Hamiltonian Symmetry in Special Relativity : Consequences of a decreasing light speed

FELIX T. SMITH

Molecular Physics Laboratory, SRI International
Menlo Park, CA, USA

ABSTRACT.In symmetric special relativity (SSR) Minkowski space-time is replaced by an expanding, spatially hyperbolic four-space. The light speed decreases with cosmological time and a new Hubble-Lorentz expansion constant $\sigma = c_0^2 H_0^{-1}$ enters the theory. A number of changes in the fundamental constants of physics and cosmology follow: (1) The logarithmic time derivative of c measures the Hubble time, and is predicted to be $-3.65 \times 10^{-11} y^{-1}$, a rate that may now be within reach of measurement. (2) The cosmologically decreasing light speed foreshortens the apparent time elapsed since the Big Bang when it is measured by light from remote sources, as compared to the historical look-back time measured locally in a some equivalent clock time. This allows ages as great as twice the Hubble time for local processes in the past history of objects such as stars or stellar clusters. (3) A number of former constants of physics lose their constancy, including energies and the Planck length and time. Among those remaining constant are masses, angular momenta and action, including Planck's constant, and the fine structure constant. (4) A new fundamental mass constant of gravitation and cosmology is identified, $m_* = (\hbar^2 H_0 / G_0 c_0)^{1/3}$, with the value $1.087(\pm 0.010) \times 10^{-28}$ kg. The product αm_* of m_* with the electromagnetic fine structure constant α is observed to account for 87% of the observed inertial mass m_e of the electron. This brings to light a new phenomenological relationship between the constants of electromagnetism and the electron on the one hand and those of gravitation and cosmology on the other.

1. Introduction

A new symmetric extension of special relativity [1], based in expanding hyperbolic position space instead of Minkowski space, has as one of its important consequences the conclusion that the speed of light systematically decreases in the cosmological expansion. The Hubble red shift is then attributed to the symmetrical combination of a decreasing light speed $c(t)$ and an expanding Hubble length scale $\rho(t) = c(t)H^{-1}(t)$, while their product $\sigma = c(t)\rho(t) = c_0{}^2 H_0{}^{-1}$ is a fundamental natural constant, the Hubble-Lorentz constant. Recent WMAP measurements establish the Hubble parameter H_0 and the age of the universe t_0 to an accuracy of 1 to 2% and make it possible to evaluate $\sigma = 3.89(\pm 0.06)\times 10^{34}\,\mathrm{m^2 s^{-1}}$.

This paper explores the consequences of this development for a number of the fundamental parameters of physics and cosmology. Some of these are currently within range of possible experimental testing, and others appear to provide theoretical support for astrophysical observations that have previously seemed to conflict with the prevailing models that rely on the constancy of c in cosmological time. A number of important new results are reported in detail below:

(a) The time dependence of c at the present epoch is given by the logarithmic derivative $\dfrac{\partial \ln c}{\partial t} = -\dfrac{H(t)}{2}$, whose present value would measure H_0 itself. The light speed c is therefore predicted to be decreasing at a current rate of about 4 parts in 10^{11} per year.

(b) The time dependence of c has the consequence that electromagnetic signals from a spatially remote source foreshorten the apparent time since an event at that source, as compared to the historical time elapsed since that event if it were measured locally. The local historical time scale is therefore longer than the cosmological time scale, providing a possible explanation for the apparently anomalous observed ages of some astrophysical objects.

(c) The Planck length and time cease to be relativistic invariants, leaving the Planck mass together with $\hbar$ and σ as true invariants. Energies are not invariant, masses are.

(d) The magnitudes of action A and of angular momentum are invariant, with a mass equivalency $m_A = A/\sigma$. A quantum of angular momentum or spin therefore carries a rest mass whose magnitude is a multiple of $m_\hbar = \hbar / \sigma = 2.71(\pm 0.04) \times 10^{-69}\,\mathrm{kg}$.

(e) The Newtonian gravitation constant as well as the electromagnetic permittivity and permeability of the vacuum all become time-dependent along with $c(t)$, but the products $\overline{G} =: G(t)/c(t) = G_0/c_0$, $\bar{\varepsilon}_o =: \varepsilon_o(t)c(t) = \varepsilon_{o(0)}c_0$ and $\bar{\mu}_o =: \mu_o(t)c(t) = \mu_{o(0)}c_0$ are invariants. The fine structure constant $\alpha = e^2 / 4\pi\varepsilon_{o(0)}c_0\hbar$ remains invariant.

(f) The time dependence of $c(t)$ and the newly established constancy of σ and $\overline{G} = G/c$ lead to the identification of a new fundamental constant of mass determined by the parameters of gravitation and the cosmological expansion, $m_* = \left(\hbar^2 H_0 / G_0 c_0\right)^{1/3}$. The recent WMAP measurements of the Hubble constant make it possible to evaluate this constant to 1% accuracy: $m_* = 1.087(\pm 0.010) \times 10^{-28}$ kg.

(g) The product of this fundamental mass constant and the fine-structure constant α of electrodynamics is the mass $m_{e*} = \alpha m_* = 7.94 \times 10^{-31}$ kg. This is seen to be very close to the observed inertial mass of the electron, $m_e = 1.148 m_{e*}$

Phenomenologically, this brings to light a new relationship connecting the constants of electromagnetism and the unit charge with the constants of gravitation and the cosmological expansion. This identity has important implications for the theory of the electromagnetic, gravitational and inertial properties of the electron. These are explored in a following paper.

2. Background

Constancy of the speed of light has commonly been assumed to be an essential postulate of special relativity. Einstein's original postulate, however, was not so constricting. It has been stated as: "*The velocity of light is independent of the motion of the light source*". As early as 1921, Pauli said about this [2], "For conciseness, we denote this by 'constancy of the speed of light', although such a designation might give rise to misunderstandings. There is no question of a universal constancy of the velocity of light in vacuo, if only because it has the constant value c only in Galilean systems of reference." It is to be expected that c will not be constant in an expanding, time-dependent, hyperbolic universe.

The Einstein postulate permits the extension of special relativity to a broader domain of kinematics than that supported by Minkowski space. If the position space is taken as hyperbolic and expanding with the Hubble relationship, the entire structure of Hamiltonian dynamics can be made explicitly covariant and the integration of quantum mechanics with relativity is broadened and simplified [1]. The result is an enlarged Symmetric Special Relativity (SSR) that conforms to the cosmological topology of the observed universe, eliminating the unlimited divergences that afflict the Minkowski world model at cosmologically remote distances and at times approaching the Hubble time.

To accomplish these goals while preserving all the physical consequences of the Lorentz transformation and related procedures requires significant changes in the formalism of SR. One of the conclusions that stem from requiring improved agreement with the broad features of the Hubble expansion is that the velocity of light is not constant in cosmological time. Instead the apparent expansion is shared symmetrically between a decreasing light speed $c(t)$ and an increasing length scale in position space $\rho(t) = c(t)H^{-1}(t)$ in such a way that their product is a constant, the Hubble-Lorentz constant, $c(t)\rho(t) = \sigma$.

A. Geometry and Kinematics in Symmetric Special Relativity

Einstein's SR altered the kinematics of Galileo and Newton by introducing the scale parameter c into velocity space. This converted the flat space of Galilean velocity and momentum into the negatively curved, hyperbolic space of velocity in SR. The simplest position-space geometry describing the observed Hubble expansion is also negatively curved and hyperbolic. It has a scale parameter that is time-dependent, $\rho(t) = cH^{-1}(t)$. The new extension of SR incorporates this scale parameter in position space. A relativistic position-velocity symmetry can then be completed by recognizing a time-dependence in $c(t)$ as well.

The symmetries of the extended form of special relativity are embodied in a new hyperbolic Poincaré group, which has the usual Lorentz and Poincaré groups as subgroups and also displays additional symmetries, including the symplectic symmetry of Hamiltonian dynamics. The new symmetry requires the cosmological expansion to be shared between a decreasing light speed $c(t)$ and the expanding length scale $\rho(t)$ in such a way that their product

$$\sigma = c(t)\rho(t) = c_0^2 H_0^{-1}. \tag{1}$$

is a fundamental natural constant, the Hubble-Lorentz constant, while retaining the basic Hubble relationship

$$H(t) = c(t)/\rho(t). \tag{2}$$

It follows that

$$\rho(t) = \sigma^{1/2}H^{-1/2}(t) \text{ and } c(t) = \sigma^{1/2}H^{1/2}(t). \tag{3}$$

Recent WMAP measurements of the Hubble constant H_0 and the age of the universe t_0 to an accuracy of 1 to 2% make it possible to evaluate $\sigma = 3.89(\pm 0.06)\times 10^{34}\,\mathrm{m^2s^{-1}}$.

The Symmetric Special Relativity (SSR) that results from these changes has not one but two Lorentzian symmetries. The first, ordinary Lorentz symmetry, describes the consequences for both position space and velocity-momentum space of a boost in velocity, i.e., a hyperbolic translation in the rapidity vector $\boldsymbol{\varepsilon}$ whose magnitude is

$$\varepsilon = \tanh^{-1}(v/c) = \tanh^{-1}\left(v/\sigma^{1/2}H^{1/2}[t]\right). \tag{4}$$

The second Lorentz-like symmetry recognizes that the Hubble effect of a translational shift in hyperbolic position space on an observed velocity is the converse of the Lorentz effect of a velocity boost on an observed length, while its effect in position space is an Einstein-like addition. The hyperbolic position variables comparable to $\boldsymbol{\varepsilon}$ comprise the separation vector $\boldsymbol{\eta}$, whose magnitude is

$$\eta = \sinh^{-1}\left(r/\rho[t]\right) = \sinh^{-1}\left(r/\sigma^{1/2}H^{-1/2}[t]\right). \tag{5}$$

There results an important relativistic position-velocity symmetry,

$$\mathbf{r} \leftrightarrow \mathbf{u}, \text{ where } \mathbf{u} = \mathbf{v}\left(1 - v^2/c^2\right)^{-1/2}, \text{ i.e., } \boldsymbol{\eta} \leftrightarrow \boldsymbol{\varepsilon}. \tag{6}$$

This position-velocity symmetry is accompanied by an equivalent Hamiltonian coordinate-momentum symmetry, provided we use the generalized coordinates and momenta defined nonrelativistically by

$$\overline{\mathbf{q}} = m^{1/2}\mathbf{r} \leftrightarrow \overline{\mathbf{p}} = m^{-1/2}\mathbf{p} = m^{1/2}\mathbf{u}. \tag{7}$$

The equivalent relativistic expressions of space and time can be written as four-vectors,

$$X\left(t,\eta\right)=\begin{pmatrix} w \\ \mathbf{r} \end{pmatrix}=\sigma^{1/2}H^{-1/2}\left(t\right)\begin{pmatrix} \cosh\eta \\ \sinh\eta\,\hat{\eta} \end{pmatrix}, \tag{8}$$

where w is the fourth, time-like, component of the space-time four-vector and reduces to $c_0H_0^{-1}+c_0\tau$ in the Minkowski limit. The relativistic velocity four-vector is

$$U\left(t,\boldsymbol{\varepsilon}\right)=\begin{pmatrix} c\left(t\right)\cosh\varepsilon \\ \mathbf{u} \end{pmatrix}=\sigma^{1/2}H^{1/2}(t)\begin{pmatrix} \cosh\varepsilon \\ \sinh\varepsilon\,\hat{\boldsymbol{\varepsilon}} \end{pmatrix}. \tag{9}$$

For further details including the extension to four-by-four tensor expressions needed for full covariance under both boosts in velocity and shifts in hyperbolic position, see reference [1].

The integration of the Hubble expansion into special relativity brings the cosmic time t as a new parameter into the structure of SSR. When t is measured from its natural origin in the Big Bang, it is a variable orthogonal to the hyperbolic three-vectors $\boldsymbol{\eta}$ and $\boldsymbol{\varepsilon}$. The coordinate system $\left(t,\boldsymbol{\eta}\right)=\left(t,\eta,\theta_{\boldsymbol{\eta}},\phi_{\boldsymbol{\eta}}\right)$ is the natural one to describe relativistic covariance under both velocity boosts, which can be expressed by $\Delta\boldsymbol{\varepsilon}$, and position shifts, expressed by $\Delta\boldsymbol{\eta}$; these boosts and shifts alter both $\boldsymbol{\eta}$ and $\boldsymbol{\varepsilon}$, but leave t invariant. This invariant cosmic proper time t, synchronized at the big bang origin, stands in notable contrast to the local, frame-dependent relative times or observer times τ_n of Minkowski space and ordinary SR,

$$\tau_n = H^{-1}(t)\cosh\eta_n - H_0^{-1} \tag{10}$$

that are associated with individual mass-points n and their position vectors

$$\mathbf{r}_n = \rho(t)\sinh\eta_n\ \hat{\boldsymbol{\eta}}_n = \sigma^{1/2}H^{-1/2}(t)\sinh\eta_n\ \hat{\boldsymbol{\eta}}_n \tag{11}$$

in individual four-vectors of the form of Eq. (8).

Both of these time variables, t and τ, are important for the evaluation of the effects of the cosmological time-dependence of c. The system of cosmological coordinates $\left(t,\boldsymbol{\eta}\right)$ ensures relativistic covariance, and is the system in which basic principles can be established. To deal with practical measurements, however, a system of coordinates $(\tau, \mathbf{r})$ must be used with a locally established origin and scale in both position and time (terrestrial or laboratory coordinates). The transformation between these two coordinate systems is given in the next Subsection.

B. Cosmological and Laboratory Coordinates

The cosmological time variable t has its natural origin at the big bang singularity of the observed Hubble expansion. Terrestrial measurements, however, require using a time variable τ with a recent origin $\tau_0 = 0$. A transformation of coordinates is required to go from the position and time coordinates $(\mathbf{r}, \tau)$ of a local measurement to the covariant coordinates of the cosmological system, i.e., the cosmic time t and the dimensionless hyperbolic vector $(\boldsymbol{\eta} = \eta, \theta_{\boldsymbol{\eta}}, \phi_{\boldsymbol{\eta}})$, supplemented by the curvature length $\rho(t)$ of Eq. (3). The angular coordinates are not affected by the transformation, and we can confine our attention to the two-dimensional transformation $(\eta, t) \leftrightarrow (r, \tau)$, with the Hubble length $\rho(t) = \sigma^{1/2} H^{-1/2}(t)$ as the auxiliary function.

The Hubble function $H(t)$ can be left unspecified in establishing the general form and properties of the coordinate transformation, provided only that H is a monotonic function of the time. The observed properties of the expansion at the present epoch are consistent with the simple reciprocal time approximation

$$H(t) \cong t^{-1}, \tag{12}$$

and this provides a convenient model and practical approximation useful everywhere except, probably, the very early stages of the cosmic expansion. When it is used the Hubble length and the light speed are given by

$$\rho(t) \cong \sigma^{1/2} t^{1/2}, \qquad c(t) \cong \sigma^{1/2} t^{-1/2}. \tag{13}$$

The cosmological-to-laboratory transformation of coordinates is derived in reference [1], and its essentials are summarized here. We first convert from a hyperbolic representation in the cosmological variables t, η by an orthogonal transformation to the coordinates w, r:

$$w = \rho(t) \cosh \eta, \tag{14}$$

$$r = \rho(t)\sinh\eta\,. \tag{15}$$

The resulting position variable r is now independent of the transformed time coordinate w. It follows that the cosmic expansion embodied in the function $\rho(t)$ does not affect the local measurement of lengths, which takes place in the physical three-space, orthogonal to w, of the three-vector $\mathbf{r}$ with its magnitude r.

If we define the cosmological time to be t_0 when $(\tau = 0, \eta = 0)$, so that $t = t_0 + \delta t$, the expanding curvature length is

$$\rho(t) = \rho_0 + (d\rho/dt)_0\,\delta t + \ldots. \tag{16}$$

To introduce a purely local time variable τ we can write w in the alternative form

$$w = \rho_0 + ac_0\tau \tag{17}$$

and require as a matching condition at $t = t_0$ and $r_0 = 0$ that the rate of a clock be the same whether measured in the local variable τ or the cosmological one t:

$$(\partial t/\partial\tau)_{t_0,r_0} = 1, \quad \text{or} \quad (\partial w/\partial t)_{t_0,r_0} = (\partial w/\partial\tau)_{t_0,r_0}\,. \tag{18}$$

Comparing the expressions for w in Eqs. (14) and (17) we find

$$ac_0 = \cosh\eta\,(d\rho/dt)_{t_0}\text{, and thence} \tag{19}$$

$$\tau = \delta t + (d\rho/dt)_0^{-1}\left[\rho(t_0 + \delta t) - \rho_0\,\mathrm{sech}\,\eta\right]. \tag{20}$$

If we expand $\rho(t_0 + \delta t)$ about t_0 this becomes:

$$\tau = \delta t + (d\rho/dt)_0^{-1}\left[\rho_0\left(1 - \mathrm{sech}\,\eta\right) + \frac{1}{2}\left(\frac{d^2\rho}{dt^2}\right)_0 \delta t^2 + \ldots\right]. \tag{21}$$

We can now apply Eqs. (20), and (21) to the simple approximation of Eq. (12) for the time dependence of the Hubble function, $H(t) = t^{-1}$. The transformation from (η, t) to (r, τ) coordinates now takes the form

$$\tau\left(t, \eta; t_0\right) = 2t_0^{1/2} t^{1/2} - 2t_0 \operatorname{sech}\eta , \tag{22}$$

$$r(t, \eta) = \sigma^{1/2} t^{1/2} \sinh\eta . \tag{23}$$

The inverse relationship between the two coordinates of time can be written as

$$t\left(\tau, r; t_0\right) = t_0 \left(1 + \frac{r^2}{\sigma t_0}\right)^{-1} + \tau \left(1 + \frac{r^2}{\sigma t_0}\right)^{-1/2} + \frac{\tau^2}{4t_0^{\,2}}, \text{where } \sigma t_0 = \rho_0^{\,2} \cong \left(c_0 H_0^{-1}\right)^2 \tag{24}$$

and its partner $\eta(\tau, r; t_0)$ follows by a simple substitution of (24) in (23),

$$\sinh\eta\left(\tau, r; t_0\right) = \frac{r}{\sigma^{1/2} t^{1/2}\left(\tau, r; t_0\right)}. \tag{25}$$

C. Fundamental Data: The Hubble Expansion Rate.

The simple linear approximation of Eq. (12) for the Hubble function has the consequence that the age of the universe since the Big Bang is identically the Hubble time, $t_0 = H_0^{-1}$. In a more detailed cosmological model the estimated age t_0 of the universe is not necessarily the same as H_0^{-1}, the reciprocal of the present value of the Hubble expansion rate, and both of them are often reported separately in the treatment of astrophysical data.

The state of information with regard to the Hubble constant has become reassuringly consistent since the measurements from the Hubble Space Telescope [4,5] and the Wilkinson Microwave Anisotropy Probe [6] have become available. The HST measurements showed $H_0 \cong 72 \left(\text{km s}^{-1}\right)\text{Mpc}^{-1}$, with error limits of ±10%. The WMAP data have narrower error limits, and agree very closely with each other, with a central value of $H_0 \cong 71 \left(\text{km s}^{-1}\right)\text{Mpc}^{-1}$ and error limits varying between ±1% and ±5%.

The best WMAP estimates of these quantities at present are those compiled by Bennett et al. [6] in their Table 3. They show a Hubble constant of $H_0 = 71^{+4}_{-3}\left(\text{km s}^{-1}\right)\text{Mpc}^{-1}$. This is equivalent to a Hubble time of $H_0^{-1} = 13.8^{+0.6}_{-0.8}\,\text{Gyr}$. They also give a separate best estimate of the age of the universe t_0. For this they report $t_0 = 1.37(\pm 0.02)\times 10^{10}\,\text{y}$. Their reported values of H_0^{-1} and t_0 are therefore essentially equal, within their error limits for t_0. I shall therefore use the same value for both time estimates,

$$H_0^{-1} \cong t_0 = 1.37(\pm 0.02)\times 10^{10}\,y.. \tag{26}$$

We can then evaluate σ as

$$\sigma = c_0^2 H_0^{-1} = 3.89(\pm 0.06)\times 10^{34}\,\text{m}^2\text{s}^{-1}. \tag{27}$$

3. Observational Consequences of the Time Dependent Light Speed

A. The Rate of Change of c

1. The Time Dependence of c as a Measure of the Hubble Parameter:

It follows from Eq. (14) that the dependence of the speed of light on the cosmological time t should be directly related to the Hubble function,

$$\frac{dc(t)}{dt} \cong -c(t)\frac{H(t)}{2}. \tag{28}$$

The present magnitude of the logarithmic rate of change of c is then predicted to be

$$\left[\frac{d\ln c(t)}{dt}\right]_0 \cong -\frac{H_0}{2} = -3.65(\pm 0.04)\times 10^{-11}\,\text{y}^{-1}. \tag{29}$$

A rate of change of this magnitude, should it be detectible in a terrestrial measurement, would be approaching the range of observability at the present time. Such a measurement would both confirm the temporal behavior of c and provide a direct means of measuring the Hubble parameter in a laboratory system. To verify the applicability of the time dependence of c as in Eq. (29) to a measurement in the laboratory system we can apply to the time dependence of c the transformation between cosmological and laboratory coordinates presented in Eqs. (22) to (25).

B. Application to the Speed of Light

Eq. (22) can be used to express the time dependence of the light speed c directly in terms of the local time variable τ based at $\left(t = t_0, \eta_0 = 0\right)$ instead of the cosmological time variable t:

$$c\left(t\right) = \sigma^{1/2} t^{-1/2} = c\left(\tau, \eta; t_0\right) = c_0 \cosh\eta \left(1 + \frac{\tau\cosh\eta}{2t_0}\right)^{-1}. \tag{30}$$

In the case of a local measurement of c we are interested only in the spatial region near $r = 0, \eta = 0$. To first order, then,

$$\left(\frac{\partial c}{\partial \tau}\right)_{\eta=0} = -\frac{c_0}{2t_0} = -\frac{c_0}{2H_0^{-1}}. \tag{31}$$

A local measurement of the dependence of c on the locally observable time τ is therefore predicted to reflect the same secular decrease as was predicted in Eq. with respect to the cosmological time variable t:

$$\left(\frac{\partial \ln c\left[\tau, \eta\right]}{\partial \tau}\right)_{\eta=0} = \left(\frac{d \ln c\left[t\right]}{dt}\right) = -\frac{H_0}{2}. \tag{32}$$

As discussed above in connection with Eq. (15), the cosmological expansion reflected in the Hubble length $\rho(t)$ is never detectible locally through the length variable r accessible to terrestrial measurement. In contrast, the cosmological dependence of the light speed $c(t)$ on the cosmological time variable t is reflected in an equivalent dependence of $c(\tau, \eta)$ on the local observer's time variable τ.

It can also be remarked that in SSR both the fundamental atomic frequencies that are used as the measuring standards for time and the fundamental constants such as the fine structure constant α are predicted to be time-independent in terrestrial measurements.

C. The Possibility of Measurement

The light speed c was measured in 1972 with a precision of 3.5 parts in 10^9 [3]. In the intervening 33 years, the present theory predicts it will have decreased by about 1 part in 10^9. If renewed measurement can improve the precision in such measurement by a factor of 10 to 100, then this theory can be tested more conclusively in times of the order of a few years. It is worth noting that an absolute measurement of c will not be needed provided its rate of change can be measured with regard to a sufficiently stable standard of length. If c is found to decrease by a rate close to this prediction, it follows that the Hubble parameter H_0 will be measurable directly in the laboratory.

The detection of a rate of change of this magnitude in c will be of technological importance for applications of the GPS system in geodesy, because this theory suggests that apparent distances as measured with the present definition of the meter will result in a rate of change of close to a millimeter a year in an earth radius.

4. Local and Cosmological Measurements of Age

A. The Effect of Velocity Boosts and Distance Shifts on the Connection between Cosmological and Local Time Scales

The connection between the local time τ as measured by a stationary clock or by a local observer and the invariant cosmological time t as given in Eqs. and above can be used also at times and locations remote from the reference origin $\left(t = t_0, r = 0; v/c = 0\right)$ or $\left(\tau = 0, \eta = 0; \varepsilon = 0\right)$ of local time. The cosmological time t is an invariant under spatial shifts measured in the hyperbolic variables $\boldsymbol{\eta} = \left(\eta, \theta_{\boldsymbol{\eta}}, \phi_{\boldsymbol{\eta}}\right)$ as well as velocity boosts in $\boldsymbol{\varepsilon} = \left(\varepsilon, \theta_{\boldsymbol{\varepsilon}}, \phi_{\boldsymbol{\varepsilon}}\right)$, but the local time variable τ depends on t_0 as well as on a position shift $\Delta\boldsymbol{\eta}$ or a rapidity boost $\Delta\boldsymbol{\varepsilon}$. To evaluate their effect on the local time observable τ we can use Eq. (22) with the replacement $\eta \to \left|\Delta\boldsymbol{\eta} \mathbin{\hat{-}} \Delta\boldsymbol{\varepsilon}\right|$, where the subtraction is performed by using the law of cosines for the sides of the hyperbolic vector triangle (see ref. [1]). We then have

$$\tau\left(t, \Delta\boldsymbol{\eta}, \Delta\boldsymbol{\varepsilon}; t_0\right) = 2 t_0^{1/2} t^{1/2} - 2 t_0 \operatorname{sech}\left|\Delta\boldsymbol{\eta} \mathbin{\hat{-}} \Delta\boldsymbol{\varepsilon}\right|. \tag{33}$$

B. Local Historical Time and Remote Cosmological Time

We can use Eq. (3) and specialize to the local condition near $r \to 0, \eta \to 0$, to illustrate how the local time

$$\tau_{\eta=0} = 2t_0\left[\left(1+\delta t / t_0\right)^{1/2} - 1\right] \tag{34}$$

diverges from the cosmological time increment of time δt increasingly as both depart from their reference point t_0. Looking back toward the origin of the cosmic expansion at $t = 0, \delta t = -t_0$, we see that local clocks will measure that remote origin as occurring at $\tau = -2t_0$. This permits a local historical time scale for the evolution of astronomical objects and radioactive materials that can be up to twice the available Hubble age of the universe when measured remotely through propagating radiation.

5. The Time-Independent Fundamental Constants and Consequences

The fact that c is no longer to be treated as a constant of nature and that the Hubble-Lorentz constant σ is such a constant requires a new assessment of a number of the other fundamental constants of atomic physics and cosmology. Among its consequences is a new quantification of some of the famous dimensionless ratios with magnitudes of the order of 10^{40} constructed from the constants of cosmology and atomic physics. This brings to light a numerical identity between two of these numbers, and establishes a firm relationship connecting the gravitational and cosmological constants $\left(G_0 / c_0\right)$ and $\sigma = c_0{}^2 H_0{}^{-1}$ and the atomic and electromagnetic constants $\hbar, m_e$ and α.

A. The Fundamental Natural Constants in SSR

1. Mass and Energy

The time dependence of c in the doubly hyperbolic universe requires abandoning the conservation of energy on a cosmological time-scale, but allows the conservation of mass and the constancy of particle masses to remain unchanged. Planck's constant h or $\hbar$ and the dimensionless fine structure constant α remain among the fundamental constants and are joined by the new Hubble-Lorentz constant σ.

2. Interactions: Newton's and Coulomb's Laws

Interaction potentials become time-dependent like other energy terms, but the corresponding mass expressions $\Delta m_{\mathrm{Coul}} = V_{\mathrm{Coul}} c^{-2}$ and $\Delta m_{\mathrm{grav}} = V_{\mathrm{grav}} c^{-2}$ must remain constant like other masses. It follows that the electrostatic permittivity ε_{o} and the Newtonian gravitational constant G both become time-dependent, but the product

$$\bar{\varepsilon}_{\mathrm{o}} = \varepsilon_{\mathrm{o}}(t) c(t) = \varepsilon_{\mathrm{o}(0)} c_0 \tag{35}$$

and its gravitational analog

$$\bar{G} = G(t) / c(t) = G_0 / c_0 = 2.22574(\pm 0.00028) \times 10^{-19} \,\mathrm{m}^2 \mathrm{kg}^{-1} \mathrm{s}^{-1} \tag{36}$$

are constant. Combining the electromagnetic identity $\varepsilon_{\mathrm{o}} \mu_{\mathrm{o}} c^2 = 1$ and Eq. (35) we get the new velocity-corrected constant of magnetic permeability

$$\bar{\mu}_o = \mu_o(t) c(t) = \mu_{o(0)} c_0. \tag{37}$$

It follows from Eq. (35) that the fine structure constant is unaffected by the time variation in $c(t)$:

$$\alpha = \frac{e^2}{4\pi \varepsilon_{\mathrm{o}}(t) c(t) \hbar} = \frac{e^2}{4\pi \bar{\varepsilon}_{\mathrm{o}} \hbar}. \tag{38}$$

Its constancy has recently been confirmed by several ultraprecise frequency measurements in atomic spectra which show that it changes by less than two parts in 10^{15} per year [7].

3. The Planck Units and Atomic Units

The Planck mass

$$m_{\mathrm{Pl}} = (\hbar c / G)^{1/2} = 2.17671(\pm 0.00014) \times 10^{-8} \,\mathrm{kg} \tag{39}$$

remains constant because it in fact contains the new constant $\overline{G} = G / c$, but the Planck units of length, time and energy become time-dependent and lose their role as natural constants.

In the same way, the atomic units of length and time vary with cosmological time, the Bohr radius a_0 increasing cosmologically as $(t/H_0^{-1})^{1/2}$,

$$a_0 = a_{00}\left(\frac{t}{H_0^{-1}}\right)^{1/2} = \left(\frac{\hbar}{m_e \alpha c_0}\right)\left(\frac{t}{H_0^{-1}}\right)^{1/2}. \tag{40}$$

and the atomic time increasing as $\left(t/H_0^{-1}\right)$,

$$t_{\mathrm{at}}\left(t\right) = \left(\frac{a_{00}}{\alpha c_0}\right)\left(\frac{t}{H_0^{-1}}\right) = \left(\frac{\hbar}{m_e \alpha^2 c_0^{\;2}}\right)\left(\frac{t}{H_0^{-1}}\right). \tag{41}$$

All the atomic units are in fact time-varying except those of mass, m_e, and of angular momentum and action, $\hbar$.

4. Constants and Semiconstants

Many of the numbers we have been accustomed to think of natural constants can now be perceived to have a slow cosmological variation comparable to that of the Hubble parameter $H(t)$. Their measured values at the present epoch are as important as ever, but their secular variation needs to be acknowledged. Such quantities I shall denote as “semiconstants”. The fundamental semiconstants include $c, G, \varepsilon_o,$ and H.

Many other accepted constants of physics including atomic constants such as the Rydberg constant, the Bohr radius and the Bohr magneton, can now be seen to be semiconstants. Each of them can be converted to a related cosmologically constant magnitude by using the appropriate power of $\left(c(t)/c_0\right) = \left(t_0/t\right)^{1/2}$ as a factor.

B. The Mass of Quantized Action and Angular Momentum

With the recognition of the Hubble-Lorentz constant σ as a fundamental constant, action A and angular momentum become connected with a related mass M as conserved quantities in the proportionality

$$A = M\sigma . \tag{42}$$

It follows from this that a quantum of angular momentum or action possesses its own mass, a multiple of the extremely minute elementary mass

$$m_\hbar = \hbar / \sigma = 2.71\left(\pm 0.04\right)\times 10^{-69}\,\text{kg}\,. \tag{43}$$

This raises the possibility that photons themselves may carry a minute rest mass of this origin.

C. Masses and Mass Ratios: The Cosmological Sequence

With the help of σ as a fundamental constant, it becomes possible to construct a constant of mass which depends only on the macrocosmic constants $\overline{G}$ and σ, i.e. on the measured parameters $\left(G_0, c_0, H_0^{-1}\right)$. Unlike the Planck mass it is independent of Planck's constant $\hbar$:

$$m_G = \frac{\sigma}{\overline{\overline{G}}} = \frac{c\sigma}{G} = \frac{c_0^{\ 3}}{G_0 H_0} = 1.75\left(\pm 0.03\right)\times 10^{51}\ \text{kg}\,. \tag{44}$$

This mass is just twice the mass of the universe of cosmological theory, the total mass within the Hubble horizon $\rho_0 = c_0 H_0^{\ -1}$, assuming a homogeneous distribution with the critical density $d_0 = 3H_0^{\ 2} / 8\pi G_0$. Under the usual assumptions, with c and G time-independent, the mass m_G increases in time with H^{-1}. In SSR, where $\overline{G}$ and σ are now constant, but G and c are not, this universal mass m_G can also be recognized as constant in time.

Comparing Eq. (44) with Eqs. (39) and (43) we see that the Planck mass is the geometric mean of this macrocosmic mass m_G and the quantal action mass $m_\hbar$:

$$m_{\text{Pl}}^{\ 2} = m_\hbar m_G\,. \tag{45}$$

From the masses m_G, m_e and $m_\hbar$ we can create three mass ratios:

$$\lambda_1 = \frac{m_\hbar}{m_e} = \frac{\hbar}{\sigma m_e} = 2.98\left(\pm 0.04\right)\times 10^{-39}\,, \tag{46}$$

$$\lambda_2 = \frac{m_e}{m_G} = 5.22\left(\pm 0.07\right)\times 10^{-84}\,,\ \text{and} \tag{47}$$

$$\lambda_3 = \lambda_1 \lambda_2 = \frac{m_\hbar}{m_G} = \left(\frac{m_\hbar}{m_{\text{Pl}}} \right)^2 = 1.55\left(\pm 0.05\right) \times 10^{-122} . \tag{48}$$

In the ratio $\lambda_1 = m_\hbar / m_e$, Eq. (46), which depends on the measured constants or semiconstants $\left(m_e, \hbar, c_0, H_0\right)$ and is independent of G_0, we see an appearance of a number close to the famous gravitational and cosmological large number $\approx 10^{-40}$. A number roughly comparable to its square is seen in ratio $\lambda_2 = m_e / m_G$ of Eq. (47). Naturally, their product, the ratio λ_3 in Eq. (48), shows a version of this cosmological number occurring to the third power.

The suggestive impression of a geometrical sequence in the three numbers λ_i can be improved to form a true geometric sequence based on the root value $k = a\lambda_1$, where we can choose a to satisfy the condition $\lambda_3 = k^3 = a^3 \lambda_1^{\,3}$. Evaluating the number a we find

$$a = \lambda_3^{\,1/3} / \lambda_1 = 8.4 \times 10^{-3} \cong 1/119 . \tag{49}$$

This is close enough to the fine structure constant α to suggest improving the sequence of the λ_n by introducing not an arbitrary number a but rather the known constant α as a factor in the first two members of the sequence, replacing m_e by m_e / α in each case:

$$k_1 = \alpha \lambda_1 = \frac{\alpha\, m_\hbar}{m_e} = \frac{\alpha \hbar H_0}{m_e c_0^{\,2}} = 2.17\left(\pm 0.03\right) \times 10^{-41} , \tag{50}$$

$$k_2 = \alpha^{-1} \lambda_2 = \frac{m_e H_0 G_0}{\alpha c_0^{\,3}} = 7.15\left(\pm 0.10\right) \times 10^{-82} , \tag{51}$$

$$k_3 = \lambda_3 = k_1 k_2 = \frac{\hbar H_0^{\,2} G_0}{c_0^{\,5}} = 15.52\left(\pm 0.40\right) \times 10^{-123} . \tag{52}$$

Two of the three ratios k_n are functionally independent, and the third is redundant. Obviously the geometric appearance of the sequence is greatly improved.

It is now natural to write this sequence in the form

$$k_1,\ k_2 = fk_1^2, \quad k_3 = fk_1^3, \tag{53}$$

where k_1 is given by Eq. (50) and

$$f = \left(\frac{m_e}{\alpha\hbar}\right)^3 \frac{\hbar c_0 G_0}{H_0} = 1.513\big(\pm 0.020\big). \tag{54}$$

Because the accuracy of our knowledge of both k_1 and f is limited by the present error limits of H_0, it is also important to present separately the ratio k_2 / k_1, an important constant which will be denoted γ_g.

$$\gamma_g = \frac{k_2}{k_1} = \frac{\lambda_2}{\alpha^2\lambda_1} = \left(\frac{G_0 m_e^2}{c_0}\right)\left(\frac{1}{\alpha^2\hbar}\right) = 3.28887\big(\pm 0.00042\big)\times 10^{-41}. \tag{55}$$

It is independent of H_0. It can therefore be evaluated to the accuracy of five significant figures with which the gravitation constant G_0 is known.

The dimensionless ratio γ_g is very close in magnitude to the ratio k_1 of Eq. (50). Both their relationship and their significant difference can be emphasized by relabeling k_1 as γ_e and displaying it in the form

$$\gamma_e = k_1 = \frac{\alpha\hbar}{m_e} = \left(\frac{e^2}{4\pi\varepsilon_o c}\right)\left(\frac{1}{\sigma m_e}\right) = 2.17\big(\pm 0.03\big)\times 10^{-41}. \tag{56}$$

These two constants are related by the factor f of Eq. (54),

$$\gamma_g = f\gamma_e \cong 1.5\gamma_e. \tag{57}$$

D. The Cosmological Large Numbers of Dirac

The pattern shown in the sequence of Eqs. (50) to (52) now provides a new and quantitative confirmation of the long-suspected relationship between several large dimensionless ratios of cosmology and atomic physics, of the order of 10^{40} in magnitude. These have been discussed in various ways since the work of Eddington and Dirac in the 1930s. Dirac's 1937-38 papers [8,9] provide a useful starting point to develop the connection. Dirac discussed this problem in the context of general relativity, but it already exists at the level of special relativity, and it can be clarified in the expanding hyperbolic space approximation of SSR.

In his discussion, Dirac pays special attention to three dimensionless ratios of atomic physics and cosmology. These are the force ratio R_{force} of the gravitational to electrostatic forces between two charged elementary particles, the mass ratio R_{mass} of an elementary particle to that of the mass of the universe m_U, and the time ratio R_{time} of a characteristic time of atomic physics to the Hubble time H_0^{-1} of the universal expansion.

For the ratio R_{force} of electric and gravitational forces, Dirac chose as the type pair an electron and a proton, but he also commented that other pairs such as two electrons could also be used. We can now prefer the two-electron case, where this force ratio is

$$R_{\mathrm{force}}\left(e,e\right)=\frac{Gm_e^{\ 2}}{\left(e^2/4\pi\varepsilon_{\mathrm{o}}\right)}=\left(\frac{G_0}{c_0}\right)\frac{m_e^{\ 2}}{\alpha\hbar}\,. \tag{58}$$

This ratio is a constant, both under the assumptions of ordinary SR and those of SSR.

It can immediately be identified as the product of the fine structure constant α and the dimensionless constant γ_g of Eq. (55):

$$R_{\mathrm{force}}(e,e)=\alpha\gamma_g\,. \tag{59}$$

For the cosmological mass ratio R_{mass}, where Eddington and Dirac compared the mass of the universe $m_U = m_G/2$ with that of a proton, I shall use instead the electron as the comparison particle. The relevant mass ratio is then

$$R_{\text{mass}}\left(e,U\right)=\frac{2m_e}{m_G}=2m_e\left(\frac{G}{c}\right)\left(\frac{H}{c^2}\right)=\frac{2m_eG_0H_0}{c_0{}^3}, \tag{60}$$

a constant under SSR but time-dependent under SR. It can be recognized as a product of the constants γ_e and γ_g,

$$R_{\text{mass}}\left(e,U\right)=2\alpha\gamma_e\gamma_g\,. \tag{61}$$

For the purpose of the time comparison, Dirac [9] used an atomic time $t_{*(\text{Dir})}$ based on the electron charge and mass, in the form

$$t_{*(\text{Dir})}=\frac{e^2}{4\pi\varepsilon_o m_e c^3}=\frac{\alpha\hbar}{m_e c_0{}^2}, \tag{62}$$

which provides a constant time scale as long as the constancy of $c=c_0$ is unquestioned. Dirac's time ratio R_{time} compares this constant atomic time $t_{*(\text{Dir})}$ with the time-varying Hubble time $H^{-1}(t)$, and is therefore itself time-dependent,

$$R_{\text{time}(\text{Dir})}=\frac{t_{*(\text{Dir})}}{H^{-1}(t)}=\frac{\alpha\hbar}{m_e c_0{}^2H^{-1}(t)}\cong\frac{\alpha\hbar}{m_e c_0{}^2 t}\,. \tag{63}$$

Making use of Eq. (56), this can be written as

$$R_{\text{time}(\text{Dir})}(t)\cong\frac{\gamma_e t_0}{t}\,. \tag{64}$$

The problem which Dirac then faced was the appearance of an approximate coincidence in magnitude at the present epoch between the values of the apparently time-dependent time ratio $R_{\text{time}(\text{Dir})}$ and the apparently time-independent force ratio R_{force} as well as the square root of the electron-to-universe mass ratio, $\left(R_{\text{mass}}\right)^{1/2}$. This led him to propose a time-varying gravitational theory in which all these ratios would vary appropriately together. This proposal has never achieved much acceptance.

In SSR, these difficulties disappear. When we acknowledge time variation of the velocity $c(t)=\sigma^{1/2}H^{1/2}(t)\cong c_0\left(t_0/t\right)^{1/2}$ and the constancy of σ instead, the Dirac standard of atomic time becomes a time scale $t_{*(\mathrm{SSR})}(t)$ which varies in proportion to the cosmic time itself:

$$t_{*(\mathrm{Dir})} \to t_{*(\mathrm{SSR})}(t) = \frac{\alpha\hbar}{m_e c^2(t)} = \frac{\alpha\hbar H^{-1}(t)}{m_e \sigma} \cong \frac{\alpha\hbar t}{m_e \sigma}. \tag{65}$$

This is to be compared with the equally time-dependent Hubble age. The time ratio is then a constant, and is in fact the very constant γ_e of Eq. (56):

$$R_{\mathrm{time(SSR)}} = \frac{t_{*(\mathrm{SSR})}(t)}{H^{-1}(t)} = \frac{\alpha\hbar}{m_e \sigma} = \gamma_e. \tag{66}$$

If we compare the ratio of forces, Eq. (59), to this ratio of times, we find in SSR that both ratios are constant, and connected by the simple equation

$$R_{\mathrm{force}}(e,e) = f\alpha R_{\mathrm{time(SSR)}} \cong \left(\frac{3\alpha}{2}\right) R_{\mathrm{time(SSR)}}. \tag{67}$$

We see from Eq. (67) that in SSR Dirac’s program of comparing the ratio of times, atomic to cosmic, with the ratio of forces, gravitational to electrostatic, when applied to an electron pair and combined with the factor α, leads to exactly the same numerical relationship between two very large numbers that is reached by the examination of mass ratios in Subsection ***C*** above.

The recent great improvement in the accuracy of our knowledge of the Hubble constant is crucial to the change from the order-of-magnitude similarity of numbers in the range of 10^{40}, which was available to Dirac, to the precision which we now see in comparing Eqs. (55) and (56). This now establishes an important identity between two ratios made up of physical constants of, apparently, very different physical nature, having to do with electromagnetism and with gravitation respectively, as applied to the electron.

There are distinct advantages now in being able to base the current work within the relativistic framework of SSR, where all the relevant ratios prove to be constant, and not in ordinary SR, where some of them appeared to be time-varying. The theoretical complications that Dirac faced in creating a self-consistent gravitational and cosmological dynamics in which the gravitational constant became time-dependent [10] are now eliminated. Instead, what had appeared to be a numerical coincidence within a few powers of ten between two or more independent dimensionless ratios, complicated by the time-dependence problem, is now replaced by a cleanly time-independent relationship, the identity of Eq. ((7), with its coefficient 1.5, defined in both partners to 1% accuracy.

E. The Electrogravitational Connection

The dimensionless constants γ_g of Eq. (55) and γ_e of Eq. (56) are related to each other both by a near equality in magnitude and by their similarity in physical structure. Each of them contains a factor that has the appearance of a potential energy term for a central field, electrostatic in one case and gravitational in the other, but with the separation length δr replaced by the velocity c in each case; the result in each case is a constant of self-action associated with the field. We can thus define the electrostatic action associated with a unit charge e as

$$A_{\mathrm{es}}\left(e\right)=\frac{e^2}{4\pi\varepsilon_{\mathrm{o}}\left(t\right)c\left(t\right)}=\frac{e^2}{4\pi\varepsilon_{\mathrm{o}}\left(t_0\right)c_0}=\alpha\hbar\,. \tag{68}$$

Similarly the gravitational action associated with a mass m can be constructed; because it is connected with an attractive interaction it has a negative sign:

$$A_{gr}(m)=-\frac{G(t)m^2}{c(t)}=-\frac{G_0m^2}{c_0}. \tag{69}$$

Its absolute magnitude appears in Eq. (55), in the case where the central mass is that of the electron.

To complete the similarity between Eqs. (55) and (56) we can convert the factor $\alpha^2\hbar$ in the denominator of (55) into the product of a mass δm_g and the expansion constant σ:

$$\alpha^2\hbar=\delta m_g\sigma\,. \tag{70}$$

This defines a mass-energy correction term δm_g associated with the gravitational field of the electron's mass, just as m_e itself is a mass resulting from the electrostatic field of the charge e. This mass can be seen to be a factor of α^2 smaller than the angular momentum mass $m_\hbar = \hbar / \sigma$, $\delta m_g = \alpha^2 m_\hbar$.

The information from Eqs. (55) and (56) can now be displayed as follows:

$$\gamma_e = k_1 = \left(\frac{e^2}{4\pi\varepsilon_o c}\right)\left(\frac{1}{m_e\sigma}\right) = \frac{\alpha\hbar}{m_e\sigma} = \frac{A_{\text{es}}(e)}{m_e\sigma} = 2.17(\pm 0.03)\times 10^{-41}, \quad (71)$$

$$\gamma_g = \frac{k_2}{k_1} = \left(\frac{Gm_e^{\,2}}{c}\right)\left(\frac{1}{(\alpha^2 m_\hbar)\sigma}\right) = \frac{|A_{\text{gr}}(m_e)|}{\delta m_g\sigma} = 3.28887(\pm 0.00042)\times 10^{-41}. \quad (72)$$

In each of these equations the structure of the product of natural constants on the left hand side can be analyzed into three constant factors, an action term representing an electrostatic or a gravitational field source, a mass factor, and the universal expansion rate σ. The structural similarity on the left is matched on the right by their close but not perfect agreement in magnitude.

The dominant feature of these numbers is obviously a very large common factor. Because of the precision with which the Hubble parameter H_0 and the age of the universe t_0 are now known, the difference between them is also significant. Its influence can be expressed by the ratio f of Eq. (54). With its help we can connect the expressions of Eqs. (71) and (72) as an identity connecting the constant γ_g of the gravitational self-interaction of the electron with a comparable constant γ_e of its electromagnetic self-interaction:

$$\gamma_g = f\gamma_e \cong 1.5\gamma_e. \quad (73)$$

This equation embodies an empirically established connection between the expressions of electromagnetics and of gravitation. It can be called an electrogravitational identity.

A related identity can be generated if we define an electromagnetic structure constant of the electron with the same dimensions as the cosmological expansion constant σ; we can call this the electromagnetic expansivity of the electron:

$$\kappa_e = \sigma\gamma_e = \frac{\alpha\hbar}{m_e} = 8.447974\left(\pm 0.000040\right)\times 10^{-7}\ \text{m}^2\text{s}^{-1}. \tag{74}$$

From an inspection of Eq. (74) it can be seen that it is useful to define also a cosmological expansivity constant

$$\kappa_G = \left(\hbar\bar{G}\sigma\right)^{1/3} = \left(\frac{\hbar G_0 c_0}{H_0}\right)^{1/3} = 9.70\left(\pm 0.10\right)\times 10^{-7}\ \text{m}^2\text{s}^{-1}. \tag{75}$$

A second form of electrogravitational identity is then

$$\kappa_G = g\kappa_e,\ \text{ where }\ g = f^{1/3} = 1.148\left(\pm 0.010\right). \tag{76}$$

This form of the identity can converted into an instructive relationship connected with the mass of the electron. The observed electron mass m_e, an inertial mass, appears in Eq. (74). Rearranging Eqs. (74) to (76) we can write

$$m_e = \frac{\alpha\hbar}{\kappa_e} = \frac{g\alpha\hbar}{\kappa_G} = g\alpha m_*,\ \text{where} \tag{77}$$

$$m_* = \frac{\hbar}{\kappa_G} = \left(\frac{\hbar^2}{\bar{G}\sigma}\right)^{1/3} = \left(\frac{\hbar^2 H_0}{G_0 c_0}\right)^{1/3} = 1.087\left(\pm 0.010\right)\times 10^{-28}\ \text{kg}. \tag{78}$$

In this expression m_* is a fundamental mass constant defined by a combination of the more general constants of gravitation and cosmology, and free of any connection with electricity and magnetism.

The product of this fundamental mass with the dimensionless electromagnetic coupling constant α,

$$m_{e*} = \alpha m_* = 7.94\left(\pm 0.08\right)\times 10^{-31}\ \text{kg}, \tag{79}$$

represents 87% of the measured inertial electron mass. If m_{e*} is a fundamental structural electron mass, then it may be asked whether the additional contribution making up the measured inertial mass,

$$\delta m_{e*} = m_e - m_{e*} = 1.17 \times 10^{-31} \text{ kg}, \tag{80}$$

may be attributable to a renormalization effect.

F. Conclusions

A reexamination of some of the fundamental constants of physics and cosmology in the light of the symmetric form of special relativity appropriate to the topology of a space-time with an open, expanding hyperbolic position space has brought to light a new relationship connecting some of the fundamental constants of electricity and magnetism, embodied in the mass of the electron and the fine structure constant, and a new fundamental mass constant m_* dependent only on Planck's constant $\hbar$, the Newtonian constant of gravitation corrected for the changing light speed of the universal expansion $\overline{G} = G_0 / c_0$, and the Hubble-Lorentz expansion constant $\sigma = c_0^{\ 2} / H_0$. This empirical relationship connects some of the expressions of gravitation and cosmology with others from electromagnetic theory in a way that has been entirely unexpected.

It can be conjectured that the fundamental mass m_* and a formula similar to (77) may play a part in the theory of the mass of some other fundamental particles.

These electrogravitational relationships have rich implications for the fundamental theory of the electromagnetic, gravitational and inertial properties of the electron. These will be explored further in a following paper.

References

[1] F. T. Smith, Annales de la Fondation L. de Broglie, **30**, 179 (2005).
[2] W. Pauli, Relativitätstheorie, (1921); Theory of Relativity, Pergamon, London (1958), p. 5.
[3] K. M. Evenson et al., Phys. Rev. Lett., **29,** 1346 (1972)
[4] Freedman, W.L., et al., Astrophys. J., **553**,47 (2001).
[5] Freedman, W.L., and Turner, M.S., Rev. Mod. Phys., **75**,1433 (2003).
[6] C. L. Bennett et al., Astrophys. J. Suppl., Ser. **143**,1 (2003).
[7] T. W. Hänsch *et al*., in *Atomic Physics 19*, L. G. Marcassa, K. Helmerson, and V. S. Bagnato, Eds., Amer. Inst. of Phys., (2005), pp. 71-78.

[8] P. A. M. Dirac, Nature, **139**, 323 and 1001 (1937).
[9] P. A. M. Dirac, Proc. Roy. Soc. **A 165**,199 (1938).
[10] P. A. M. Dirac, Proc. Roy. Soc. **A 338**, 439 (1974) and Proc. Roy. Soc. **A 354**, 19 (1979).

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