

The Fine-structure Constant and the Torsion Potential

HWEI-JANG YO

Department of Physics,
National Cheng-Kung University, Tainan 701, Taiwan

ABSTRACT. The possible variation of the fine-structure constant, α , has inspired many people to work on modifications and/or generalizations of the current “standard” theories in which the electromagnetic field is involved. Here we first point out the amazing similarity between Bekenstein’s model, describing the variation of α by a varying charge, and the Hojman-Rosenbaum-Ryan-Shepley torsion potential model. This observation invites us to consider a geometric theory of gravity in which a varying α originates from another kind of dynamic quantity of spacetime, i.e., vector torsion. Since the vector torsion field is weak and also not strongly coupled with fermions it is difficult to detect it directly. The detection of a time-varying α could thus provide some promising evidence for the existence of torsion.

P.A.C.S.: 03.65.Bz; 04.20.-q

There has been a long history of people searching for the variation of various constants, see eg, [1]. By checking how *constant* those constants are, it in fact questions the correctness of the theories where those constants are defined. The variation of a constant implies new degrees of freedom or dynamical variables which have been ignored in the theory. Thus it indicates either that a modification and/or a generalization for the related theory is needed, or that the theory is only an effective one which should come from some more fundamental theory.

The fine-structure constant, α , is one of the constants which has been examined extensively. The different measurements appear to converge to the conclusion: α was smaller in the past, i.e., $\Delta\alpha/\alpha < 0$ [2]. This result fueled the effort of establishing new models incorporating the variation

of α , as well as the examinations of the possible variation of the other constants.

Among various theoretical models leading to the prediction of such a variation, Bekenstein [3] proposed the following framework to incorporate a varying α . With a certain set of assumptions, the Maxwell equations are modified to account for the effect of the variation of the elementary charge, $e = \epsilon(\vec{x})e_0$ where $\epsilon(\vec{x})$ is a dimensionless scalar field. The vector potential A_μ follows the gauge transformation law $A'_\mu \leftarrow A_\mu + \epsilon^{-1}\chi_{,\mu}$, rather than the usual $A'_\mu \leftarrow A_\mu + \chi_{,\mu}$. The electromagnetic tensor generalizes to

$$F_{\mu\nu} = \frac{1}{\epsilon} [(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}]. \quad (1)$$

The dynamics of ϵ is given from the action:

$$S_\epsilon = -\kappa \int \epsilon^{-2} \epsilon_{,\mu} \epsilon^{,\mu} \sqrt{-g} d^4x \quad (2)$$

where κ is a parameter, and the electromagnetic action is the usual quadratic form of $F_{\mu\nu}$ from eqn (1):

$$S_{\text{EM}} = -\frac{1}{16\pi} \int F_{\mu\nu} F^{\mu\nu} \sqrt{-g} d^4x. \quad (3)$$

It has been shown to be a viable theory for explaining the α variation [4].

However, three conventionally fundamental concepts which are deeply believed in science are violated and/or modified in this model: (a) charge conservation, (b) local gauge invariance, and (c) minimal coupling. At a first glance, it is not obvious why one has to change so many physical laws only to obtain a varying α . Assuming that this idea is on the right track, here we would like to exploit the possible linkage of Bekenstein's model with the geometry of spacetime. If one considers this idea from a geometric point of view, it is easy to recognize how similar the structure of this model is with the Hojman-Rosenbaum-Ryan-Shepley (hereafter HRRS) torsion potential model[5]. In the HRRS model, the Einstein-Cartan theory of gravity is considered. This theory allows non-symmetric connection coefficients $\Gamma^\mu{}_{\nu\sigma}$, therefore it has a nonzero torsion tensor $T_{\nu\sigma}{}^\mu$ [10]

$$T_{\nu\sigma}{}^\mu = \Gamma^\mu{}_{\sigma\nu} - \Gamma^\mu{}_{\nu\sigma}. \quad (4)$$

The torsion tensor field, $T_{\mu\nu}{}^\sigma = T_{[\mu\nu]}{}^\sigma$, is then determined by the gradient of a scalar field $\phi(\vec{x})$,

$$T_\mu \equiv T_{\mu\nu}{}^\nu = \phi_{,\mu}, \quad T_{\mu\nu}{}^\sigma = \frac{2}{3}T_{[\mu}{}^\sigma\delta_{\nu]} = \frac{1}{3}(\delta_\nu^\sigma\phi_{,\mu} - \delta_\mu^\sigma\phi_{,\nu}), \quad (5)$$

where T_μ is the vector torsion. With such an assumption the Einstein-Cartan theory of gravity, is now allowed to propagate. In torsion field, which plays originally an algebraic role in the order to be compatible with torsion, the form of local gauge invariance in the HRRS model is modified—exactly into the one in Bekenstein’s model. Such a similarity suggests to us a bold assumption, i.e., $\phi = \ln \epsilon$. Then the electromagnetic field tensor $F_{\mu\nu}$, which is defined as the *covariant derivative* of the the vector potential A_μ with the affine connection including Christoffel symbol *and* torsion, in the HRRS model is identical to the definition (1), as follows:

$$F_{\mu\nu} \equiv A_{\nu;\mu} - A_{\mu;\nu} = A_{\nu,\mu} - A_{\mu,\nu} + T_\mu A_\nu - T_\nu A_\mu = \frac{1}{\epsilon}[(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}]. \quad (6)$$

It is noteworthy that a similar situation has happened when the variation of the gravitational constant was considered as evidence for the existence of torsion [6], which also leads to the possibility of re-interpreting the Brans-Dicke theory using the torsion potential idea [7].

The HRRS model has been shown to conflict with the result of the Eötvös-Dicke-Braginsky experiments in our solar system [8]. However, the failure probably comes from its restrictive form of the gravitational Lagrangian density L_g , not necessarily from the concept of torsion potential. Some generalizations of the HRRS have been proposed by adding a potential term of ϕ in the Lagrangian density, in order to be compatible with the experiments, eg, see [9]. Here we would like to point out another possibility. In the HRRS model, only the affine scalar curvature in the Einstein-Cartan theory is considered in L_g . The affine scalar curvature is decomposed into one Riemannian scalar curvature term and three quadratic torsion terms (plus a total derivative of torsion term which can be neglected in the action.) Therefore, the torsion terms and the Riemannian scalar curvature in fact share the same parameter in L_g , i.e., the gravitational constant. The situation looks like a coincidence when a new type of dynamic field is introduced into a theory. In general, we will expect that a new parameter is assigned to this new field for its strength. This is the case in the Poincaré gauge theory of gravity [10], in

which the gravitational Lagrangian is a combination of one scalar curvature, three quadratic torsion terms, and six quadratic curvature terms: $L_g \sim R + 3 \times T^2 + 6 \times R^2$, schematically. There are totally ten parameters in the theory, one for each term. Therefore, for the HRRS model, we would like to propose a new type of its generalization, i.e.,

$$L_g = R + aT_\mu T^\mu + V(\phi), \quad (7)$$

where $T_\mu T^\mu$ is the kinetic part of ϕ and a is the assigned parameter, and $V(\phi)$ is the potential term of ϕ . This type of gravitational Lagrangian is the same as the one in the model of a varying α [4].

With the new Lagrangian density (7), it becomes possible for the torsion potential theory to be consistent with the solar experiments. In [8], it argues that in the HRRS model the acceleration difference due to the torsion potential effect is at least about $10^4 \sim 10^5$ times larger than the solar experiments. With our relaxed version of HRRS torsion potential model, we can make a quick fix on the inconsistency by allowing the parameter $a \geq 10^5$ and neglecting the potential term. Here we only point out that a torsion potential model could be viable by adding the kinetic term $aT_\mu T^\mu$ and the potential term $V(\phi)$. The detailed comparison between this model and the experiments needs further investigations.

Once the Bekenstein's model and the generalization of the HRRS model (7) are merged together, analogous to the metric tensor which dynamically determines the value of the interval, the torsion potential could be understood as dynamically determining the vacuum impedance¹ Ω of spacetime, with $\Omega \propto e^{2\phi}$. The physical meaning of a dynamic Ω is interpreted to a varying charge in Bekenstein's model. The usual gauge transformation law of electromagnetic field (which might imply the premetric character of electromagnetism [11]) is incompatible with the Einstein-Cartan theory. The usual type of definition means that photons are decoupled from torsion. Since torsion is a geometric quantity of spacetime generated by the spin of matter, we would rather argue that spinning particles, including photon, should interact with torsion. Moreover a form of minimal coupling is preserved in our model, with the vector torsion defined as the gradient of the torsion potential.

It is generally believed that the source of torsion comes from spin. Thus a torsion field could be generated in the early Universe in which a

¹the vacuum impedance $\Omega \equiv \sqrt{\mu_0/\epsilon_0}$ where μ_0 is the permeability of free space, and ϵ_0 is the permittivity of free space

strong spin field was formed by the highly aligned matter of high density. Due to the property of isotropy and homogeneity of the Universe, once the primordial torsion was produced, and evolved with the expansion of the Universe, it should be weak and mainly time-dependent at present, this is just what we expect of the torsion potential, $\phi = \phi(t)$. Thus the temporal component of the vector torsion will be dominant: $T_\mu \sim \delta_\mu^0 T_0(t)$. Therefore, we expect the time variation of the torsion and the torsion potential will be much larger than their spatial variation, and likewise for α .

It will make a big difference if the scalar field in Bekenstein's model is regarded as the torsion potential, i.e., $\phi = \ln \epsilon$. Since torsion is a geometrically intrinsic character of spacetime, its existence will affect the definition of parallel transport, and consequently affect the physical laws, i.e., the behavior of matter and fields in spacetime. However, as we know, a fermion is strongly coupled to the totally antisymmetric torsion but only indirectly linked with vector torsion T_μ [12], thus most of the matter in the Universe will be relatively insensitive to the existence of T_μ , only a boson like a photon could be coupled to T_μ . This should explain why the vector torsion is so elusive. However, with the technological improvement in precision measurements, the *tiny* variations of the physical constants might shed a light and show us how to establish a new theory of gravity with torsion.

The authors is grateful to Prof. James M. Nester and Prof. Chopin Soo for their helpful suggestions and discussions. This work was supported in part by the National Science Council of the R.O.C. (Taiwan) under grant No. NSC95-2112-M-006-017-MY2.

References

- [1] P.A.M. Dirac, "The cosmological constants," Nature **139**, 323 (1937); P.J. Peebles and R.H. Dicke, "Cosmology and the Radioactive Decay Ages of Terrestrial Rocks and Meteorites," Phys. Rev. **128**, 2006 (1962).
- [2] J.P. Uzan, "The fundamental constants and their variation: observational and theoretical status," Rev. Mod. Phys. **75**, 403 (2003).
- [3] J.D. Bekenstein, "Fine-structure constant: Is it really a constant?" Phys. Rev. **D25**, 1527 (1982).
- [4] H.B. Sandvik, J.D. Barrow, and J. Magueijo, "A simple Cosmology with a Varying Fine Structure Constant," Phys. Rev. Lett. **88**, 031302 (2002).

- [5] S. Hojman, M. Rosenbaum, M.P. Ryan, and L.C. Shepley, "Gauge invariance, minimal coupling, and torsion," *Phys. Rev.* **D17**, 3141 (1978).
- [6] R.T. Rauch, "Time-Dependent Gravitational Coupling and Torsion," *Phys. Rev. Lett.* **52**, 1843 (1984).
- [7] H.H. Soleng, "Scalar-tensor theory of gravitation with spin-scalar-torsion coupling," *Class. Quantum Grav.* **5**, 1489 (1988).
- [8] W.T. Ni, "Hojman-Rosenbaum-Ryan-Shepley torsion theory and Eötvös-Dicke-Braginsky experiments," *Phys. Rev.* **D19**, 2260 (1978).
- [9] N.J. Poplawski, "Propagating torsion in the Einstein frame," *J. Math. Phys.* **47**, 112504 (2006).
- [10] F.W. Hehl, P. von der Heyde, G.D. Kerlick, and J.M. Nester, "Spin and Torsion in General Relativity: Foundations and Prospects," *Rev. Mod. Phys.* **48**, 393 (1976).
- [11] F.W. Hehl and Y.N. Obukhov, "Dimensions and units in electrodynamics," *Gen. Rel. Grav.* **37** 733 (2005).
- [12] I.L. Shapiro "Physical aspects of the space-time torsion," *Phys. Rep.* **357** 113 (2002).

(Manuscript reçu le 14 septembre 2007)