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Dirac's principle in multimode interference of independent sources

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ABSTRACT. The extended Dirac's principle describes the interference between different particles as an effect of the multiparticle system with itself. In this paper we present a novel example, based on the detection of particles emitted in multimode states by independent sources, which illustrates in a simple way the necessity of extending the original Dirac's criterion.

1 Introduction

Interference effects are ubiquitous in quantum mechanics. At the beginning of modern quantum theory Dirac enunciated a criterion for the existence of interference effects between photons [1]:

> Each photon interferes only with itself. Interference between two different photons never occurs.

Originally Dirac formulated the principle only for photons, but soon it was extended to the case of massive particles.

Later, following the analysis of some new developments in quantum theory it was realized that the criterion should be interpreted in a more subtle way. These developments refer to situations where the wave function describing a multiparticle system cannot be separated into the product of the wave functions of the particles composing the complete system. There are two well-known scenarios where these states appear. One is by preparing the particles in an initially entangled state. In the language of wave functions this property translates into the impossibility of separating the wave function of the complete system. The other scenario is related to the spin-statistics connection. The multiparticle wave function of bosons or fermions must be symmetrized or antisymmetrized resulting in non separable wave functions. In both scenarios it is impossible to speak about the properties of any of the particles as an independent entity. One is tempted to interpret the interference effects associated with these systems as interferences between different particles. However, as the properties of the individual particles are only defined within a larger entity, the complete multiparticle system (the only entity that is defined from the quantum point of view), we really observe interferences of the complete system. According to Silverman, this extended Dirac's criterion can be expressed as [2]:

> A system interferes only with itself.

The extended formulation reflects the fundamental property of entangled systems of loosing the individuality of the particles within the multiparticle system, the only *single entity* from a quantum point of view.

Recently, it has been discussed in the literature other experimental scenario where the extended interpretation must be used, in spite of the fact that the situation refers to non-entangled photons emitted by independent sources [3]. Two sources emit independently photons, which are detected at two different positions. When the composed paths (the paths of the two photons) between the two sources and the two detectors are indistinguishable we must add the probability amplitudes obtaining interferences in the joint detection probability. Once more we do not face interferences between both photons, since the probability amplitudes that we add are those of composed paths. The composed paths are not properties of the individual photons, but a property of the two-photon system.

We present in this paper a new example of multiparticle system for which it is necessary the extended interpretation. It is based on a recently proposed arrangement [4], in which two sources independently emit particles in non-entangled multimode states. When the particles have common modes the probability of detecting only one of the two particles shows interference effects that must be associated with the complete multiparticle system.

2 Interferences in multimode states

We briefly describe the ideal arrangement considered in this paper, which has been presented in detail in Ref. 4 (see Fig 1). Two different sources independently emit indistinguishable particles. By the matter of simplicity we restrict our considerations to bosons (the case of fermions can be developed along very similar lines, but needs from a more elaborated and lengthy discussion of some technical aspects [4]). In the region of overlapping of the two beams we place at a fixed position a detector. We study the detections that occur at a given time t. We concentrate on the cases where only one of the two particles is detected, disregarding the events when the two particles are detected simultaneously. We assume, by simplicity, that the detector can distinguish between one- and two-particle detection events (see Ref. 4 for a realistic arrangement with this property).

The particles are emitted by the sources in the state

$$|I>=\int d^3\mathbf{p}\int d^3\mathbf{q}\eta(\mathbf{p})\mu(\mathbf{q})\hat{a}^+(\mathbf{p})\hat{a}^+(\mathbf{q})|0>, \tag{1}$$

where $\mathbf{p}$ and $\mathbf{q}$ represent the momenta of the particles, η and μ are the complex distributions of momenta, $\hat{a}^+(\mathbf{p})$ is the creation operator of a particle with momentum $\mathbf{p}$ and $|0>$ refers to the vacuum state in Fock's space. Moreover, by simplicity, we have assumed both particles to be in the same state of spin. The extension to particles in different states is straightforward.

The form of Eq. (1) follows directly from the independent nature of the two sources. When the momenta distributions are non-trivial, i. e., when they are different from zero for several values of the momentum we have a multimode distribution. We assume the modes to be plane waves, i. e., the mode of momentum $\mathbf{p}$ is $(2\pi\hbar)^{-3/2}\exp{(i\mathbf{p}.\mathbf{r}/\hbar)}$. We assume both mode distributions to be normalized to unity:

$$\int|\eta(\mathbf{p})|^2d^3\mathbf{p}=1=\int|\mu(\mathbf{p})|^2d^3\mathbf{p}. \tag{2}$$

Using the commutation relations (both particles are identical bosons), $\hat{a}(\mathbf{p})\hat{a}^+(\mathbf{q})-\hat{a}^+(\mathbf{q})\hat{a}(\mathbf{p})=\delta^3(\mathbf{p}-\mathbf{q})$, and the property of the annihilation operators $\hat{a}(\mathbf{p})|0>=0$, we have

$$<I|I>=1+\int d^3\mathbf{p}\int d^3\mathbf{q}\eta^*(\mathbf{p})\mu^*(\mathbf{q})\eta(\mathbf{q})\mu(\mathbf{p}). \tag{3}$$

To obtain this relation we have also used the normalization conditions (2) and the usual normalization of the vacuum, $<0|0>=1$.

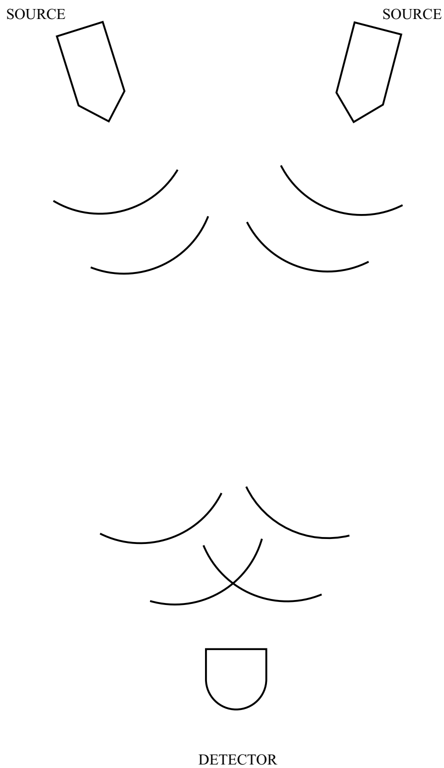

Figure 1: Schematic representation of the experiment. The two sources emit particles in multimode states characterized by the complex mode distributions μ and η. Both beams come together and mix at the position of the detector.

The probability of detecting only one of the two particles at point **r** and time t is given by [5]:

$$P(\mathbf{r},t) = \frac{< I|\hat{\psi}^{+}(\mathbf{r},t)\hat{\psi}(\mathbf{r},t)|I >}{< I|I >}, \qquad (4)$$

where $\hat{\psi}(\mathbf{r},t)$ is the Schrödinger field operator, which in the plane wave representation for the modes is given by

$$\hat{\psi}(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^{3/2}} \int d^3\mathbf{Q} \exp\left(i\mathbf{Q}.\mathbf{r}/\hbar\right) \exp\left(-iE(\mathbf{Q})t/\hbar\right)\hat{a}(\mathbf{Q}), \qquad (5)$$

where $E(\mathbf{Q}) = \mathbf{Q}^2/2m$ is the energy of the mode $\mathbf{Q}$.

Equation (4) can be evaluated in the same way done for the denominator (Eq. (3)). Interchanging the parameters $\mathbf{Q}$, $\mathbf{q}$ and $\mathbf{p}$ where necessary the result is

$$\begin{aligned} P(\mathbf{r},t) = \frac{1}{< I|I >}\frac{1}{(2\pi\hbar)^3} \int d^3\mathbf{p} \int d^3\mathbf{q}\eta^*(\mathbf{p})\mu^*(\mathbf{q}) \times \\ (\exp\left(-i(\mathbf{p}.\mathbf{r} - E(\mathbf{p})t)/\hbar\right)\eta(\mathbf{q}) \int d^3\mathbf{Q} \exp\left(i(\mathbf{Q}.\mathbf{r} - E(\mathbf{Q})t)/\hbar\right)\mu(\mathbf{Q}) + \\ \exp\left(-i(\mathbf{q}.\mathbf{r} - E(\mathbf{q})t)/\hbar\right)\eta(\mathbf{p}) \int d^3\mathbf{Q} \exp\left(i(\mathbf{Q}.\mathbf{r} - E(\mathbf{Q})t))/\hbar\right)\mu(\mathbf{Q}) + \\ \exp\left(-i(\mathbf{p}.\mathbf{r} - E(\mathbf{p})t)/\hbar\right)\mu(\mathbf{q}) \int d^3\mathbf{Q} \exp\left(i\mathbf{Q}.\mathbf{r} - E(\mathbf{Q})t)/\hbar\right)\eta(\mathbf{Q}) + \\ \exp\left(-i(\mathbf{q}.\mathbf{r} - E(\mathbf{q})t)/\hbar\right)\mu(\mathbf{p}) \int d^3\mathbf{Q} \exp\left(i(\mathbf{Q}.\mathbf{r} - E(\mathbf{Q})t)/\hbar\right)\eta(\mathbf{Q})) \end{aligned} \qquad (6)$$

We rearrange these expressions in a more tractable way. We introduce the following notation:

$$\alpha_{\mu\eta} = \frac{\int d^3\mathbf{Q}\mu^*(\mathbf{Q})\eta(\mathbf{Q})}{< I|I >}. \qquad (7)$$

From the definition and Eq. (2) they follow easily relations $\alpha_{\eta\eta} = \alpha_{\mu\mu} =< I|I >^{-1}$ and $\alpha^*_{\mu\eta} = \alpha_{\eta\mu}$. These coefficients correspond to the integrals in Eq. (6) that do not contain any dependence on **r** or t.

On the other hand, we introduce the time and position dependent functions:

$$P_{\eta\mu}(\mathbf{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3\mathbf{p}\int d^3\mathbf{q}\exp\left(i((\mathbf{q}-\mathbf{p}).\mathbf{r}-(E(\mathbf{q})-E(\mathbf{p}))t))/\hbar\right)\eta^*(\mathbf{p})\mu(\mathbf{q}). \tag{8}$$

Note that $P^*_{\eta\mu} = P_{\mu\eta}$.

Combining all these expressions, we obtain for the detection probability:

$$P(\mathbf{r},t) = \alpha_{\mu\eta}P_{\eta\mu}(\mathbf{r},t) + \alpha_{\eta\eta}P_{\mu\mu}(\mathbf{r},t) + \alpha_{\mu\mu}P_{\eta\eta}(\mathbf{r},t) + \alpha_{\eta\mu}P_{\mu\eta}(\mathbf{r},t). \tag{9}$$

Using the properties of the $\alpha's$ and $P's$ above remarked we have finally the equation

$$P(\mathbf{r},t) = \alpha_{\eta\eta}P_{\mu\mu}(\mathbf{r},t) + \alpha_{\mu\mu}P_{\eta\eta}(\mathbf{r},t) + 2Re(\alpha_{\mu\eta}P_{\eta\mu}(\mathbf{r},t)), \tag{10}$$

where $Re(\xi)$ denotes the real part of the complex expression ξ.

Equation (10) shows the existence of an interference phenomenon. The detection probability is composed of three terms. $P_{\mu\mu}(\mathbf{r},t)$ and $P_{\eta\eta}(\mathbf{r},t)$ represent the contributions, up to normalization factors, that one would obtain if only one of the two sources would emit particles. On the other hand, the third term $2Re(\alpha_{\mu\eta}P_{\eta\mu}(\mathbf{r},t))$ has the typical form of an interference term. It introduces a non trivial deviation with respect to the probability one would obtain if the detection events would be independent for both sources: the probability is not the sum of the probabilities corresponding to each source.

We emphasize that the interference phenomenon is observed at any fixed point $\mathbf{r}$. We are not dealing with spatially extended interferences fringes, for whose observation we would need to move the detector. The interference effects manifest at every point by the deviation of the detection probability from that corresponding to the two sources emitting independently. The form of the third term depends on the product of the two mode distributions. The interference phenomenon is clearly dependent on the existence of common modes: when there are no common modes the product $\eta^*(\mathbf{p})\mu(\mathbf{p})$ is zero, becoming null the function $P_{\eta\mu}$ and the interference term.

3 Discussion

The interference effects found in the previous section are explained in terms of the existence of common modes. The detector cannot distinguish if the common modes belong to one or other of the particles. In presence of alternatives that cannot be distinguished quantum mechanics leads to interference effects.

This is a novel manifestation of the extended Dirac's principle. The interference effects cannot be associated with one or other of the particles or with the interaction between them, but with the existence of common modes, which is not a property of the particles composing the system, but of the complete system. We face a self-interference effect of the complete system.

We note that the example proposed in this paper cannot be interpreted as a new manifestation of the spin-statistics connection. Although this connection has been used in the commutation relations it is clear that, in absence of common modes, the connection does not produce interference effects. We also remark that in our example the state is a non-entangled one, as in Ref. 3. The fundamental difference between both arrangements is that in that case the interference is associated with indistinguishable two-photon paths, whereas in our example the cause of the interferences is the existence of common modes.

Acknowledgments

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