There could be the different ways of embedding the large extra-dimensions cycles according some new symmetries, what can give us new phenomena in neutrino physics, such as a possible new  $SO(1,1)$  boost at high energies of neutrino [20]. The embedding the new symmetries (ternary,...)open the window into the extra-dimensional world with  $D > 3 + 1$ , gives us renormalizable theories in the space-time with  $\text{Dim}=5,6,\dots$  similarly as Poincare' symmetry with internal gauge symmetries gave the renormalizability of quantum field theories in  $D=4$  [10].

## 8 Majorana-Diraco genesis and proton/electron non-stability

The idea of Majorana-Diraco genesis starts from Dirac/Majorana equations and is related to the further attempts to solve the baryon asymmetry of universe linking such question with an origin  $Q_{em}$  charge symmetry. We would like to explore the idea of the complex structure of Dirac fermions and propose that the Dirac fermions in  $D=4$  have been produced from real fermions of higher dimensions due to a mechanism which we plan to study in future. But now we can just discuss some remarkable facts which could support our conjecture:

$$|Q_p + Q_e| < 10^{-21} \quad (21)$$

which can indicate that the proton ( quarks) and electron can have the unique origin after Big Bang. Also, one can suppose that the total electromagnetic charge of our universe is equal zero.

This conjecture suggests an existence in high dimensional space a class of "real" Majorana fermions connected with our fermions through a new interaction which is based on a new symmetry beyond binary Cartan-Lie algebras. ( These fermions can have the other structures.) Such an interaction can predict a non-stability of the electrons or protons. We can illustrate a scale of such an interaction, taking for example, two extra dimensions. To make estimations we are in the situation in which was E.Fermi before discovery Yang-Millz interactions [8]. So we will follow to his ideas which he used for construction the

four-fermion weak theory with coupling constant  $G_F$ , which dimension is  $[G_F] = [M^{-2}]$ .

So we can start from the multi-fermion  $D = 6$  Fermi Lagrangian and the corresponding Fermi constant  $G_{FS}$ , which could have dimension:

$$[G_{FS}] = [M^{-p}], \quad p = 3, 4, 5, 6, \quad (22)$$

where  $p$  depends on the  $n$ -arity of a new interaction.

In our opinion, this dimension of coupling constant corresponds to a ternary gauge interaction, whose propagator could have a form like  $[P(q)_p + M_S^p]^{-1}$ , where  $P(q)_p$  is a polynomial of the transferred momentum  $q$  of degree  $p = 3, 4, \dots$ . The form of such a propagator corresponds to a relativistic equation of the “gauge boson” flying in  $D = 6$ . We can propose that such a “gauge boson” will membrane origin, but not of string. So, the gauge variant of the Fermi  $D = 6$  interactions could have a membrane origin and we can call them membrane gauge bosons. This is a completely new peculiarity of high-dimensional interactions based on ternary gauge symmetries. So for the tree-level calculations of the quark or charged-lepton decays into neutral real bulk fermions  $\nu_S$

$$q \mapsto n \nu_S, \quad e^\pm \mapsto n \nu_S, \quad (23)$$

taking the propagator of degree four one can get the following estimate for the partial width:

$$\Gamma(e \rightarrow n \nu_S) \sim g_S^4 \cdot \frac{m_{e/q}^9}{M_S^8}, \quad (24)$$

where  $m_{e/q}$  is the electron mass and  $M_S$  is the membrane gauge boson mass.

To obtain the lower boundary for  $M_S$  one can compare the partial width for electron decay with the lifetime of the muon getting in  $D_4$ -Fermi interactions. For the estimations one can take the following limit of the electron life-time:  $\tau_e \leq 10^{25} \text{ years} \approx \pi \cdot 10^7 \cdot 10^{25} \text{ sec}$ . This upper boundary gives the following estimate for

$$M_S \geq \left(\frac{g_2}{g_S}\right)^{1/2} \cdot 10 \cdot M_W. \quad (25)$$

Table 2: *Electron decay.*

<i>Process</i>	<i>Ref</i>	$\tau(y)$
$Ga^{71} \rightarrow Ge^{71}$	<i>Norman et al</i>	$4 \times 10^{26}$
$e \rightarrow \nu_e + \gamma$	<i>Avignone et al</i>	$2 \times 10^{25}$
$e \rightarrow \nu_e + \gamma$	<i>Belli et al</i>	$2 \times 10^{26}$
$e \rightarrow any$	<i>Reusser et al</i>	$3 \times 10^{23}$
$e \rightarrow any$	<i>Belli et al</i>	$2 \times 10^{24}$

From this, it is clear that the  $M_S$  can be also in the  $\sim O(10)TeV$  region. This mass scale is consistent with the upper boundary getting from the searching the proton decay. Note that in this approach one can study the proton decay problem in  $B$ - physics, in  $\mu$ - and  $\tau$ - decays also! There is also an interesting question about the spin structure of a new real bulk fermions,  $\nu_S$ . Thank to the ternary symmetries they could have the  $1/3$  spin structure. On algebraic language it means that a map of the internal ternary symmetry of new fermions into the symmetry of the ambient space-time is a nontrivial triple covering like in the case of the electron, its spin  $1/2$  structure is related with the double covering maps:  $SU(2) \mapsto SO(3)$  and/or  $SL(2, C) \mapsto SO(3, 1)$  [4].

And what is very interesting that we have got the universal mechanism of the decays of the all known us quarks  $u, d, s, c, b, t$  and charged leptons, electron, positron, etc. and we can get a relation between the time life of proton and electron.

To estimate  $M_S$  we can start from the electron data which one can see from the Table 2

To get the lower boundary for  $M_S$  et compare the partial width for electron decay with the life time of muon getting in  $D_4$ -Fermi interactions:

$$\frac{\Gamma(\mu \rightarrow e \nu \bar{\nu})}{\Gamma(e \rightarrow \nu \nu_S)} = \left(\frac{g_S}{g}\right)^4 \frac{m_\mu^5}{m_e^9} \frac{M_S^8}{M_W^4} \approx 10^{38}. \quad (26)$$

In this expression we took for electron life time the following lower boundary:  $\tau_e \leq 10^{25} years \geq \pi 10^7 \cdot 10^{25} sec$  This upper boundary gives the following estimate for

$$M_S \geq \left(\frac{g_2}{g_S}\right)^{1/2} \cdot 10 \cdot M_W. \quad (27)$$



The next interesting estimation we can make for muon decay:

$$\frac{\Gamma(\mu \rightarrow n\nu_s)}{\Gamma(\mu \rightarrow e + \nu + \bar{\nu})} \leq 10^{-19} \left(\frac{g_S}{g_2}\right)^4, \quad (28)$$

if we take into account the lower boundary  $M_S > 10M_W$  which we have got from electron decay data. One compare with the experimental possibilities in searches for the rare decay of muon  $\mu^+ \rightarrow e + e^+e^-$  in which the result is  $Br(\mu^+ \rightarrow e^+e^+e^-) < 2 \cdot 10^{-12}$ .

For the  $\tau$  lepton decay the branching ratio of charge violation decay can be more:

$$B = \frac{\Gamma(\tau \rightarrow n\nu_s)}{\Gamma(\tau \rightarrow \mu + e + \nu)} \leq 5 \cdot 10^{-15} \left(\frac{g_S}{g_2}\right)^4. \quad (29)$$

The estimations for the branching ration of the quarks can be done similarly to the lepton cases. Just there could be a question of calculation of the processes for hadrons, for example, for protons. For proton decay such estimation of the branching ratio can be done similarly to the estimation of the proton decay in the Pati-Salam Grand Unified Theory with low unification scale  $O(10^{3-4} GeV)$ . Of course, the processes which are going in D=6 Fermi interactions are going more and more intensively with growing masses of the decaying particles. For example, for b-quark the branching ration could be near the experimental possibilities,

$$B(b \rightarrow n\nu_S) \leq 10^{-8-10}. \quad (30)$$

Apart from charge violation decays we can expect also the processes with violation of the  $CPT$ -invariance. The most interesting process could be related with measuring of the  $K^0$ - $\bar{K}^0$  mass difference:

$$\delta_m = |m - \bar{m}| \sim \frac{m^5}{M_S} < 10^{-15} GeV. \quad (31)$$

From which one can see that the  $M_S$  can be also in  $TeV$  region.

## 9 Conclusions

We started our discussion from Majorana neutrino and step by step want to come to an idea of an existence of completely new physics at energy

scale  $O(1 - 10)TeV$ . Mathematically, the Majorana neutrino has no the binary complex structure, but it has a ternary complex structure which is directly related to the origin of three quark-lepton families. The new ternary structure of Majorana neutrino can give a lot of new interesting physics. This physics could be related with new interactions what we called by "neutrino light" [40] and these new interactions could be related to the dark matter/energy problems. As the ideas suggested the new ambient geometry of our space-time and Majorana neutrino is living in this space-time, we can check the our ideas by measuring of the neutrino velocity. The progress what made in the construction of the new neutrino detectors in Gran Sasso Underground Laboratory gives us the independent possibilities to check is neutrino of Majorana nature or not. The other possibility to check the Majorana nature of neutrino can be related with the proton decay problem. In the scheme of Majorana-Diraco genesis the proton non-stability problem could be solved also in the *electron*,  $\mu$ -,  $\tau$ - lepton and  $b$ - meson rare decay experiments ( see below):

1. Majorana neutrino and new space-time geometry
2. Complexification and global  $U(1)$  symmetries
3. Majorana neutrino in cosmology with extra dimensions  $D > 4$
4. Poincare' duality:  $CPT$ -invariance and  $Q_{em}$ -conservation
5. Dark symmetry in the SM. Ternary symmetry beyond Lie.
6. Baryo-, Lepto-, Majorana-Diraco genesis
7. New interactions and proton/electron non-stability

The Majorana nature of the neutrino gave a lot of new and non-ordinary ideas for the probing of some of which I would like to express

my thanks to U. Aglietti, V. Ammosov, I. Antoniadis, P. Aurenche, V. Dvoeglazov, M. Baldo-Ceolin, G. Bélanger, G. Costa, P. Chankowski, G. Girardi, A. De Rujula, A. Dubrovskiy, J. Ellis, L. Fellin, H. Klapdor-Kleingrothaus, I. Krivosheina, L. Lipatov, A. Liparteliani, A. Masiero, R. Obergfell, M. Ruggier, A. Sabio-Vera, S. Sychkov, J-B. Zuber. It is pleasure for me to thank Z.Berezhiani and the participants of the X Summer Institute 2005 and XI Summer Institute 2006 in Gran-Sasso for the nice possibilities to discuss the ideas of this work.

## References

- [1] E. Majorana, *Teoria simmetrica dell'elletrone e del positrone*, Nuovo Cimento **14** (1937) 171.
- [2] J.C. Maxwell, *An elementary treatise on electricity*, 2nd ed.Clarendon Press, Oxford, (1988).
- [3] P.A.M. Dirac, *The quantum theory of electron*, Proc. Roy. Soc. **A117** (1928) 610.
- [4] C. Chevalley, *The algebraic theory of spinors* Columbia University Press, New York, (1954).
- [5] H.V. Klapdor-Kleingrothaus, A. Dietz, H.I.Harney and I.V. Krivosheina, hep-ph/0201231 and Mod. Phys. Lett. A **16** (2001) 2409-2420, Found. Phys. **31** (2002) 1181-1123 (Corrigenda 2003).
- [6] H.V. Klapdor-Kleingrothaus, I.V. Krivosheina, A. Dietz et al., *Phys. Lett. B* **586** (2004) 198-212.
- [7] H.V. Klapdor-Kleingrothaus, A. Dietz, I.V. Krivosheina et al., *Nuclear Instr. and Methods A* **522** (2004) 371-406.
- [8] E. Fermi, *Tentativo di una teoria dei raggi  $\beta$* , Ricerca Scientifica, (1933)(also, Z. Phys., 1934).
- [9] G. Volkov, *Geometry of Majorana neutrino and new symmetries*, hep-ph/0607334, talk at the II-nd workshop "‘ Dark Matter, Neutrino in nuclear physics’", 3-9 September, Paris, 2006.
- [10] N.N. Bogoljubov, Shirkov, Introduction in Quantum Field Theory.
- [11] C.Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
- [12] Ryder L.H. *Quantum Field Theory* ( Cambridge University Press , Cambridge) 1996.
- [13] V.V. Varlamov, *Universal Coverings of Orthogonal Groups*, Advances in Applied Clifford Algebras **14**, (2004) 81-167.
- [14] V.A. Monich,B.V. Struminsky, G. Volkov, *Oscillation and CP Violation in the Horizonatal Superweak Gauge Scheme* Phys.Lett.**B104**,382,(1981), JETP Lett.**34**,213,(1981), Zh.Eksp.Teor.Fiz.**34**,222-225,(1981).  
G.G. Volkov, A.G. Liparteliani, V.A. Monich, Yu.P. Nikitin

- . *Transitions between neutral mesons in multi-quark schemes* Nucl.Phys.(B195),425,(1982), Erratum-ibid.**B206**496,(1982), Sov.J.Nucl.Phys.**33**, 235,(1981), Yad.Fiz.**33**, 446-461,(1981).
- A.N. Amaglobeli, A.R. Kereselidze, A.G. Liparteliani, G.G. Volkov, . *The supersymmetric vector-like horizontal model with intermediate symmetry breaking scale*, Phys.Lett.**B237**,417,(1990).
- A.A. Maslikov, S.M. Sergeev, G.G. Volkov, *Grand unified string theories with non-Abelian gauge gauge family symmetry in the free fermion formulation*, Phys.Rev.**D50**,7440-7449,(1994).
- A. Maslikov, I. Naumov, G. Volkov, *The fermion generation problem in the GUST in the free world-sheet fermion formulation* Int.J.Mod.Phys.**A11**,1117-1168,(1996)
- [15] A.Maslikov and G. Volkov, *Towards a low-energy quark-lepton family non-Abelian gauge symmetry*, preprint ENSLAPP-A-382-92, May 1992. 40pp., Annecy-Le-Vieux (1991), A.G. Liparteliani, G.G. Volkov (Serpukhov, IHEP),. *Discrete symmetries in weak interactions*, preprint-IFVE-79-58, Apr 1979. 11pp.
- [16] P. Ramond, R.G. Roberts and G. G. Ross, Nucl.Phys. **B406**, 19,(1993)
- [17] V. Rubakov, Large and infinite extra dimensions, Phys. Usp. **44** (2001) 871.
- [18] I.Antoniadis, *Phys. Lett.* **B246** (1990) 377.
- [19] N. Arkani-Hamed, S. Dimopoulos and G.Dvali, *Phys. Lett.* **B429** (1998)263  
I.Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.Dvali, *Phys. Lett.* **B436** (1998) 256
- [20] V. Ammosov and G. Volkov, *Can neutrino probe extra dimension*, Padua preprint DFPD-00/TH/39, hep-ph/0008032 (2000).
- [21] A.G. Kurosh, *Multioperator rings and algebras*, Russian Math.Surveys, **24** (1969), 1-13.
- [22] Y. Nambu, *Generalized Hamiltonian mechanics*, Phys. Rev. **D7** (1973) 2405–2412.
- [23] V.T. Filippov, *n-ary Lie algebras*, Sibirskii Matematicheskii Zhurnal, **26**, 6 (1985) 126–140.
- [24] A. Capelli, C. Itzykson and J.–B. Zuber, The ADE classification of minimal and  $A_1^{(1)}$  conformal invariant theories, Commun. Math. Phys. **184** (1987), 1-26, MR **89b**, 81178.
- [25] L. Takhtajan, *On foundations of generalized Nambu mechanics*, Comm. Math. Phys., **160**, (1994), 295-315.
- [26] N. Fleury, M. Rausch de Traubenbergn et R.M. Yamaleev, *J. Math. Anal. and Appl.*, **180**, (1993), 431.
- [27] M. Raush de Traubenbergn, *Clifford algebra in physics*, hep-th/0506011 (2005) and *Algèbres de Clifford supersymétrie et symétrie  $Z_n$  – Applications en théorie des champs*, Orsay report LPT 97-02 (1997).

- [28] P. Hanlon and M. Wachs, *On Lie  $k$ -algebras*, Adv. in Math. **113** (1995), 206-236.
- [29] P.W. Michor and A.M. Vinogradov,,  *$n$ -ary Lie and associative algebras*, Rend. Sem. Mat. Univ. Pol. Torino **53**,3 (1996).
- [30] M.R. Bremner, *Identities for ternary commutator*, J. Algebra 206,(2), (1998), 615-623, MR1637092 (99h:16071).
- [31] R. Kerner, *Ternary algebraic structures and their applications in physics*, math-ph/0011023, *Proc. Conference ICGTMP "Group-23"*, Dubna, 2000.
- [32] V.Abramov, R. Kerner, B. Le Roy , *Hypersymmetry: a  $Z_3$ -graded generalization of supersymmetry* J.Math.Phys. 38 (1997) 1650-1669,hep-th/9607143
- [33] M. Berger, *Sur les groupes d'holonomie homogène des variétés à connexion affine et des variétés riemanniennes*, Bull. Soc. Math. France **83** (1955) 279–310.
- [34] F. Anselmo, J. Ellis, D. Nanopoulos and G.Volkov, *Towards an algebraic classification of Calabi–Yau Manifolds: Study of  $K3$  spaces*, Phys. Part. Nucl. **32** (2001) 318–375, and Fiz. Elem. Chast. Atom. Yadra., **32** (2001) 605–698.
- [35] Candelas and A. Font, *Duality between the web of heterotic and type IIA vacua*, Nucl. Phys. **B511** (1998) 295-325, hep-th/9603170.
- [36] G. Volkov, *Hunting for the new symmetries in Calabi–Yau jungles*, Int. J. Mod. Phys. **A19** (2004) 4835–4860, hep-th/0402042.
- [37] L. Lipatov, A. Sabio-Vera, V. Velighanin and G. Volkov, *Reflexive numbers and Berger graphs from Calabi-Yau spaces* Int. J. Mod. Phys. **A25** (2006) –, hep-th/0501101 (2005) .
- [38] G.Volkov, *From Galois theory to the ultra-radical high energy physics*, preprint LAPTH-958-02 (2002).
- [39] A. Dubrovsky and G. Volkov, *Ternary numbers and algebras. Reflexive numbers and Berger graphs*. hep-th/0608073, (2006).
- [40] [www.neutrinolight.org](http://www.neutrinolight.org)
- [41] G. Volkov, talk at the OPERA Meeting, 10 January 2006.

(Manuscrit reçu le 20 septembre 2006)