Effective Dynamics of Electric and Magnetic Electroweak Bosons and Leptons with Partonic Substructure for CP-Symmetry Breaking

H. Stumpf

Institute of Theoretical Physics, University Tübingen, Germany

ABSTRACT . Based on the assumption that electroweak bosons, leptons and quarks possess a substructure of elementary fermionic constituents, in a previous paper it was demonstrated that under CP-symmetry breaking "electric" and "magnetic" electroweak bosons coexist, where the latter transmit magnetic monopole interactions. In this paper the calculation is extended to the derivation of the effective theory for electroweak bosons and leptons. The dynamical law of the fermionic constituents is assumed to be given by a relativistically invariant nonlinear spinor field theory with local interaction, canonical quantization, selfregularization and probability conservation. The effective theory is derived by means of weak mapping theorems where owing to CP-violation SU(2)-symmetry is simultaneously broken. An associated effective Lagrangian yields a $SU(2) \otimes U(1)$ gauge theory for vanishing magnetic vector potential and boson masses.

1 Introduction

The concept of electric and magnetic electroweak bosons is employed to formulate the electroweak theory for both electric and magnetic charges without topological construction. In selfdual electrodynamics a distinction of electric and magnetic photons is impossible,[1]. But in nonabelian gauge theories with broken symmetries selfduality cannot be defined. So in this case one can proceed from the assumption that electric and magnetic electroweak bosons are independent quantities, where the difference between both species manifests itself by different vector potentials.

While the electric electroweak bosons are those particles which appear in the conventional electroweak Standard Model, the magnetic electroweak bosons are physical strangers. This depends on the fact that the sources of magnetic electroweak bosons, the magnetic monopoles and dyons have not been discovered so far,[1],[2].

In spite of this negative result, in the last decades a great number of theoretical papers was published, dealing with models of monopoles with very large masses which experimentally are out of reach for their production by accelerators,[2]. On the other hand low mass magnetic monopoles have been assumed to participate in low energy nuclear reactions,[3]. So one should look for monopole models, the effects of which are accessible to ordinary experimental technique.

Indeed, two decades ago Lochak proposed a massless neutrino, carrying a magnetic charge in its ground state or in excited states,[4],[5],[6], and he described the electro-magnetic action of it by means of a magnetic vector potential introduced by Cabibbo and Ferrari,[7] In this approach any topological property is avoided in contrast to the theory of conventional monopoles. Furthermore Lochak demonstrated that in de Broglie's photon theory magnetic photons can be derived which are to be associated with the magnetic vector potential,[8].

The latter approach has to be improved: In de Broglie's photon theory only magnetic *or* electric photon states can be calculated, i.e. theoretically these states cannot exist simultaneaously and the whole theory is referred to single particle states,[9],chap.1. Thus a field theoretic version of de Broglie's and Lochak's discoveries is required which leads to an extended electroweak Standard Model as an effective theory for electric and magnetic electroweak bosons as well as for fermions. This was advocated by Lochak,[6].

To treat these problems we use a model which is based on a relativistically invariant nonlinear spinor field theory with local interaction, canonical quantization, selfregularization and probability interpretation. This model implies that in the sense of de Broglie and of Heisenberg the present "elementary" particles are assumed to possess a fermionic substructure. The model is expounded in detail in [9].

As the appearance of magnetic bosons is closely related to the existence of magnetic monopoles or dyons, it is obvious that special efforts are needed to theoretically as well as to experimentally detect these particles so far unknown.

By purely theoretical reasoning it was demonstrated in [10] that in the above spinor field model electric and magnetic electroweak boson states can coexist if the CP-symmetry of the vacuum is violated. This finding is in accordance with the phenomenological observation that the existence of magnetic monopoles and dyons implies CP-symmetry breaking,[7],[11],[12]. Therefore theoretically one can assume that CP-symmetry breaking is a crucial condition for the discovery of magnetic monopole effects and it is the task to propose scenarios to realize this symmetry breaking in practice.

For instance in quantum electrodynamics one can directly observe the influence and the modification of the physical vacuum in finite volumes by the Casimir effect, [13]. A similar effect of the modification of the vacuum may occur, if in finite volumes an electric discharge is set off leading to a plasma state which generates symmetry breaking. Corresponding experiments have been carried out by Urutskoev et al.[3]. A theoretical discussion of this symmetry breaking mechanism will be given elsewhere.

In the following we do not bother about the time intervall of the discharge, but simply consider the field dynamics if CP-symmetry is broken. With respect to the theoretical treatment of this case it has to be emphasized that in our model the method of introducing CP-violation is completely different from the corresponding method in the conventional theory. While in the Standard Model the CP-symmetry breaking is formally introduced by quark mass matrices with complex parameters,cp. [14], chap.26, in our approach this symmetry breaking is effected by an appropriate change of the vacuum. Mathematically this indicates the transition to a new inequivalent field representation which is a common method in algebraic field theory successfully applied in solid state physics, cf. [15],[16].

In consequence of this difference of the methods, the results differ considerably too. While the formal phenomenological method of the Standard Model is intended to explain the decay of K-mesons, the algebraic method of the model under consideration leads to a completely new formulation and structure of the whole theory due to the new inequivalent vacuum. The physical consequences of this algebraic approach are remarkable, but this will be discussed elsewhere.

As our exposition is based on the results of the preceding papers [10],[17], it is unavoidable that for brevity we have to refer to these results without giving renewed deductions. In particular we skip the formulation of the algebraic representation of the basic spinor field itself, a clear exposition of which was given in [18], sect. 2. Furthermore in our calculations no use was made of the decomposition into left-handed and right-handed fermions for simplicity. Insofar the model under consideration is a simplified version of the mathematical structure of the Standard Model. This is justified as already in this version the crucial effects of CP- symmetry breaking can be demonstrated.

2 Algebraic representation of effective theories

Effective theories are generated if in the functional formulation of the algebraic Schroedinger representation of the spinor field [18], sect.2, mappings on other appropriate functional spaces are performed. The effective boson dynamics was treated in [18], sect.3, while the mathematical foundation of the effective boson-fermion dynamics was developed in a previous paper, [19]. Without going into details we refer to this paper and give only the relevant final formulas.

In the spinor field model it is assumed that electroweak gauge bosons possess a substructure consisting of two partons, while leptons and quarks are formed by three partons. We will explicitly evaluate this representation for the case of CP-symmetry breaking.

Let the functional states of the corresponding effective field theory be defined by

$$\begin{aligned} |\mathcal{P}(b,f;a)\rangle := \sum_{n=2}^{\infty}\sum_{m=0}^{\infty}\sum_{r=0}^{\infty} \delta_{n,2m+3r} \frac{1}{(m!)(r!)} \\ \tilde{\varrho}(k_1...k_m, q_1...q_r|a) b_{k_1}...b_{k_m} f_{q_1}...f_{q_r}|0\rangle_{BF} \end{aligned} \tag{1}$$

where b, ∂^b and f, ∂^f are the functional sources of the phenomenological bosons and fermions, respectively, while the $\tilde{\varrho}$-functions represent the matrix elements of the effective (phenomenological) boson-fermion theory.

Then the mapping leads to an effective functional energy operator $\tilde{\mathcal{H}}$, [19], eq.(82) with the eigenvalue equation

$$E|\mathcal{P}(b,f)\rangle = \tilde{\mathcal{H}}(b,\partial^b,f,\partial^f)|\mathcal{P}(b,f)\rangle \tag{2}$$

The energy operator $\tilde{\mathcal{H}}$ can be decomposed into leading terms, higher order terms and quantization terms which result from the quantization terms in $\mathcal{H}_F$ by the mapping. As we are only interested in the dynamical structure of the effective theory, we exclude the quantization terms from our discussion.

With respect to the higher order terms, estimates were performed in [9],sect.8.7 and [20]. For heavy spinor field masses m_i these terms are tiny and can be omitted from the physical discussion, in particular in the low energy range. It would exceed the scope of this paper to discuss this calculation scheme and its estimates explicitly. So in this paper we treat only the leading terms which are physically relevant.

Decomposing $\tilde{\mathcal{H}}$ into

$$\tilde{\mathcal{H}} = \mathcal{H}_f + \mathcal{H}_b + \mathcal{H}_{bf} \tag{3}$$

for the leading terms the following expressions result from the mapping theorems, [19]:

$$\begin{aligned}
\mathcal{H}_f :=& K^f_{qp} f_q \partial^f_q + M^f_{qp} f_q \partial^f_p \\
\mathcal{H}_b :=& K^b_{kl} b_k \partial^b_l + M^b_{kl} b_k \partial^b_l + W_1^{kl_1l_2} b_k \partial^b_{l_1} \partial^b_{l_2} \\
\mathcal{H}_{bf} :=& W_2^{qlp} \partial^b_l f_q \partial^f_p + W_4^{q_1q_2p_1p_2} R^k_{q_1q_2} b_k \partial^f_{p_1} \partial^f_{p_2}
\end{aligned} \tag{4}$$

with, see [19], eq.(66):

$$\begin{aligned}
K^f_{qp} :=& 3R^q_{II'I_1} K_{I_1I_2} C^p_{I_2II'} \\
M^f_{qp} :=& -9W_{I_1I_2I_3I_4} F_{I_4K} (R^q_{IKI_1} C^p_{I_2I_3I} - R^q_{II'K} C^p_{I_2I_3I'}) \\
K^b_{kl} :=& 2R^k_{II_1} K_{I_1I_2} C^l_{I_2I} \\
M^b_{kl} :=& -6W_{I_1I_2I_3I_4} F_{I_4K} R^k_{KI_1} C^l_{I_2I_3} \\
W_1^{kl_1l_2} :=& 4W_{I_1I_2I_3I_4} R^k_{II_1} C^{l_1}_{I_4I} C^{l_2}_{I_2I_3} \\
W_2^{qlp} :=& 3W_{I_1I_2I_3I_4} R^q_{II'I_1} C^p_{I_4II'} C^l_{I_2I_3} \\
W_4^{q_1q_2p_1p_2} :=& -54W_{I_1I_2I_3I_4} F_{I_4K} R^{q_1}_{II'I_1} R^{q_2}_{K_1K_2K} C^{p_1}_{I'K_1K_2} C^{p_2}_{I_2I_3I}
\end{aligned} \tag{5}$$

In these expressions the quantities $C^l_{II'}$ and $R^n_{II'}$ symbolize the boson states and their duals, while $C^q_{II'I''}$ and $R^p_{II'I''}$ are the fermion states and their duals both of which will be explicitly introduced in section 3. The other quantities appearing in (5) are the terms of the original spinor field which determine the structure and the numerical values of the effective theory defined by the energy operator, see [18],sect.2.

3 Boson and fermion basis states

For the evaluation of the effective theory the states of the composite particles are required. In order to get an optimal adaption to the structure of the physical particles these states should be derived from corresponding solutions of generalized de Broglie-Bargmann-Wigner (GBBW)-equations. For details see [9],[21],[22].

For the bosons the exact vector boson state solutions of the GBBW-equations have been derived for the case of CP-symmetry breaking in a previous paper, [10]. Hence for the introduction of appropriate test functions all group theoretical properties can be adopted from the exact solutions.

For the fermion states no exact solutions of the corresponding GBBW-equations are known. Thus an idea has to be borrowed from phenomenology, how by CP-symmetry breaking these states should be group theoretically modified.

If in the Standard model several generations of fermions are taken into account then there is no reason for the fermion mass matrices to be diagonal. Indeed suitable mass matrices lead to CP-symmetry breaking effects. But, and this is very important: if the neutrinos are massless or nearly massless, then the CP-symmetry breaking terms cannot affect the lepton part of the mass matrix. Only the quark mass matrix is remarkably affected by these symmetry breaking terms, [23],p.116, [2],p.47.

Owing to this obvious difference in the behavior of lepton and quark states under CP-symmetry breaking we confine ourselves to the treatment of the lepton states only as for these states the full group theoretical information of the CP-invariant theory can be used.

For electroweak bosons the superspin-isospin part is given by a singlet and a triplet matrix representation and defined by the following sets of symmetric and antisymmetric matrices:

$$S^l = \begin{pmatrix} 0 & \sigma^l \\ (-1)^{l+1}\sigma^l & 0 \end{pmatrix}; \qquad T^l = \begin{pmatrix} 0 & \sigma^l \\ (-1)^l\sigma^l & 0 \end{pmatrix} \tag{6}$$

for the triplet, and

$$S^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}; \qquad T^0 = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \tag{7}$$

for the singlet.

For CP-symmetry breaking an eigenfunction reads in the basis (6),(7), see [10],

$$\varphi^{\kappa_1\kappa_2}_{\alpha_1\alpha_2}(x_1,x_2|k,l)_{i_1i_2} = (S^l + T^l)^S_{\kappa_1\kappa_2}\varphi_{\alpha_1\alpha_2}(x_1,x_2|k)_{i_1i_2} \tag{8}$$

where the superscript S characterizes the superspin-isospin representation referred to the spinor fields ψ and ψ^c, see eq. (33) for the phenomenological fields, while φ is given by

$$\begin{aligned}\varphi_{\alpha_1\alpha_2}(x_1,x_2|k,l)_{i_1i_2} =& exp[-i\frac{k}{2}(x_1+x_2)] \times \\ & \{A^l_\mu\chi^\mu_{\alpha_1\alpha_2}(x_1-x_2|k)_{i_1i_2} + G^l_\mu\zeta^\mu_{\alpha_1\alpha_2}(x_1-x_2|k)_{i_1i_2}\}\end{aligned} \tag{9}$$

With $p_+ := p + k/2$ and $p_- := p - k/2$ the following definition holds

$$\begin{aligned}\chi^\mu_{\alpha_1\alpha_2}(x|k)_{i_1i_2} :=& \frac{2ig}{(2\pi)^4}\lambda_{i_1}\lambda_{i_2}\int d^4p\ \exp(-ipx) \\ & [S_F(p_+,m_{i_1})\gamma^\mu S^{CP}_F(p_-,m_{i_2})C]_{\alpha_1\alpha_2}\end{aligned} \tag{10}$$

where S^{CP}_F is the CP-symmetry breaking Feynman propagator and where the function ζ^μ is obtained by replacing γ^μ by $\gamma^5\gamma^\mu$ in (10).

For the spin orbit parts (9) it can be shown that A^l_μ and G^l_μ have to be identified with the phenomenological electric and magnetic vector potential of the boson state. In addition in a free single particle state (8) only the free electroweak field tensors can occur which can be identified with terms of the kind

$$k_\nu A^a_\mu\Sigma^{\nu\mu}C \equiv F^a_{\nu\mu}\Sigma^{\nu\mu}C; \qquad \varepsilon^{\mu\nu\varrho\lambda}k_\varrho G^a_\lambda\Sigma_{\nu\mu}C \equiv F_a^{\nu\mu}\Sigma_{\nu\mu}C \tag{11}$$

If for simplified test functions all terms which contain p_μ vectors are neglected ($\equiv$ s-wave approximation) then (9) can be replaced by the expression

$$\begin{aligned}\varphi_{\alpha_1\alpha_2}(x_1,x_2|k)_{i_1i_2} =& \exp[-i\frac{k}{2}(x_1+x_2)]\{A^l_\mu(\gamma^\mu C)_{\alpha_1\alpha_2}\omega(x_1-x_2|k)_{i_1i_2} \\ & +G^l_\mu(\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2}\vartheta(x_1-x_2|k)_{i_1i_2} \\ & +F^l_{\nu\mu}(\Sigma^{\nu\mu}C)_{\alpha_1\alpha_2}\varrho(x_1-x_2|k)_{i_1i_2}\}\end{aligned} \tag{12}$$

For solutions of the GBBW-equations the relations between the vectors A, G and the field strength tensor F are fixed. However, for test functions we consider the quantities A, G and F as unconstrained, freely variable quantities which can be adapted to interactions described in terms of the effective field equations. As a consequence the wave functions (8) have to be decomposed into three independent parts, associated to the field variables A, G and F.

For the evaluation of the effective theory the single time wave functions are needed and owing to the independence of A, G and F this leads to three types of testfunctions

$$
\begin{aligned}
C^A_{Z_1 Z_2}(\mathbf{r}_1,\mathbf{r}_2|\mathbf{k},l,\mu) &:= (S^l+T^l)^S_{\kappa_1\kappa_2}\exp[-i\frac{\mathbf{k}}{2}(\mathbf{r}_1+\mathbf{r}_2)](\gamma^\mu C)_{\alpha_1\alpha_2} f^A(\mathbf{r}_1-\mathbf{r}_2)_{i_1 i_2}\\
C^G_{Z_1 Z_2}(\mathbf{r}_1,\mathbf{r}_2|\mathbf{k},l,\mu) &:= (S^l+T^l)^S_{\kappa_1\kappa_2}\exp[-i\frac{\mathbf{k}}{2}(\mathbf{r}_1+\mathbf{r}_2)](\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2} f^G(\mathbf{r}_1-\mathbf{r}_2)_{i_1 i_2}\\
C^F_{Z_1 Z_2}(\mathbf{r}_1,\mathbf{r}_2|\mathbf{k},l,\mu,\nu) &:= (S^l+T^l)^S_{\kappa_1\kappa_2}\exp[-i\frac{\mathbf{k}}{2}(\mathbf{r}_1+\mathbf{r}_2)](\Sigma^{\mu\nu} C)_{\alpha_1\alpha_2} f^F(\mathbf{r}_1-\mathbf{r}_2)_{i_1 i_2}
\end{aligned}
\tag{13}
$$

with $Z := (i,\alpha,\kappa)$. According to [22] the duals are defined by $R := \lambda^{-1}_{i_1}\lambda^{-1}_{i_2}C^+$ which need not be explicitly represented here.

Concerning the fermion states, their group theoretical analysis has been performed in several papers for unbroken CP-symmetry.[22],[24-27].

In this case the permutation group representations play an essential role in the construction of appropriate wave functions. For obtaining lepton states we adopt the group theoretical representations of test functions from Pfister, [26], being based on the theory of representations of the permutation group elaborated by Kramer et.al., [28].

The group theoretical analysis of the three parton problem must guarantee that the resulting test functions possess quantum numbers which coincide with those of the leptons in phenomenological theory. This can only be achieved by using mixed representations of the permutation group. Such mixed representations are generated by the application of Young operators C_{ik}. For two dimensional representations these operators are defined by the relations, [26],[28]

$$
\begin{aligned}
C_{11}^{[21]} &:= \frac{1}{2}(1-P_{12})\frac{1}{3}(2+P_{13}+P_{23})\\
C_{22}^{[21]} &:= \frac{1}{2}(1+P_{12})\frac{1}{3}(2-P_{13}-P_{23})\\
C_{12}^{[21]} &:= \frac{1}{2}(1-P_{12})\frac{\sqrt{3}}{3}(P_{23}-P_{13})\\
C_{21}^{[21]} &:= \frac{1}{2}(1+P_{12})\frac{\sqrt{3}}{3}(P_{23}-P_{13})
\end{aligned}
\tag{14}
$$

where P_{ik} means transposition which interchanges arguments with index i and k. These operators will be applied to superspin-isospin states and separately to spin-orbit states.

The use of the Young operators allows to start with products of test wave functions which are not antisymmetrized from the beginning. For lepton states these products have to be formed by superspin-isospin testfunctions Θ^j and spin-orbit test functions $\Omega \otimes \psi$.

The superspin-isospin testfunctions are responsible for the definition of the phenomenological quantum numbers for isospin and charge and for the fermion number, while the spin orbit test- functions should lead to the spin 1/2 of the leptons, to generation numbers and (or) internal excitation levels. The latter two possibilities will not be further pursued in this investigation.

After rearrangements the general expression of Young combinations leads to the following antisymmetric test functions for leptons, [26], owing to $[21] \times [21] \rightarrow [111]$:

$$
\begin{aligned}
C^{\kappa_1\kappa_2\kappa_3}_{\alpha_1\alpha_2\alpha_3}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3|\mathbf{k},j,n) := exp[-i\mathbf{k}\frac{1}{3}(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)] \times \\
[(C_{11}\Theta^j_{\kappa_1\kappa_2\kappa_3})C_{22}\{\Omega^n_{\alpha_1\alpha_2\alpha_3}\psi(\mathbf{r}_2-\mathbf{r}_1,\mathbf{r}_3-\mathbf{r}_2)\} \\
-(C_{21}\Theta^j_{\kappa_1\kappa_2\kappa_3})C_{12}\{\Omega^n_{\alpha_1\alpha_2\alpha_3}\psi(\mathbf{r}_2-\mathbf{r}_1,\mathbf{r}_3-\mathbf{r}_2)\}]
\end{aligned}
\tag{15}
$$

A structurally transparent representation of the superspin-isospin parts in (15) can be derived if charge conjugated spinors of the original spinor field are transformed into G-conjugated spinors by

$$
\psi^D_{\kappa\alpha i} := \psi^G_{\kappa\alpha i} = G_{\kappa\kappa'}\psi^C_{\kappa'\alpha i}
\tag{16}
$$

with $G := \mathbf{1} \oplus c$ and $c = -i\sigma^2$.

To avoid confusion with te magnetic vector potential G, the superscript D (decomposition) is employed in (16). If one decomposes the index κ into the double index (Λ, A) the following scheme is obtained

$$\begin{array}{l}(C_{11}\Theta^1)^D := \chi^{1/2}_{1/2}(r_2)\otimes(1,1,1);\ \ (C_{21}\Theta^1)^D := \chi^{1/2}_{1/2}(r_1)\otimes(1,1,1) \equiv e^+ =: \psi_1\\ (C_{11}\Theta^2)^D := \chi^{1/2}_{-1/2}(r_2)\otimes(1,1,1);(C_{21}\Theta^2)^D := \chi^{1/2}_{-1/2}(r_1)\otimes(1,1,1) \equiv \bar\nu_e =: \psi_2\\ (C_{11}\Theta^3)^D := \chi^{1/2}_{1/2}(r_2)\otimes(2,2,2);\ \ (C_{21}\Theta^3)^D := \chi^{1/2}_{1/2}(r_1)\otimes(2,2,2) \equiv \nu_e =: \psi_3\\ (C_{11}\Theta^4)^D := \chi^{1/2}_{-1/2}(r_2)\otimes(2,2,2);(C_{21}\Theta^4)^D := \chi^{1/2}_{-1/2}(r_1)\otimes(2,2,2) \equiv e^- =: \psi_4\end{array} \tag{17}$$

with

$$\begin{array}{l}\chi^{1/2}_{1/2}(r_1) := (\frac{2}{3})^{1/2}\delta_{1A_1}\delta_{1A_2}\delta_{2A_3} - (\frac{1}{6})^{1/2}[\delta_{2A_1}\delta_{1A_2} + \delta_{1A_1}\delta_{2A_2}]\delta_{1A_3}\\ \chi^{1/2}_{1/2}(r_2) := (\frac{1}{2})^{1/2}[\delta_{1A_1}\delta_{2A_2} - \delta_{2A_1}\delta_{1A_2}]\delta_{1A_3}\\ \chi^{1/2}_{-1/2}(r_1) := -(\frac{2}{3})^{1/2}\delta_{2A_1}\delta_{2A_2}\delta_{1A_3} + (\frac{1}{6})^{1/2}[\delta_{2A_1}\delta_{1A_2} + \delta_{1A_1}\delta_{2A_2}]\delta_{2A_3}\\ \chi^{1/2}_{-1/2}(r_2) := (\frac{1}{2})^{1/2}[\delta_{1A_1}\delta_{2A_2} - \delta_{2A_1}\delta_{1A_2}]\delta_{2A_3}\\ (1,1,1) := \delta_{1\Lambda_1}\delta_{1\Lambda_2}\delta_{1\Lambda_3};\qquad (2,2,2) := \delta_{2\Lambda_1}\delta_{2\Lambda_2}\delta_{2\Lambda_3}\end{array} \tag{18}$$

The quantum numbers of these states coincide with the phenomenological quantum numbers and the last column in (17) corresponds to the phenomenological spinor fields ψ^D_{phen} afterwards. If the latter are transformed into the phenomenological S-representation the positions of $\bar\nu$ and e^+ must be interchanged. Then $(\bar\nu, e^+)$ is charge conjugated to $(\bar\nu, e^-)$ and these arrays in columns are in agreement with the phenomenological notation. In the following we suppress the index "*phen*".

In defining the spin tensor we expect to obtain lepton fields $l^j_\alpha(x)$ in the effective theory which are *not* eigenstates of the free Dirac operator for definite **k**-vector, i.e. owing to the interactions these effective lepton fields must be general spinor fields which excludes a representation by free fields. Hence the spin tensor Ω is not allowed to be constructed by means of eigensolutions to **k**-vectors.

Furthermore as the leptons are assumed to occupy the ground states of the three-parton system, the spin tensor as well as the orbit functions must show the highest possible invariance under symmetry operations, which for these parts of the wave functions are the little group operations with all discrete transformations. This leads to the spin tensor and its charge conjugated counterpart

$$\Omega^n_{\alpha_1\alpha_2\alpha_3} = C_{\alpha_1\alpha_2}\xi^n_{\alpha_3}, \qquad \bar{\Omega}^n_{\alpha_1\alpha_2\alpha_3} = C_{\alpha_1\alpha_2}C_{\alpha_3\alpha}\xi^n_{\alpha} \tag{19}$$

where ξ^n_α are the four unit spinors $\delta_{\alpha n}$, n = 1,2,3,4 , while C is invariant under rotations and the discrete operation PC ,see [29],p.110 . The orbit part is assumed to have s-wave character which automatically is invariant under parity transformations.

4 Effective canonical equations of motion

By means of the test functions of section 3 the effective functional energy operator $\tilde{\mathcal{H}}$ has been calculated in detail in [17]. Due to these calculations it is convenient to replace the general decomposition in eq. (3) by

$$\tilde{\mathcal{H}} = \mathcal{H}_f + \mathcal{H}^1_b + \mathcal{H}^2_b + \mathcal{H}^3_b + \mathcal{H}^1_{bf} + \mathcal{H}^2_{bf} \tag{20}$$

where the various terms of (20) are defined by equations [17], (106), (45), (48), (55), (73) and (104) in the order of equation (20).

In these terms the general functional (source) operators b and ∂^b for bosons are decomposed into a set of operators associated to the various field quantities, i.e. $b := \{b^A, b^G, b^E, b^G\}$, etc.

To be in conformity with the phenomenological field definitions of section 5, it is necessary to carry out a canonical transformation of the functional algebra for the G-fields and E-fields which is defined by

$$\begin{aligned} b^G_{la}(\mathbf{z}) = ib^G_{la}(\mathbf{z})';\partial^G_{la}(\mathbf{z}) = -i\partial^G_{la}(\mathbf{z})' \\ b^E_{la}(\mathbf{z}) = -b^E_{la}(\mathbf{z})';\partial^E_{la}(\mathbf{z}) = -\partial^E_{la}(\mathbf{z})' \end{aligned} \tag{21}$$

while the other algebra elements for the A-fields and the B-fields remain unchanged.

After having performed this transformation of (20) we omit the primes of the new sources in (21) for brevity. With (21) the explicit expressions for the various terms of (20) read

$$\mathcal{H}^2_f = \int d^3z f(\mathbf{z}|B_1b_1\alpha_1)[-i(\gamma^0\gamma^k)\partial^z_k + m\gamma^0]_{\alpha_1\alpha_2}\partial^f(\mathbf{z}|B_1b_1\alpha_2) \tag{22}$$

$$\begin{aligned}\mathcal{H}_b^1 =& i\int d^3z\, b_{la}^A(\mathbf{z})[c_1\varepsilon_{lkm}\partial_k^z\partial_{ma}^G(\mathbf{z}) - c_2\partial_{la}^E(\mathbf{z})]\\ &-i\int d^3z\, b_{la}^G(\mathbf{z})[c_1\varepsilon_{lkm}\partial_k^z\partial_{ma}^A(\mathbf{z}) - c_3\partial_{la}^B(\mathbf{z})]\\ &+i\int d^3z\, b_{la}^E(\mathbf{z})[\varepsilon_{lkm}\partial_k^z\partial_{ma}^B(\mathbf{z}) + c_2\partial_{la}^A(\mathbf{z})]\\ &-i\int d^3z\, b_{la}^B(\mathbf{z})[\varepsilon_{lkm}\partial_k^z\partial_{ma}^E(\mathbf{z}) + c_3\partial_{la}^G(\mathbf{z})]\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{H}_b^2 =& -i\int d^3z \hat{f}^A c_4 b_{la}^E(\mathbf{z})\partial_{la}^A(\mathbf{z})\\ &+i\int d^3z \hat{f}^G c_4 b_{la}^B(\mathbf{z})\partial_{la}^G(\mathbf{z})\end{aligned} \tag{24}$$

$$\begin{aligned}\mathcal{H}_b^3 =& \bar{\eta}_{abc}\varepsilon_{lkm}\{64\hat{f}^A\int d^3z\,[k_1 b_{l,a}^A(\mathbf{z})\partial_{k,b}^A(\mathbf{z})\partial_{m,c}^G(\mathbf{z}) + k_2 b_{l,a}^E(\mathbf{z})\partial_{k,b}^A(\mathbf{z})\partial_{m,c}^B(\mathbf{z})\\ &-k_2' b_{l,a}^B(\mathbf{z})\partial_{k,b}^A(\mathbf{z})\partial_{m,c}^E(\mathbf{z}) - k_3 b_{l,a}^G(\mathbf{z})\partial_{k,b}^A(\mathbf{z})\partial_{m,c}^A(\mathbf{z})]\\ &+64\hat{f}^G\int d^3z\,[k_4 b_{l,a}^A(\mathbf{z})\partial_{k,b}^G(\mathbf{z})\partial_{m,c}^A(\mathbf{z}) + k_5 b_{l,a}^E(\mathbf{z})\partial_{k,b}^G(\mathbf{z})\partial_{m,c}^E(\mathbf{z})\\ &+k_5' b_{l,a}^B(\mathbf{z})\partial_{k,b}^G(\mathbf{z})\partial_{m,c}^B(\mathbf{z}) + k_6 b_{l,a}^G(\mathbf{z})\partial_{k,b}^G(\mathbf{z})\partial_{m,c}^G(\mathbf{z})]\}\end{aligned} \tag{25}$$

$$\begin{aligned}\mathcal{H}_{bf}^1 =& -K_1\int d^3z(\gamma^0\gamma^k)_{nm}(T^0\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial_{k0}^A(\mathbf{z})\partial_{mj}^f(\mathbf{z})\\ &+iK_1\int d^3z(\gamma^0\gamma^k\gamma^5)_{nm}(S^0\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial_{k0}^G(\mathbf{z})\partial_{mj}^f(\mathbf{z})\\ &+\frac{1}{3}K_1\sum_{b=1}^{3}\int d^3z(\gamma^0\gamma^k)_{nm}(T^b\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial_{kb}^A(\mathbf{z})\partial_{mj}^f(\mathbf{z})\\ &-i\frac{1}{3}K_1\sum_{b=1}^{3}\int d^3z(\gamma^0\gamma^k\gamma^5)_{nm}(S^b\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial_{kb}^G(\mathbf{z})\partial_{mj}^f(\mathbf{z})\end{aligned} \tag{26}$$

$$\begin{aligned}\mathcal{H}_{bf}^2 =& iKt(0)^4\int d^3z\,\Theta^n_{B_1b_1,B_2b_2}[-2\,f^E(\gamma^kC)^+_{\mu_1\mu_2}b^E(\mathbf{z}|n,k)\\ &+f^B i(\gamma^5\gamma^kC)^+_{\mu_1\mu_2}b^B(\mathbf{z}|n,k)]\partial^f(\mathbf{z}|B_1,b_1,\mu_1)\partial^f(\mathbf{z}|B_2,b_2,\mu_2)\end{aligned} \tag{27}$$

It should be emphasized that the input of equations (22)-(27) is solely the spinor field model, [17], Sect.2 , [18], Sect.2 and its sets of single bosonic and single fermionic bound states of the preceding section.

A physical interpretation of the associated effective functional energy equation (2) can be achieved by considering the classical limit of this equation. In this classical limit the system is described by its classical equations of motion. These equations of motion can be exactly derived from (2) if correlations in the matrix elements are suppressed. For details of the corresponding deduction we refer to [18],sect.5 for instance.

In the field part of this set of equations the quantities E_{la} and B_{la} , l = 1, 2, 3 and a = 0, 1, 2, 3 , represent the $SU(2) \otimes U(1)$ field strengths, while A_{la} and G_{la} are the "electric" and "magnetic" vector potentials in temporal gauge. This "gauge" can be selfconsistently justified as a general constraint, even if the original $SU(2)$ invariance is broken. Such vector potentials were introduced by Cabbibo and Ferrari, [7] in electrodynamics and the following set of equations represents an electroweak generalization of this approach,

$$
\begin{aligned}
i\dot{A}_{la}(\mathbf{z}) =& ic_1 \varepsilon_{lkm} \partial_k^z G_{ma}(\mathbf{z}) - ic_2 E_{la}(\mathbf{z}) \\
&+ \bar{\eta}_{abc} \varepsilon_{lkm} [\hat{f}^A k_1 A_{kb}(\mathbf{z}) G_{mc}(\mathbf{z}) + \hat{f}^G k_4 G_{kb}(\mathbf{z}) A_{mc}(\mathbf{z})]
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
i\dot{G}_{la}(\mathbf{z}) =& -ic_1 \varepsilon_{lkm} \partial_k^z A_{ma}(\mathbf{z}) + ic_3 B_{la}(\mathbf{z}) \\
&+ \bar{\eta}_{abc} \varepsilon_{lkm} [-\hat{f}^A k_3 A_{kb}(\mathbf{z}) A_{mc}(\mathbf{z}) + \hat{f}^G k_6 G_{kb}(\mathbf{z}) G_{mc}(\mathbf{z})]
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
i\dot{E}_{la}(\mathbf{z}) =& i \varepsilon_{lkm} \partial_k^z B_{ma}(\mathbf{z}) + i(c_2 - \hat{f}^A c_4) A_{la}(\mathbf{z}) \\
&+ \bar{\eta}_{abc} \varepsilon_{lkm} [\hat{f}^A k_2 A_{kb}(\mathbf{z}) B_{mc}(\mathbf{z}) + \hat{f}^G k_5 G_{kb}(\mathbf{z}) E_{mc}(\mathbf{z})] \\
&- iK' \Theta^a_{B_1 b_1, B_2 b_2} \hat{f}^E (\gamma^l C)^+_{\mu_1 \mu_2} \psi_{B_1 b_1 \mu_1}(\mathbf{z}) \psi_{B_2 b_2 \mu_2}(\mathbf{z})
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
i\dot{B}_{la}(\mathbf{z}) =& -i \varepsilon_{lkm} \partial_k^z E_{ma}(\mathbf{z}) - i(c_3 - \hat{f}^G c_4) G_{la}(\mathbf{z}) \\
&+ \bar{\eta}_{abc} \varepsilon_{lkm} [-\hat{f}^A k_2' A_{kb}(\mathbf{z}) E_{mc}(\mathbf{z}) + \hat{f}^G k_5' G_{kb}(\mathbf{z}) B_{mc}(\mathbf{z})] \\
&+ i(K'/2) \Theta^a_{B_1 b_1, B_2 b_2} \hat{f}^B (i\gamma^5 \gamma^l C)^+_{\mu_1 \mu_2} \psi_{B_1 b_1 \mu_1}(\mathbf{z}) \psi_{B_2 b_2 \mu_2}(\mathbf{z})
\end{aligned}
\tag{31}
$$

The factor 64 in (25) has been included in the definition of the constants k_i in (28)-(31)

For the fermion fields the following equations of motion can be derived

$$
\begin{aligned}
i\dot{\psi}_{\alpha l}(\mathbf{z}) =& [-i(\gamma^0\gamma^k)_{\alpha\beta}\partial_k^z + \gamma^0_{\alpha\beta}m]\psi_{\beta l}(\mathbf{z}) \\
& -K_1[(\gamma^0\gamma^k)_{\alpha\beta}(T^0\gamma^5)_{ln}A_{k0}(\mathbf{z}) - i(\gamma^0\gamma^k\gamma^5)_{\alpha\beta}(S^0\gamma^5)_{ln}G_{k0}(\mathbf{z})]\psi_{\beta n}(\mathbf{z}) \\
& +\frac{1}{3}K_1\sum_{b=1}^{3}[(\gamma^0\gamma^k)_{\alpha\beta}(T^b\gamma^5)_{ln}A_{kb}(\mathbf{z}) - i(\gamma^0\gamma^k\gamma^5)_{\alpha\beta}(S^b\gamma^5)_{ln}G_{kb}(\mathbf{z})]\psi_{\beta n}(\mathbf{z})
\end{aligned}
\tag{32}
$$

where the indices l, n refer to the phenomenological numeration of the lepton states. This means that the field quantities $\psi_{\alpha,l}$ are superspinors of the phenomenological theory and ought not to be confused with the spinor field operators of the basic spinor field model in the background. The sets of antisymmetric and symmetric matrices T^a , S^a, $a = 0$, 1, 2, 3 represent the underlying $SU(2) \otimes U(1)$ group structure. They are given by equations (6) and (7).

In (30) and (31) the four dimensional index κ is splitted into the double index $\kappa = (B, b)$ and in the S-representation the phenomenological superspinors are defined by

$$
\psi^S_{Bb\alpha i}(x) = \begin{pmatrix} \psi_{b\alpha i}(x); B = 1 \\ \psi^c_{b\alpha i}(x); B = 2 \end{pmatrix}
\tag{33}
$$

By technical reasons of the calculation in [17], aside from charge conjugated spinors also G-conjugated spinors were introduced, see eq. (16), and this definition is also applied to the phenomenological theory. There it is likewise indicated by the superscript D (decomposition). For instance the formula [17], eq.(92) of the superspin-isospin part of the current calculation is formulated in D-representation. This formula is central for the physical interpretation of the current expressions in eqs. (30), (31), but it is beyond the scope of this paper to give an outline of its derivation. Therefore we only discuss the evaluation of this formula.

To calculate this D-representation we start with the S-representation. According to the construction of formula [17], eq.(92), the tensor Θ^n on the left hand side of [17], eq.(92) is the superspin-isospin part of the boson dual function $R^k_{q_1q_2}$ in [17], eq.(74). Thus we start first with the superspin-isospin parts of the original boson functions $C^k_{q_1q_2}$, construct their duals and transform these duals from the S- into the D-representation. The original superspin-isospin basis set of the $C^k_{q_1q_2}$ functions reads for the case of CP-symmetry breaking, cp. [17], eq.(27), or eq.(8), respectively.

$$(\Theta^a)^S_{\kappa_1\kappa_2} := \frac{1}{2}(T^a + S^a)^S_{\kappa_1\kappa_2} := \frac{1}{2}(i\sigma^2 + \sigma^1)_{B_1B_2} \otimes \sigma^a_{b_1b_2} \qquad a = 0, 1, 2, 3 \tag{34}$$

Its dual set $\tilde{\Theta}^n$, n = 0, 1, 2, 3, is given by

$$(\tilde{\Theta}^n)^S_{\kappa_1\kappa_2} = (\tilde{\Theta}^n)^S_{B_1b_1B_2b_2} = \frac{1}{2}(i\sigma^2 + \sigma^1)_{B_1B_2}(\sigma^n)^T_{b_1b_2} \tag{35}$$

Owing to the properties of the Pauli-algebra one easily verifies that the duality relations

$$(\tilde{\Theta}^n)^S_{\kappa_1\kappa_2}(\Theta^{n'})^S_{\kappa_1\kappa_2} = \frac{1}{4}(i\sigma^2+\sigma^1)_{B_1B_2}(i\sigma^2+\sigma^1)_{B_1B_2}(\sigma^n)^T_{b_1b_2}\sigma^{n'}_{b_1b_2} = 2\delta_{nn'} \tag{36}$$

are satisfied. In eq. (36) the state normalization is omitted, because it is irrelevant, see below. In the next step we transform the tensor (35) from the S- into the D-representation.

The transformation law of the superspin-isospin part (34) of the boson functions $C^k_{q_1q_2}$ is defined by the relation

$$(\Theta^n)^S_{\kappa_1\kappa_2} = G_{\kappa_1\kappa_1'}G_{\kappa_2\kappa_2'}(\Theta^n)^D_{\kappa_1'\kappa_2'} \tag{37}$$

with the transformation matrix

$$G := \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -i\sigma_2 \end{pmatrix} \tag{38}$$

The duality relation (36) has to be invariant under the change of the representation. This means that

$$(\tilde{\Theta}^n)^S_{\kappa_1\kappa_2}(\Theta^{n'})^S_{\kappa_1\kappa_2} = (\tilde{\Theta}^n)^D_{\kappa_1\kappa_2}(\Theta^{n'})^D_{\kappa_1\kappa_2} \tag{39}$$

has to hold which leads to the transformation law for the dual set

$$(\tilde{\Theta}^n)^S_{\kappa_1\kappa_2} = G^{-1}_{\kappa_1\kappa_2'}G^{-1}_{\kappa_2\kappa_2'}(\tilde{\Theta}^n)^D_{\kappa_1'\kappa_2'} \tag{40}$$

Then one obtains with (35), (38) and the inverse of (40)

$$(\tilde{\Theta}^n)^D_{\kappa_1\kappa_2} = \frac{1}{2}(i\sigma^2 + \sigma^1)_{B_1B_2}[(\sigma^n)^T c^T]_{b_1b_2} \equiv \delta_{B_11}\delta_{B_22}[(\sigma^n)^T c^T]_{b_1b_2} \tag{41}$$

In consequence of eq.(41) equation (92) of [17] must be corrected by replacing $(\Theta^n)^D$ by $(\tilde{\Theta}^n)^D$. This yields the revised formula

$$S^{A_1a_1A_2a_2}_{B_1b_1B_2b_2}(\tilde{\Theta}^n)^D_{[A_1]a_1[A_2]a_2} = (\bar{\Theta}^n)^D_{B_1b_1B_2b_2} := (\tilde{\Theta}^n)^D_{B_2b_1B_1b_2} \tag{42}$$

where $\bar{\Theta}^n$ is an auxiliary tensor defined by $\tilde{\Theta}^n$ on the right hand side of (42). Therefore in [17] all following equations have to be corrected in accordance with this correction. This includes the correction of the current expressions in equations (30) and (31). For instance, in eq. (30) the electric current has to be replaced by

$$j^a_l = (\bar{\Theta}^a_{B_1b_1B_2b_2})^D(\gamma^lC)^+_{\mu_1\mu_2}\psi_{B_1b_1\mu_1}\psi_{B_2b_2\mu_2} \tag{43}$$

To evaluate this expression for the phenomenological Dirac spinors (17), their definition by means of the quantum numbers B, b has to be given. The decomposition into these two quantum numbers can be formally applied, but their meaning depends on the representation. By their construction the phenomenological spinors (17) are referred to the D-representation. Therefore we formally introduce the B- b-numeration by the definition

$$\psi^D_{1,1,\mu} \equiv e^+_\mu; \quad \psi^D_{1,2,\mu} \equiv \bar{\nu}_\mu; \quad \psi^D_{2,1,\mu} \equiv \nu_\mu; \quad \psi^D_{2,2,\mu} \equiv e^-_\mu \tag{44}$$

Then with (41) and definition (42) equation (43) reads

$$\begin{aligned} j^a_l &= \frac{1}{2}\delta_{2B_1}\delta_{1B_2}[(\sigma^a)^Tc^T]_{b_1b_2}(\gamma^lC)^+_{\mu_1\mu_2}\psi^D_{B_1b_1\mu_1}\psi^D_{B_2b_2\mu_2} \\ &= \frac{1}{2}[(\sigma^a)^Tc^T]_{b_1b_2}(\gamma^lC)^+_{\mu_1\mu_2}\psi^D_{2b_1\mu_1}\psi^D_{1b_2\mu_2} \end{aligned} \tag{45}$$

The phenomenological fields in S-representation are defined by $\psi^D_{2,1,\mu} \equiv \psi^S_{2,1,\mu} \equiv \nu_\mu$ and $\psi^D_{2,2,\mu} \equiv \psi^S_{2,2,\mu} \equiv e^-_\mu$ and their charge conjugated counterparts. The latter can be generated by the transformation $\psi^D_{1,b,\mu} = c^T_{b,b'}\psi^S_{1,b',\mu}$. Therefore (45) can be rewritten into the form

$$\begin{aligned} j^a_l &= \frac{1}{2}[(\sigma^a)^Tc^Tc^T]_{b_1b_2}(\gamma^lC)^+_{\mu_1\mu_2}\psi^S_{2,b_1,\mu_1}\psi^S_{1,b_2\mu_2} \\ &= -\frac{1}{2}(\sigma^a)^T_{b_1b_2}(\gamma^lC)^+_{\mu_1\mu_2}\psi_{b_1\mu_1}\psi^c_{b_2\mu_2} \end{aligned} \tag{46}$$

With (γ^lC) its Hermitean conjugate is symmetric too. Thus (45) reads equivalently

$$j^a_l = -(\psi^c_{b_2\mu_2})^T(\sigma^a)_{b_2b_1}(\gamma^lC)^+_{\mu_2\mu_1}\psi_{b_1\mu_1} \tag{47}$$

In the last step one uses $(\psi^c)^T = \bar{\psi} C^T$ and obtains from (47) the $U(1)$ and $SU(2)$-currents

$$j_l^a \equiv -\frac{1}{2} \bar{\psi}_{b_1 \mu_1} \sigma^a_{b_1 b_2} \gamma^l_{\mu_1 \mu_2} \psi_{b_2 \mu_2} \tag{48}$$

In the same way one can proceed to get the magnetic currents J_l^a. The factors (1/2) will be absorbed in the coupling constants, i.e. normalization of the states is irrelevant.

In the next step we rearrange the Dirac equation (32) into the conventional form. For the interpretation of (32) it is important to realize that the $(T\gamma^5)$ and $(S\gamma^5)$ matrices in (32) arise from matrix elements between two three-parton states which characterize the superspin-isospin part of the composite leptons, see [17], eqs.(68),(69). As the lepton states are constructed in the D-basis of parton spinors, the latter matrix elements have to be calculated in this basis. The calculation yields for $a = 1, 2, 3$

$$(S^a \gamma^5)^D_{ln} = \begin{pmatrix} \sigma^a & 0 \\ 0 & -\sigma^a \end{pmatrix}; \qquad (T^a \gamma^5)^D_{ln} = \begin{pmatrix} \sigma^a & 0 \\ 0 & \sigma^a \end{pmatrix} \tag{49}$$

and for $a = 0$

$$(S^0 \gamma^5)^D_{ln} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & \mathbf{1} \end{pmatrix}; \qquad (T^0 \gamma^5)^D_{ln} = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix} \tag{50}$$

In this representation the indices l, n = 1,2, are referred to the spinors $\psi_1^D := e^+$ and $\psi_2^D := \bar{\nu}$, while l, n = 3,4, correspond to the G-conjugated spinors $\psi_3^D := \nu$ and $\psi_4^D := e^-$.

The substitution of (49) and (50) into equation (32) shows that this equation can be decomposed into two separate equations for ψ_1, ψ_2 and ψ_3, ψ_4. In particular for $(\psi_3, \psi_4) \equiv (\nu, e^-)$ one obtains after multiplication of eq. (32) with γ^0 in spin-space the following equation

$$\begin{aligned} &[-i\gamma^\mu \partial_\mu + m]\psi_l + \frac{1}{2}[g\sigma^a_{ln}\gamma^k A_{ka} + g'\sigma^0_{ln}\gamma^k A_{k0}]\psi_n \\ &+ i\frac{1}{2}[g\sigma^a_{ln}(\gamma^k\gamma^5)G_{ka} + g'\sigma^0_{ln}(\gamma^k\gamma^5)G_{k0}]\psi_n = 0 \end{aligned} \tag{51}$$

where the Pauli matrices are referred to the two-dimensional state space defined by (ψ_3, ψ_4). The corresponding equation for (ψ_1, ψ_2) is redundant and will not be explicitly given for the sake of brevity.

It is interesting to note that the coupling of the magnetic vector-potential G^μ in (51) coincides for $a = 0$ with the coupling in Lochak's monopole equation, [6].

Finally we rearrange the field equations into their final form. Neglecting for simplicity the coupling between $SU(2)$-fields and $U(1)$-fields from [17] eq. (53) it follows $\bar{\eta}_{abc} := i\varepsilon_{abc}$. Furthermore we define $c_1 = 1$, $\hat{f}^A = \hat{f}^G$, $k_2 = k_2'$, $k_5 = k_5'$, $(c_2 - \hat{f}^A c_4) =: \mu_A$ and $(c_3 - \hat{f}^A c_4) =: \mu_G$, and express the current coupling constants g_e and g_m by the original constants in eqs. (30) and (31).

Substitution of these definitions and canceling out i yields for equations (28)-(31) the following set of field equations

$$\begin{aligned}\dot{A}_{la}(\mathbf{z})=&-\varepsilon_{lkm}\partial_k^z G_{ma}(\mathbf{z}) - c_2 E_{la}(\mathbf{z})\\ &+\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_1 A_{kb}(\mathbf{z})G'_{mc}(\mathbf{z}) + k_4 G_{kb}(\mathbf{z})A_{mc}(\mathbf{z})]\end{aligned} \tag{52}$$

$$\begin{aligned}\dot{G}_{la}(\mathbf{z})=&-\varepsilon_{lkm}\partial_k^z A_{ma}(\mathbf{z}) + c_2 B_{la}(\mathbf{z})\\ &-\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_3 A_{kb}(\mathbf{z})A_{mc}(\mathbf{z}) - k_6 G_{kb}(\mathbf{z})G_{ma}(\mathbf{z})]\end{aligned} \tag{53}$$

$$\begin{aligned}\dot{E}_{la}(\mathbf{z})=&\varepsilon_{lkm}\partial_k^z B_{ma}(\mathbf{z}) + g_e j_l^a + \mu_A A_{la}\\ &\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_2 A_{kb}(\mathbf{z})B'_{mc}(\mathbf{z}) + k_5 G_{kb}(\mathbf{z})E'_{mc}(\mathbf{z})]\end{aligned} \tag{54}$$

$$\begin{aligned}\dot{B}_{la}(\mathbf{z})=&-\varepsilon_{lkm}\partial_k^z E_{ma}(\mathbf{z}) + ig_m J_l^a - \mu_G G_{la}\\ &-\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_2 A_{kb}(\mathbf{z})E_{mc}(\mathbf{z}) - k_5 G_{kb}(\mathbf{z})B'_{mc}(\mathbf{z})]\end{aligned} \tag{55}$$

For $a = 0$, all terms with ε_{abc} vanish, i.e., one gets the $U(1)$ field equations.

To complete the theory of vector fields their constraints have to be formulated (electric and magnetic Gauss law). In the canonical version of the theory these constraints need not be postulated, but can be derived from eqs. (52)-(55) in combination with the spinor equation (51), cf. for instance [9], section 8.2. This will not be done here, because it is not along the lines of our investigation.

5 Effective Lagrangian density

To draw physical conclusions from the above results it is advantageous to express them in form of an effective Lagrangian as in phenomenology the Lagrangians are the central quantities for the evaluation of the theory.

To apply the Lagrange formalism the definition of the electroweak field tensor in terms of the vector fields is required. In the literature this definition is not uniform. We follow the definition used in the treatment of gauge theories by differential forms, [30],eq.(4.6),[31],p.70, which reads for antisymmetric $F^a_{\mu\nu}$

$$E^a_k = -F^a_{0k}; \qquad B^a_k = \frac{1}{2}\varepsilon_{kij}F^a_{ij} \tag{56}$$

where the metric is defined by $\eta = diag(1, \text{-}\ 1, \text{-}\ 1, \text{-}\ 1)$.

This definition of the fields is consistent with that used in section 4. Furthermore for the currents the following definitions hold

$$j^a_\mu := \bar\psi\sigma^a\gamma_\mu\psi = (j^a_\mu)^+; \qquad J^a_\mu := \bar\psi\sigma^a\gamma^5\gamma_\mu\psi = (J^a_\mu)^+ \qquad a = 0,1,2,3 \tag{57}$$

where the minus sign in (48) is absorbed in the coupling constant.

To describe the effective field dynamics we postulate the following Lagrangian density

$$\begin{aligned}\mathcal{L}:=&-\frac{1}{4}F^a_{\mu\nu}\eta^{\mu\varrho}\eta^{\nu\kappa}F^a_{\varrho\kappa} + \frac{i}{2}[\bar\psi\gamma^\mu\partial_\mu\psi + (\partial_\mu\bar\psi)\gamma^\mu\psi] - m\bar\psi\psi \\ &-g_\chi A^a_\mu j^\mu_a - ig_\pi G^a_\mu J^\mu_a + \frac{1}{2}\mu_A^2 A^a_\mu\eta^{\mu\varrho}A^a_\varrho + \frac{1}{2}\mu_G^2 G^a_\mu\eta^{\mu\varrho}G^a_\varrho\end{aligned} \tag{58}$$

where $g_x = g_\chi, g_\pi$ takes the value g_x for $a = 0$, and g'_x for $a = 1,\ 2,\ 3$.

In (58) the field strength tensor is given by

$$\begin{aligned}F^a_{\mu\nu}:=&\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \varepsilon_{\mu\nu\varrho\sigma}\eta^{\varrho\varrho'}\eta^{\sigma\sigma'}\partial_{\varrho'}G^a_{\sigma'} \\ &+\varepsilon^{abc}(g_1 A^b_\mu A^c_\nu + g_2 G^b_\mu G^c_\nu + g_3\varepsilon_{\mu\nu\varrho\sigma}\eta^{\varrho\varrho'}\eta^{\sigma\sigma'}A^b_{\varrho'}G^c_{\sigma'})\end{aligned} \tag{59}$$

To compare equations (52)-(55) with equations resulting from (58),(59) the constants in the former equations have to be fixed. In [17],eq.(54) their values are expressed by the formation of various scalar products of the space parts of the boson wave functions. As these scalar products (with inclusion of their regularization) are defined in auxiliary space they can adopt positive and negative values in contrast to the norm expressions in physical state space.

While the algebraic structure of the boson wave functions (and of course of the fermion wave functions, too) is strictly set up, the space parts of these wave functions can be chosen only with a certain degree of arbitrariness which reflects the lack of information about the influence of the field theoretic vacuum on the space structure of these states. Therefore without using selfconsistent calculation schemes for the boson wave functions the corresponding scalar products [17], eq.(54) represent parameters of the theory which can be adapted in order to get plausible results. In the present case we define

$$g_1 = \hat{f}^A k_1 = \hat{f}^A k_2 = \hat{f}^A k_3 = \hat{f}^G k_4 = -\hat{f}^G k_5 = \hat{f}^G k_6 \tag{60}$$

Theorem1: If relations (60) are satisfied, and the masses and coupling constants are adapted, then the set of equations (52)-(55) is identical with the set resulting from (58),(59). The same holds for the corresponding fermion equations.

For the sake of brevity we skip the proof.

Addendum: The effective field theory defined by the Lagrangian density (58) is limited to a finite range of energy. Above a certain energy threshold it loses its meaning and has to be modified by formfactors etc.. In this way one does not encounter the divergence difficulties of conventional field theories with Lagrangian of the type (58) and fields (59).

In the mean time a paper appeared which supplies this paper by detailed calculations and application to nuclear reactions [32].

Furthermore one directly realizes that for vanishing G^a_μ and vanishing boson masses μ_A and μ_G, one obtains the Lagrangian of a $SU(2) \otimes U(1)$ gauge theory. A discussion of the discrete Symmetries of (58) and (59) will be given elsewhere.

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(Manuscrit reçu le 7 juin 2006)