Transmogrifying Torsion

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ABSTRACT. The Oxford English Dictionary defines "transmogrify" as "to alter or change in form or appearance; to transform (utterly,grotesquely, or strangely)." Hence, "transmogrification" is a "strange or grotesque transformation".

Since the late seventies, a particular form for torsion as the derivative of a scalar field has been a recurring theme in various studies. Yet the reason for this strange transformation has been ignored. We will present a metric-torsion theory of gravity coupled to both Abelian and non-Abelian gauge fields and examine the various interpretations for the stangely transformed torsion field.

1 Introduction

Since it was put forward as a viable field to be included in a geometric theory of gravity [1], torsion has had many rebirths in diverse forms. Torsion as a possible field in a theory of gravity gained respectability after the pioneering work of Sciama and Kibble [2], who argued that gravity is a gauge theory with the Lorentz (Poincare) groups as the gauge groups.

However, torsion has always had a problem vis a vis its dynamics because the simplest extensions to GR restricted torsion to be a non-dynamic field. Such non-dynamic torsion fields could then be eliminated from the equations altogether. To provide dynamics to the torsion field, it was necessary to add explicit terms to the Lagrangian of GR containing derivatives of torsion. Unfortunately, there were no compelling physical or geometrical reasons for choosing particular types of torsion derivatives. Indeed, some early results examining the number of possible terms that could be included suggested a ridiculously large number

(≈200!). This would have entailed the introduction of an equally large number of new coupling parameters, badly scarring the beautiful marble edifice of the general theory of relativity.

The ideas advocated by Cartan and theories constructed by Sciama and Kibble envisaged torsion as arising from a non-symmetric connection and are called, generically, metric-torsion theories of gravity. These are to be distinguished from other theories of gravity containing torsion, which are called teleparallel theories of gravity.

More recently, there has been a resurgence of interest in torsion, largely because of its appearance in some string theories. The torsion that appears in these theories is usually a totally antisymmetric tensor. In metric torsion theories, in particular, the Einstein-Cartan-Sciama-Kibble(ECSK)[3] theory, when coupling spinor fields, one finds that only the totally antisymmetric part of the torsion tensor couples to the spinorial matter fields. This perhaps is the reason for the rekindling of interest in torsion theories of gravity.

A distinctive feature of all torsion theories of gravity has been the non-coupling of gauge fields to torsion (at least through the usual minimal coupling procedure for introducing gauge fields). If one attempts to include gauge fields in torsion theories of gravity through this minimal coupling procedure, one immediately faces the prospect of violation of gauge invariance. An additional feature is the appearance of parity violating fermionic interactions. Various arguments have been put forward for this non-democratic nature of the interactions of the torsion field and the need, therefore of not coupling torsion to gauge fields. Gauge fields, in particular, the Maxwell field is given a separate identity. We have always been of the opinion that torsion must interact with all fields. In this spirit, being a new field coming into the theory, it is torsion that must adjust to fit into the theory with all its rules for consistent couplings. With this in mind, we shall try to construct a theory of gravity with torsion in interaction with gauge fields.

2 Gauge Fields and Torsion

Since the torsion tensor $T^{\mu}_{\alpha\beta}$ is the antisymmetric part of the connection $\Gamma^{\mu}_{\alpha\beta}$, we write

$$T^{\mu}_{\nu\alpha} = \Gamma^{\mu}_{\alpha\nu} - \Gamma^{\mu}_{\nu\alpha}. \tag{1}$$

The connection is

$$\Gamma^{\sigma}_{\mu\alpha} = \left\{{}^{\sigma}_{\mu\alpha}\right\} - \frac{1}{2}(T^{\sigma}{}_{\cdot\,\mu\alpha} - T^{\sigma}_{\mu\,\cdot\,\alpha} - T^{\sigma}_{\alpha\,\cdot\,\mu}). \tag{2}$$

The electromagnetic field considered as an Abelian gauge field has its field strength tensor given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

in flat Minkowski spacetime. The traditional procedure for curved spacetimes is to generalize the partial derivative to the spacetime covariant derivative $\partial_\mu \longrightarrow \nabla_\mu$, where ∇_μ is defined by

$$A_{\nu;\mu} = \nabla_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\alpha_{\nu\mu} A_\alpha. \tag{4}$$

Hence the field strength tensor is given by

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu. \tag{5}$$

It is here that problems arise when the connection is not symmetric and torsion does not vanish. It is easy to see that

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - A_\sigma T^\sigma_{\mu\nu}, \tag{6}$$

thus, the A_σ term in $F_{\mu\nu}$ renders it non covaraint under the gauge transformation

$$A_\nu = A_\mu - \frac{1}{q}\partial_\mu \Lambda. \tag{7}$$

A suggestion was made by Hojman et.al. [4] that the coupling of the electromagnetic field to charged matter fields be modified from

$$D_\mu = \partial_\mu - iqA_\mu \tag{8}$$

to

$$D_\mu = \partial_\mu - iqb^\alpha_\mu A_\alpha, \tag{9}$$

where q is the electric charge and b^α_μ is to be determined by requiring that the modified $F_{\mu\nu}$ transforms gauge covariantly. This requirement then restricts the form of the torsion field. They found that if

$$b^\alpha_\mu = \delta^\alpha_\mu e^{-\phi}, \tag{10}$$

and

$$T^\alpha_{\mu\nu} = \delta^\alpha_\nu \partial_\mu \phi - \delta^\alpha_\mu \partial_\nu \phi, \tag{11}$$

then torsion can be coupled to the Abelian gauge fields while preserving the gauge invariance of the action. The total current is also conserved as required. The extension to non-Abelian gauge fields was carried out by Mukku et.al [5] who found that for a non-Abelian gauge field, the minimal coupling procedure for the gauge fields is modified to

$$\mathbf{D}_\mu = \partial_\mu \mathbf{1} - ige^{-\phi}\mathbf{A}_\mu, \tag{12}$$

where now, the gauge fields are Lie algebra valued and g is the non-Abelian gauge parameter. It is interesting to note that the torsion is unchanged and is still given by (11).

It has also been shown that the gauge field strength tensor, $\mathbf{F}_{\mu\nu}$ is gauge covariant under both infinitesimal and finite gauge transformations[5]. Therefore, the usual Lagrangian for Yang-Mills fields constructed out of $\mathbf{F}_{\mu\nu}$ is completely gauge invariant. In the process of the generalization, it was seen [5] that the modifications suggested in [4] could be reinterpreted by saying that the gauge parameter has become a function of spacetime. It is easy to see this. From (12), assuming the gauge coupling parameter $g = g(x)$, we can evaluate the curvature for the gauge connection $\mathbf{A}_\mu$:

$$\mathbf{F}_{\mu\nu} = [\mathbf{D}_\mu, \mathbf{D}_\nu] \tag{13}$$

This easily gives

$$\mathbf{F}_{\mu\nu} = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu - ig(x)[\mathbf{A}_\mu, \mathbf{A}_\nu] - [\frac{1}{g(x)}(\partial_\nu g(x))\delta^\alpha{}_\mu - \frac{1}{g(x)}(\partial_\mu g(x))\delta^\alpha{}_\nu]\mathbf{A}_\alpha \tag{14}$$

comparing with the form of $\mathbf{F}_{\mu\nu}$ in a Riemann-Cartan spacetime,

$$\mathbf{F}_{\mu\nu} = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu - ig(x)[\mathbf{A}_\mu, \mathbf{A}_\nu] - \mathbf{A}_\alpha T^\alpha{}_{\mu\nu}, \tag{15}$$

it is clear that gauge covariance of $\mathbf{F}_{\mu\nu}$ will be preserved if the torsion tensor is given by

$$T^\alpha{}_{\mu\nu} = \frac{1}{g(x)}(\partial_\nu g(x))\delta^\alpha{}_\mu - \frac{1}{g(x)}(\partial_\mu g(x))\delta^\alpha{}_\nu. \tag{16}$$

Making the identification $g(x) = ge^{-\phi}$ reduces the torsion tensor to that given in (11).

When considering the coupling of such gauge fields to charged matter fields ψ, the following gauge transformation laws hold:

$$\psi(x) \longrightarrow U\psi(x) \tag{17}$$

the gauge potential transforms as:

$$\mathbf{A}_\mu \longrightarrow \frac{-i}{g(x)}(\partial_\mu U)U^{-1} + U\mathbf{A}_\mu U^{-1} \tag{18}$$

while the field strength tensor transforms covariantly as explained above:

$$\mathbf{F}_{\mu\nu} \longrightarrow U\mathbf{F}_{\mu\nu}U^{-1} \tag{19}$$

The Lagrangian for the coupled gravity plus gauge fields is

$$\mathcal{L} = \frac{c^3}{16\pi G}R(\Gamma) - \frac{1}{4}Tr(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}). \tag{20}$$

The Tr indicates a trace over internal gauge group indices. Since,

$$\Gamma^\alpha_{\beta\gamma} = \{^{\ \alpha}_{\beta\gamma}\} - \delta^\alpha_\gamma \partial_\beta \phi + g_{\beta\gamma}\partial^\alpha \phi, \tag{21}$$

we can evaluate $R(\Gamma)$ to get

$$\mathcal{L} = \frac{c^3}{16\pi G}[R(\{\}) - 6\partial_\mu\phi\partial^\mu\phi] - \frac{1}{4}Tr(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}). \tag{22}$$

The field equations for the non-Abelian gauge fields are of interest to examine current conservation. To find them, we follow the usual methodology. Consider a variation of only the gauge field Lagrangian:

$$\delta S_{gauge} = -\int Tr(\mathbf{F}_{\mu\nu}\delta\mathbf{F}^{\mu\nu})\sqrt{-g}d^4x \tag{23}$$

Equation (15) tells us that

$$\delta\mathbf{F}^{\mu\nu} = \{\partial^\mu\delta\mathbf{A}^\nu - ig(\delta\mathbf{A}^\mu)\mathbf{A}^\nu - ig\mathbf{A}^\mu(\delta\mathbf{A}^\nu)\} - \{\mu \longleftrightarrow \nu\} - g^{\mu\rho}g^{\nu\sigma}T^\gamma{}_{\rho\sigma}\delta\mathbf{A}_\gamma \tag{24}$$

Hence,

$$\begin{aligned}\delta S_{gauge} = -\int Tr(\mathbf{F}_{\mu\nu}\{\{\partial^\mu\delta\mathbf{A}^\nu - ig(\delta\mathbf{A}^\mu)\mathbf{A}^\nu - ig\mathbf{A}^\mu(\delta\mathbf{A}^\nu)\}\\ -\{\mu \longleftrightarrow \nu\} - g^{\mu\rho}g^{\nu\sigma}T^\gamma{}_{\rho\sigma}\delta\mathbf{A}_\gamma\})\sqrt{-g}d^4x\end{aligned} \tag{25}$$

Using the antisymmetry of $\mathbf{F}_{\mu\nu}$, and the cyclic property of the trace and carrying out a partial integration (assuming the boundary term vanishes), this simplifies to

$$\delta S_{gauge} = 2\int Tr(\{\mathcal{D}^\mu\mathbf{F}_{\mu\nu}\}\delta\mathbf{A}^\nu)\sqrt{-g}d^4x + \int Tr(\mathbf{F}^{\mu\nu}T^\alpha{}_{\mu\nu}\delta\mathbf{A}_\alpha)\sqrt{-g}d^4x \tag{26}$$

where we have used $\mathcal{D}$ to denote a combined spacetime plus gauge covariant derivative with respect to the Christoffel connection:

$$\mathcal{D}_\mu \mathbf{F}_{\alpha\beta} = \partial_\mu \mathbf{F}_{\alpha\beta} - \{^\rho{}_{\alpha\mu}\}\mathbf{F}_{\rho\beta} - \{^\rho{}_{\beta\mu}\}\mathbf{F}_{\alpha\rho} - ig[\mathbf{A}^\mu, \mathbf{F}_{\alpha\beta}] \tag{27}$$

Finally, substituting the expression for the torsion from equation(11), we get the following equations of motion for the gauge field:

$$\mathcal{D}_\nu \mathbf{F}^{\mu\nu} - \mathbf{F}^{\mu\nu}\partial_\nu \phi = 0. \tag{28}$$

Notice that the current $\mathbf{J}_\mu = \mathbf{F}^{\mu\nu}\partial_\nu\phi$ is automatically conserved. If any other matter fields are added to the dynamics, the total current would be conserved. It is interesting to note here that the gauge field equations can be rewritten as

$$\mathcal{D}_\nu(e^{-\phi}\mathbf{F}^{\mu\nu}) = 0. \tag{29}$$

This then provides us with a fully consistent, gauge invariant theory with torsion. Torsion has taken a simple form and the traditional decomposition of the torsion tensor into its irreducible components, shows that the totally antisymmetric part and the vectorial component are both absent in this theory. It is only the scalar component which survives the requirement of gauge invariance. Let us proceed to the gravitational action itself when torsion is non-vanishing.

3 Einstein-Cartan Actions

Einstein, in his own inimitable way, found the simplest possible action for the general theory of relativity. Luckily, as we have seen above, coupling of gauge fields to torsion while maintaining gauge invariance provides us with some limited dynamics for the torsion. Hence, when considering metric-torsion theories of gravity, one can still use Einstein's action containing torsion without worrying about its dynamics. Einstein's Lagrangian is normally obtained by requiring it to be linear in the curvature. For a symmetric connection, one of the Bianchi identities is

$$\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}(\{\}) = 0 \tag{30}$$

and this precludes a term such as $\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\sigma\rho}(\{\})$ entering the Lagrangian in general relativity. However this is no longer zero for non-symmetric connections.

We shall make use this while requiring the action to be linear in curvature, we can generalize Einstein's Lagrangian to

$$\mathcal{L} = R(\Gamma) + \beta\eta^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}(\Gamma). \tag{31}$$

This is a parity violating Lagrangian first introduced by R.Hojman et.al.[6]. It is interesting to note that a study by Nieh et.al. [7] on Bianchi and Gauss-Bonnet identities in Riemann-Cartan geometries provides *ex post facto* justification for Lagrangians of the type given in (31). In particular, the Gauss-Bonnet identity found in [7] is

$$\sqrt{-g}\epsilon^{\mu\nu\lambda\rho}(R_{\mu\nu\lambda\rho} + \frac{1}{2}T^{\alpha}{}_{\mu\nu}T_{\alpha\lambda}{}^{\nu}) = \partial_\mu(\sqrt{-g}\epsilon^{\mu\nu\lambda\rho}T_{\nu\lambda\rho}) \tag{32}$$

Putting everything together, a simple Lagrangian for a metric-torsion theory of gravity coupled to gauge fields can be written down as:

$$\mathcal{L} = \frac{1}{16\pi G}[R(\{\}) + \beta\eta^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}(\Gamma) - 6\partial_\mu\phi\partial^\mu\phi] - \frac{1}{4}Tr(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}). \tag{33}$$

where $\eta^{\mu\nu\rho\sigma} = \frac{\epsilon^{\mu\nu\rho\sigma}}{\sqrt{-g}}$ is a tensor constructed out of the Levi-Civita tensor density. Examining the dimensions of the torsion field, we see that as it stands, the torsion potential, ϕ is dimensionless. To give it the right dimensions and to be able to identify it as a regular scalar field, we can carry out a rescaling of ϕ as follows:

$$\tilde{\phi} = \sqrt{\frac{3}{G}}\phi \tag{34}$$

The Lagrangian reduces to

$$\mathcal{L} = \frac{1}{16\pi G}[R(\{\}) + \beta\eta^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}(\Gamma) - 2G\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}] - \frac{1}{4}Tr(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}). \tag{35}$$

Correspondingly, equation(12) is modified to

$$\mathbf{D}_\mu = \partial_\mu\mathbf{1} - ige^{-\sqrt{\frac{G}{3}}c^2\tilde{\phi}}\mathbf{A}_\mu, \tag{36}$$

Thus far we have seen that gauge invariance has played a central role in fixing the various interactions in our theory with torsion. In fact if we expand the Riemann-Cartan tensor into the Riemann tensor and the torsion, or if we use a simpler method and examine the Gauss-Bonnet identity of Nieh et.al. given in equation(32), it becomes clear that for the special form of torsion that we have found above (eqn.(11)), the parity violating term introduced above is actually zero.

Gauge invariance has reduced this Riemann-Cartan theory with parity violating terms to general relativity coupled to gauge fields along with a scalar field which has unusual interactions with the gauge fields.

In fact we shall now show that the Lagrangian given above can be recast in a form that provides the dynamics for Ashtekar gravity coupled to a dilaton and gauge fields.

Let us define a tetrad $e^{\mu}{}_I$ (the dual is then $e_{\mu}{}^I$) with determinant e and a connection $A_{\mu IJ}$. The curvature of this connection is denoted by $G_{\mu\nu IJ}$. In these expressions, greek indices represent the spacetime while capital Latin indices represent the "internal" or local Lorentz indices. Corresponding to the local Lorentz symmetry, we have a Minkowski metric η_{IJ} while the spacetime metric is given by

$$g_{\mu\nu} = \eta_{IJ} e_{\mu}{}^I e_{\nu}{}^J \tag{37}$$

The gravitational part of the action from (35) can be rewritten in terms of these tetrads as follows:

$$S_{grav} = \int \mathcal{L}_{grav} \sqrt{-g} d^4x = \frac{c^3}{16\pi G} \{ \int \frac{1}{2} \epsilon_{IJKL} e^I \wedge e^J \wedge G^{KL} + \beta \int e^I \wedge e^J \wedge G_{IJ} \} \tag{38}$$

where ϵ_{IJKL} is related with the Levi-Civita tensor density through $\epsilon_{\mu\nu\rho\sigma} = \epsilon_{IJKL} e^I{}_{\mu} e^J{}_{\nu} e^K{}_{\rho} e^L{}_{\sigma}$.

Instead of the coefficient of the parity violating term being β, let us write $-1/\gamma$ then the action reduces to:

$$S_{grav} = -\frac{1}{16\pi G\gamma} \int e^I \wedge e^J \wedge (G_{IJ} - \frac{\gamma}{2} \epsilon_{IJKL} G^{KL}) \tag{39}$$

This is the action suggested by Holst [8] for deriving the Barbero [9] Hamiltonian for Ashtekar gravity. While Barbero himself realized that the additional term he was considering was similar to having a connection with torsion, Holst treats the additional term as simply adding zero (from the Bianchi identity in GR).

The parameter γ is called the Immirzi parameter. It appears in the spectra of geometric operators in loop quantum gravity. In particular, the area spectrum in loop quantum gravity is found to be proportional to this parameter and spoils the excitement of having found (the long time suspected) discreteness at the Planck scale. Attempts have also been made to find its value by evaluating the area law for black holes in loop quantum gravity and comparing with the Beckenstein-Hawking law [10]

From our perspective, it is clear that the immirzi parameter is a measure of parity violation in the present theory.

Before we write down the complete action for the theory sketched above, we shall carry out one more transformation. This will allow us to provide another transformation for the torsion potential (ϕ) to be treated as a dilaton field.

Concentrating once again on the gauge field Lagrangian and in particular, the field strength tensor $\mathbf{F}_{\mu\nu}$, as expressed in equation(14), we let $g(x) \rightarrow g e^{-\phi(x)}$ and carry out a field redefinition:

$$\mathbf{A}_\mu \longrightarrow e^{-\phi}\mathbf{A}_\mu \tag{40}$$

allows us to write the modified $\mathbf{F}_{\mu\nu}$ in the following form:

$$\mathbf{F}_{\mu\nu} \longrightarrow e^{-\phi}\mathbf{F}_{\mu\nu} \tag{41}$$

Accordingly, this field redefinition rewrites the gauge field Lagrangian as:

$$\mathcal{L}_{gauge} = -\frac{1}{4}e^{-2\sqrt{\frac{G}{3}}c^2\tilde{\phi}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \tag{42}$$

where we have also substituted the scaled scalar field from equation(34).

Now we are ready to write down the complete action for our metric-torsion theory of gravity coupled to non-Abelian gauge fields in a completely consistent, gauge invariant manner.

$$S_{grav}{=}-\frac{1}{16\pi G\gamma}\{\int e^I\wedge e^J\wedge(G_{IJ}-\frac{\gamma}{2}\epsilon_{IJKL}G^{KL}) + \int 2G\gamma\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi}\} - \int\frac{1}{4}e^{-2\sqrt{\frac{G}{3}}\tilde{\phi}}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} \tag{43}$$

Can we justify the Lagrangian for the gravitational field including torsion that we are advocating?. We have seen above that it provides an action that seems to be all the rage specially amongst those who have strung themselves up and also those who have woven themselves into tangles!

Let us recall that invariant variational methods allowed Lovelock [11] to show that the Einstein-Hilbert Lagrangian was unique. For GR, since the metric is the only dynamical variable, Lovelock examined Lagrangians such as $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, g_{\mu\nu,\rho}, g_{\mu\nu,\rho\sigma})$ and required that they be linear in the second derivatives of the metric. The result is that the Einstein-Hilbert Lagrangian is unique in four dimensions. Dropping the requirement of being linear in the second derivatives of the metric Lovelock discovered new Lagrangians which have been much discussed recently as Lovelock gravity.

From the seventies onwards, the quest for a gravity theory with dynamical torsion has lead many to study Lagrangians of the form $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, g_{\mu\nu,\rho}, g_{\mu\nu,\rho\sigma}, T^{\mu}{}_{\nu\rho}, T^{\mu}{}_{\nu\rho,\sigma})$. Unfortunately, these studies gave a rather large number of possible terms and are not therefore a worthwhile prospect without additional physical inputs on torsion interactions.

It would be an important exercise to study Lagrangians of the form $\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \Gamma^{\mu}{}_{\nu\rho}, \Gamma^{\mu}{}_{\nu\rho,\sigma})$ while requiring linearity in $\Gamma^{\mu}{}_{\nu\rho,\sigma}$ with a non-symmetric connection $\Gamma^{\mu}{}_{\nu\rho}$. One could hope that equation(31) arises as the simplest (and only!) possible Lagrangian. The group of Lovelock and Hanno Rund have studied such problems for the case of symmetric connections [12]

4 Conclusions

In this paper, we have shown that a simple metric-torsion theory of gravity can be constructed while adhering to Einstein's principle of "being simple but not too simple". The problem of gauge invariant coupling of torsion to gauge fields has been shown to be not only possible but that it leads to a torsion potential which has different interpretations. One interpretation is that the potential, being a scalar field, plays the role of the dilaton. The other interpretation, albeit a little controversial, is that the gauge coupling parameter becomes spacetime dependent. The gauge field equations exhibit the requisite gauge invariance.

It is interesting to see that over the years, while torsion has gained importance as a possible contributor to a theory of the gravitational field, there is also a been an increase in ignorance of torsion theories of gravity. The following comment by a referee exemplifies this fact. The comment is, "It is argued that....when the field $\phi(x)$ becomes constant, torsion vanishes, and the subsequent evolution of the universe should be governed by general relativity. However, for a constant $\phi(x)$, not only torsion, *but the connection itself vanishes*, which leads to the vanishing of curvature as well....". On the same paper, a second referee, states "...In order to check the consistence(sic!) of the formalism, also the field equations should be discussed. For example, are the Yang-Mills equations gauge invariant when $g = g(x)$?..." .

Yet everyday, there are hundreds of papers published which discuss dilatons and Yang-Mills fields with similar interactions! One of the reasons for being rather explicit in this paper has been to show that metric-torsion theories of gravity are still very relevant today. Another has been to show that torsion takes a special form as a *consequence of requiring gauge invariance in the theory.* Having obtained the special form of the torsion, it can be reinterpreted as either a dilaton field or equally consistently, as transforming the gauge parameter into a function on spacetime.

Experimental observations seem to rule out the existence of torsion at the present epoch. There has been a suggestion from Hanson and Regge [13] that perhaps we live in a spacetime where torsion is expelled from most of the spacetime into torsion bubbles in a form analogous to the expulsion of the magnetic flux from a type II superconducting region into vortices.

Clearly, future directions for the study of the type of Lagrangians given above would be to examine such solutions to the field equations.

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