The Dynamical de Broglie Theory

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ABSTRACT. Nobody appears to have ever asked the question of how de Broglie's theroy [1] could be proven generally valid and developed from physical laws. Therefore, we will start from the classical Newton's second law (NSL) to rebuilt the therory of special relativity (SRT) [2]. This enables us to derive de Broglie relations from NSL without any of the well-known contradictions between SRT and de Broglie's quantum wave..

KEY WORDS. Newton's second law, de Broglie matter-wave, special relativity theory.

1 Introduction

The SRT has been presented as a unique solution, yet tens of alternative theories are put forward to replace what is called the SRT.

The SRT has been known since its onset as a unifying theory. It unified space and time, matter and energy, and it became the basis for other unifying theories.

The alternative theories of SRT, however, show the shortcomings of SRT as a unifying theory. The SRT has removed the barrier between matter and energy, but it created a new barrier which cannot be transcended according to some of these theories. This barrier separates what is known as non-relativistic from the relativistic physics domain. The physical laws appropriate for non-relativistic physics cannot transcend this barrier and hence they form classical physics. The physical laws appropriate for relativistic physics can also cover the non-relativistic physics domain through the known approximation of the Lorentz Transformation (LT), and the LT will become a Galilean transformation where appropriate. The more suitable method is to start with the laws of classical physics and make them conducive to all particle velocities, i.e. to expand the appropriateness of these laws to deal with relativistic domain. This cannot be achieved unless we go back to the invariance of physical laws among inertial frames regardless of the coordinate transformations. In this way, we could derive the SRT starting from a mechanical base [3] instead of restricting the formation of SRT to the electromagnetic base [2] alone. When we do this, we expand the appropriateness of NSL by describing the moving particle as a wave. Thus, we derive de Broglie relations from NSL without any of the well-known contradictions between SRT and de Broglie quantum wave theory.

2 Energy (Mass), Momentum, Velocity and Force Transformation Relations

Let us consider two moving inertial system S and S' with a relative velocity u // ox between them. The initial law in classical mechanics is NSL and its power expression:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \text{ (a)} \qquad \frac{d\varepsilon_t}{dt} = \mathbf{Fv} \text{ (b)} \tag{1}$$

Where Eq. (1b) can be derived on Newtonian mechanical grounds [3], but in classical mechanics the kinetic energy is $\varepsilon_t = T = \frac{mv^2}{2}$ with $m = m_0$. In his second paper for establishing SRT, Einstein [4] proposed the famous equation $\varepsilon_t = mc^2$. However, many text books on SRT often devote great effort to discussing the process of elastic collision between two particles for deriving $\varepsilon_t = mc^2$ and the relativistic mass $m = \gamma m_0$. Let us try to establish a law for total energy and relativistic mass without using the LT or the original idea of Einstein. As demonstrated in [3], according to the relativity principle NSL implies the fact of “mass being variable”. Depending on this, the quantity for total energy ε_t is determined.

On proper understanding of relativistic mechanics as a modification (correction) of classical mechanics, it is seen that the same correction can be obtained if we go back to NSL and take the change of mass, rather than the scale of space-time as in SRT. So we will demonstrate that the Eqs. (1) and the relativity principle, are more natural for describing the physics of relativistic mechanics. And we can now go further to get the relativistic mass relation as well as all of the SRT's relations by this approach.

The Cartesian components of Eq. (1) in frame S are:

$$\frac{dp_x}{dt} = F_x \text{ (a)}, \quad \frac{dp_y}{dt} = F_y \text{ (b)}, \quad \frac{dp_z}{dt} = F_z \text{ (c)} \qquad (2)$$

And

$$\frac{d\varepsilon_t}{dt} = F_x v_x + F_y v_y + F_z v_z \text{ (d)}$$

Applying the relativity principle to Eq. (2), we have

$$\frac{dp'_x}{dt'} = F'_x \text{ (a)} \quad \frac{dp'_y}{dt'} = F'_y \text{ (b)} \quad \frac{dp'_z}{dt'} = F'_z \text{ (c)} \qquad (3)$$

And

$$\frac{d\varepsilon'_t}{dt'} = F'_x v'_x + F'_y v'_y + F'_z v'_z \text{ (d)}$$

So following a similar approach to that used in [3,5], we can obtain the relativistic transformation equation for momentum, energy, and velocity, as well as the relativistic force transformation:

$$p'_x = \gamma\left(p_x - \frac{u}{c^2}\varepsilon_t\right) \text{ (4a)}, \; p'_y = p_y \text{ (4b)}, \; p'_z = p_z \text{ (4c)}, \; \varepsilon'_t = \gamma(\varepsilon_t - up_x) \qquad \text{(4d)}$$

$$F'_x = F_x - \frac{\frac{u}{c^2}F_y v_y}{\left(1 - \frac{uv_x}{c^2}\right)} - \frac{\frac{u}{c^2}F_z v_z}{\left(1 - \frac{uv_x}{c^2}\right)} \text{ (5a)}, \; F'_y = \frac{F_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \text{ (5b)},$$

$$F'_z = \frac{F_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \text{ (5c)}$$

$$v'_x = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \text{ (6a) }, \quad v'_y = \frac{v_y}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \text{ (6b) }, \quad v'_z = \frac{v_z}{\gamma\left(1 - \frac{uv_x}{c^2}\right)} \qquad \text{(6c)}$$

We may write Eqs. (6) as

$$\frac{v'_x}{\sqrt{1 - \frac{v'^2_x}{c^2}}} = \frac{v_x - u}{\sqrt{1 - \frac{u^2}{c^2}}\sqrt{1 - \frac{v^2}{c^2}}} \qquad (7)$$

I use Eq. (6b) into (4b) to get

$$m' = \gamma m(1 - \frac{uv_x}{c^2}) \qquad (8)$$

Multiplying the Eq. (7) with m_0, and comparing it with (8), we deduce

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ (a) }, \quad m' = \frac{m_0}{\sqrt{1 - \frac{v'^2}{c^2}}} \text{ (b) } \qquad (9)$$

From Eq. (1b), the total energy is given by

$$\begin{aligned} d\varepsilon_t &= Fvdt = d\left(mv\right)v \\ &= v^2 dm + mvdv \end{aligned} \qquad (10)$$

And from Eq. (9a), we have

$$dm = \frac{mvdv}{c^2\left(1 - \frac{v^2}{c^2}\right)} \qquad i.e. \qquad m\,v\,dv = c^2\left(1 - \frac{v^2}{c^2}\right)dm \qquad (11)$$

Substituting Eq. (11) in Eq. (10), we get

$$d\varepsilon_t = c^2 dm \tag{12}$$

By integration, from v_1 to v_2, we get

$$\varepsilon_t = mc^2\Big|_1^2 \tag{13}$$

In the particular case, if $v_1 = 0$ and $v_2 = v$, then ε_t should equal the kinetic energy ε_k, i.e.

$$\varepsilon_k = mc^2\Big|_1^2 = mc^2 - m_0 c^2 \tag{14}$$

So the quantities mc^2 and $m'c^2$ are the total energy ε_t and ε'_t in frames S and S' respectively.

It is simple to prove, that Eqs. (4) and (9) lead to

$$\varepsilon'^2 - c^2\boldsymbol{P}'^2 = \varepsilon^2 - c^2\boldsymbol{P}^2 = m_0^2 c^4 \tag{15a}$$

Or

$$\varepsilon_t^2 = c^2\boldsymbol{P}^2 + m_0^2 c^4 \ \text{(15b)} \qquad \varepsilon_t'^2 = c^2\boldsymbol{P}'^2 + m_0^2 c^4 \tag{15c}$$

The dynamics of a moving particle are built into SRT to accommodate the LT. Therefore, we introduced an alternative path to derive all of the mentioned relations in this section, which are not based on LT.

Depending on this formulation we continue to derive the de Broglie relations for particle-wave duality, but without SRT or any relativistic assumptions as required by the LT.

3 de Broglie Theory and SRT

After the creation of the electromagnetic theory of light, it becomes possible to formulate the laws of corpuscular properties of radiation and the wave properties of corpuscular as

$$\varepsilon_t = hf = \hbar w, \; p = \frac{h}{\lambda} = \hbar k \tag{16}$$

de Broglie [1], postulated the validity of relation (16) for a particle with rest mass m_0 through his hypothesis the “periodic phenomenon”, i.e.

$$hf_0 = m_0 c^2 \tag{17}$$

When Eq. (17) is written with respect to the frame S , then Eq. (17) takes the form

$$hf = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{18}$$

According to Eq. (17), he found

$$f = \frac{f_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19a}$$

However, as is well known in SRT, if the clock has a frequency f_0 in the rest frame of the particle, its frequency, according to the so-called time dilation, when it is moving at a velocity v in frame S is

$$f = f_0 \sqrt{1 - \frac{v^2}{c^2}}\,, \tag{19b}$$

Evidently, Eq. (19b) is just the opposite of Eq. (19a). Indeed, accounting for time dilation leads to the slow down of the "moving clock" frequency, Eq. (19b), whilst accounting for the energy increase of a "moving particle" yields an increased frequency, Eq. (19a). Thus, it is clear that some additional assumption is needed to overcome such a fundamental contradiction. To find the way out of this paradox de Broglie assumed that f in Eq. (19b) is not the frequency of a clock moving with the particle but the frequency of a wave accompanying the particle propagating with velocity v_p in the direction of motion. The fact that its velocity $v_p = \frac{c^2}{v}$ is necessarily greater than light speed c, shows that it cannot represent transport of energy. It is a "phase wave".

We see now how de Broglie came to an important relationship existing between the velocity of a body in motion and a phase wave, i.e.:

$$v_p v = c^2 \quad \text{i.e.} \quad v_p = \frac{\varepsilon_t}{p} = \frac{w}{k} \;\rangle\; c \qquad (20a)$$

Then de Broglie proved a theorem that "The group velocity of phase waves equals the velocity of its associated body i.e.:

$$v_{gr} = v$$

As is well known that as usual the phase velocity of a wave is

$$v_p = \frac{\varepsilon_t}{p} = \frac{w}{k} \qquad (20b)$$

Whereas the particle velocity equals the group velocity of wave

$$v = v_g = \frac{dw}{dk} \qquad (20c)$$

Since the appearance of de Broglie theory, which was formulated through SRT's relationships, an obviously contradiction between them was raised. For instance, the well-known de Broglie relation $\lambda = \frac{h}{p}$, where λ is the wave length of a de Broglie wave associated with a particle having momentum p, is in contrast with Einstein's total energy relation, $\varepsilon_t = mc^2$ [6]. Another contradiction is that Eq. (20a) is now different from the mechanical velocity v for the same particle, therefore the superluminal velocity v_p is said to be devoid of any physical meaning [7]. Although, from this the Lorentz transformation for wave vector and frequency, i.e.

$$k' = \gamma\left(k - \frac{uw}{c^2}\right) = \gamma k\left(1 - \frac{u}{c^2}\frac{w}{k}\right), \quad w' = \gamma(w - uk) = \gamma w\left(1 - \frac{uk}{w}\right)$$

are expressed by the phase velocity as

$$k' = \gamma k\left(1 - \frac{uv_p}{c^2}\right) \text{(21a)}, \; w' = \gamma w\left(1 - \frac{u}{v_p}\right) \qquad \text{(21b)}$$

For a light wave $v = c$, Eq (21b) is a longitudinal Doppler shift formula, while for a matter wave Eq (21b) is none a longitudinal Doppler shift formula. So the union of SRT and de Broglie's wave formalism has always been precarious concerning the different velocities of the same particle.

Due to the difference between phase velocity and group velocity of de Broglie waves, de Broglie then, through Eq. (20a) developed the concept of a wave associated with material particles and removes the point – particle phenomena to a matter – wave phenomena in question. A scientific controversy was subject to mach discussion and the attempts so far. The first of these attempts can be attributed to J. Wesely [8], who supposed a real wave function instead of the complex wave function in traditional quantum theory. And he could prove that the phase velocity equals the particle velocity. Another attempt in this context is M. Wolff [9]. The wave – structure of the moving electron is analyzed on the spherical waves by Wolff. He formulates the SRT free from the usual contradiction. And then he concludes the compatibility between SRT and de Broglie theory.

4 Derivation of de Broglie relations for the Moving Duality on Dynamical Basis

Recently, R. Ferber [10], it has been showed that Eq. (20a) is a result of using LT, and not a result of de Broglie's hypothesis.

Therefore to deal with these contradictions, we must re-derive all relations in section 3 without LT. To remove the kinematical contradiction in de

Broglie formalism, we derive first the well-known de Broglie relation $\lambda = \frac{h}{p}$ on a dynamical basis, so starting from Eq. (15b), i.e.

$$\varepsilon_t^2 = c^2 p^2 + m_0^2 c^4$$

The last relation yields to:

$$\varepsilon_t d\varepsilon_t = c^2 p dp, \quad \text{i.e.;} \quad d\varepsilon_t = v dp$$

Using Plank – Einstein relation, that $\varepsilon_t = h\nu = \hbar w$, yields

$$dw = \frac{v}{\hbar} dp$$

Now using the definition of the group velocity, i.e.; Eq. (20c), we have

$$dk = \frac{dp}{\hbar}$$

By integration, $v = 0$ i.e. $k = 0$, we get

$$k = \frac{p}{\hbar} \quad \text{i.e.} \quad \lambda = \frac{h}{p} \tag{22}$$

We can now remove the contradiction in Eq. (21b) if we take into consideration that Eq. (13) could be written as [6]:

$$\varepsilon_t = mv^2 + m_0 c^2 \sqrt{1 - \frac{v^2}{c^2}} \tag{23}$$

The new form of total energy ε_t, Eq. (23), is very important, because it shows us a new hidden variable, which is the relative kinetic energy

$$\varepsilon_v = mv^2 \tag{24}$$

Eq. (24) takes the name "the relative kinetic energy" because it is related to total energy ε_t as

$$\varepsilon_v = \frac{v^2}{c^2} mc^2 = \frac{v^2}{c^2} \varepsilon_t$$

Now in addition to the right relation $\varepsilon_t = hf = mc^2$, we can specify the following relation

$$\varepsilon_v = hf = mv^2 \tag{25}$$

Eq. (25) helps us to prove that $v_p = v$, if we substitute both Eqs. (25) and (22) into Eq. (20b)

$$v_p = \frac{w}{k} = \frac{mv^2 / \hbar}{mv / \hbar} = v \tag{26}$$

Now setting Eqs. (22) and (20b) in Eq. (4a), so we have

$$k' = \gamma k(1 - \frac{uv_p}{c^2})$$

Using Eq. (26) in the last relation, we get

$$k' = \gamma k(1 - \frac{uv_x}{c^2})$$

Or

$$kv_x' = \gamma kv_x(1 - \frac{u}{v_x})$$

According to Eqs. (26) and (20b) the velocity of a moving duality in frames S and S' are $w = v_x k$, $w' = v'_x k'$, so we have :

$$w' = \gamma w(1 - \frac{u}{v_x}) \tag{27}$$

Eq. (27) is now a longitudinal Doppler shift formula for a moving matter-wave duality, and reduced for a longitudinal Doppler shift formula for a photon-wave duality, i.e. for $v = c$ [11].

de Broglie's derivation of Eq. (22) was for relativistic velocity. But we may derive de Broglie relation, Eq. (22), for non-relativistic velocities.

From Eq. (23) we have, by definition, the kinetic energy as

$$\varepsilon_k = \varepsilon_t - m_0 c^2 = mv^2 - m_0 c^2 (1 - \sqrt{1 - \frac{v^2}{c^2}}) \tag{28}$$

And for non-relativistic velocities, we expand the roots in the last relation. Finally, for non-relativistic velocities Eq. (28) reduces to:

$$\varepsilon_t = \varepsilon_k + m_0 c^2 = \frac{1}{2} m_0 v^2 + m_0 c^2 \tag{29}$$

We can now derive Eq. (22) for the case of non-relativistic velocities also. As we see from Eq. (29), the differentiation leads to:

$$d\varepsilon_t = vdp = \frac{p}{m_0} dp$$

Using the Planck–Einstein relation, i.e. $\varepsilon_t = h\nu = \hbar w$, yields

$$dw = \frac{v}{\hbar} dp$$

Now using the definition of the group velocity , i.e.; Eq. (20c), we have

$$dk = \frac{dp}{\hbar}$$

By integration, $v = 0$ i.e. $k = 0$, we get

$$k = \frac{p}{\hbar} \quad \text{i.e.} \quad \lambda = \frac{h}{p} \tag{30}$$

In this case, i.e. for non-relativistic velocities, the particle has sufficiently low energy that we may neglect relativistic effects (as one says always in SRT), so de Broglie's speculation applies for the particle.

5 Conclusion

The incompatibility between SRT and particle dynamics arise because the LT and its kinematical effects have the primacy over the physical laws in deriving the relativistic dynamical quantities and in the interpretation of relativistic phenomena [3,5,6 and 11].

Therefore there exists an inconsistency between Einstein’s special relativity and de Broglie quantum wave, and it has never been resolved from the viewpoint of relativistic physics. A more suitable method to deal with the contradictions is to start with the laws of classical physics and make them applicable to all particle velocities; i.e., expand the appropriateness of these laws to deal with the relativistic domain. When we do this, we expand the appropriateness of Newton’s Second Law by describing the moving particle as a wave. Thus, we derive the de Broglie relationships from NSL without any contradictions.

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