

On Torsion Fields in Higher Derivative Quantum Gravity

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ABSTRACT. We consider axial torsion fields which appear in higher derivative quantum gravity. It is shown, in general, that the torsion field possesses states with two spins, one and zero, with different masses. The first-order formulation of torsion fields is performed. Projection operators extracting pure spin and mass states are given. We obtain the Lagrangian in the framework of the first order formalism and energy-momentum tensor. The effective interaction of torsion fields with electromagnetic fields is discussed. The Hamiltonian form of the first order torsion field equation is given.

1 Introduction

It is obvious nowadays that the Standard Model (SM) should be extended by including gravity. However, classical General Relativity (GR) based on the Einstein-Hilbert action

$$S = -\frac{1}{\kappa^2} \int d^4x \sqrt{-g} R, \quad (1)$$

where R is a scalar curvature of space-time, is a nonrenormalizable theory. The requirement of renormalizability of the theory of gravity leads to the Lagrangian with quadratic terms in curvature (R^2 theory of gravity) [1]. Although the quantum theory of gravity becomes a renormalizable theory, the new difficulty appears - nonunitarity. This fact is connected with higher derivative equations of motion. Field equations in R^2 theory of gravity are forth-derivative equations. Now much attention is paid to higher derivative quantum field theories. For example, the higher

derivative scalar field theory was considered in [2], and higher derivative fermionic field equations were discussed in [3], [4]. The nonunitarity is the main obstacle to the consistent theory of quantum gravity. Possibly this difficulty will be solved by developing more a fundamental unified theory (the theory of everything, M-theory, the string theory, etc.) Originally, GR was applied for macroscopic objects, and the energy-momentum tensor of matter is the source of gravitational field characterized by the metric tensor $g_{\mu\nu}$ (the Riemann space-time). But it is possible to construct the microscopic theory of gravity where the spin of elementary particles is a source of another characteristic of space-time – torsion. Torsion fields are additional degrees of freedom that are independent of metric. This natural extension of gravity is gravity with torsion that is described by the Riemann-Cartan space-time U_4 [5]. The gravity of macroscopic objects can then be considered by averaging the microscopic equations over all volume. Therefore, we can naturally include torsion fields in the quantum theory of gravity. Torsion fields interact with the spin of particles and may contribute to physical observations.

It should be mentioned that the quantum gravity plays a very important role in the investigation of early universe cosmology. In cosmology, there are some puzzles such as dark energy, positive cosmological constant etc. Possibly, the torsion gravity may solve some problems in cosmology, and therefore, the generalization of metric gravity by including torsion is justified.

In this paper, we pay attention to torsion fields which appear in higher derivative quantum gravity. As the gravitation interaction (connected with the metric tensor $g_{\mu\nu}$) between elementary particles is extremely weak, one can consider the flat Minkowski space-time with torsion.

The paper is organized as follows. In section 2 we outline the geometry of space-time with torsion and write down the action of higher derivative quantum gravity with torsion. The free torsion field in flat space-time is considered in Sec. 3. We formulate the first order wave equation for the torsion neutral field in the 11-dimensional matrix form. It is shown that the torsion field which appears in higher derivative quantum gravity possesses two spins, one and zero, with different masses. When the mass of the spin-zero state approaches to infinity, one comes to the Proca equation. Projection operators extracting pure spin and mass states are given. We obtain the Lagrangian in the framework of

the first order formalism and the energy-momentum tensor. The possible effective interaction of the torsion field with external electromagnetic fields is discussed in Sec.4. The Hamiltonian form of an equation is given. We make a conclusion in Sec.5.

2 Higher derivative quantum gravity

2.1 Connection and torsion

Space-time represents the four-dimensional manifold X_4 with points x^μ , $\mu = 0, 1, 2, 3$, and x^0 is the time. The parallel transfer of a contravariant vector V^μ is given by [5]

$$dV^\mu = -\Gamma_{\alpha\beta}^\mu V^\alpha dx^\beta, \quad (2)$$

where $\Gamma_{\alpha\beta}^\mu$ is the affine connection with 64 components. The affine connection can be written as a sum of the symmetric part $\Gamma_{(\alpha\beta)}^\mu$ and the antisymmetric part $\Gamma_{[\alpha\beta]}^\mu$:

$$\Gamma_{\alpha\beta}^\mu = \Gamma_{(\alpha\beta)}^\mu + \Gamma_{[\alpha\beta]}^\mu, \quad \Gamma_{[\alpha\beta]}^\mu = \frac{1}{2} \left(\Gamma_{\alpha\beta}^\mu - \Gamma_{\beta\alpha}^\mu \right) \quad (3)$$

so that the antisymmetric part transforms as a tensor but the symmetric part not. The antisymmetric part of the connection $S_{\alpha\beta}^{\cdot\mu} \equiv \Gamma_{[\alpha\beta]}^\mu$ is called the Cartan torsion tensor and it possesses 24 components. In GR only the $\Gamma_{(\alpha\beta)}^\mu$ is explored with the additional requirements $\Gamma_{[\alpha\beta]}^\mu = 0$, $\nabla_\alpha g_{\mu\nu} = 0$, and the covariant derivative ∇_α is defined as

$$\nabla_\alpha V^\beta = \partial_\alpha V^\beta + \Gamma_{\alpha\mu}^\beta V^\mu. \quad (4)$$

The symmetric part of the connection, in GR, is named the Christoffel symbol and is expressed via the metric tensor:

$$\left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} = \frac{1}{2} g^{\mu\lambda} (\partial_\alpha g_{\lambda\beta} + \partial_\beta g_{\lambda\alpha} - \partial_\lambda g_{\alpha\beta}). \quad (5)$$

The square of the infinitesimal interval is given by $ds^2 = -g_{\mu\nu} dx^\mu dx^\nu$. In the Minkowskian flat space-time $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$.

Now, we imply that the affine connection $\Gamma_{\alpha\beta}^\mu$ is not symmetric (connection with torsion), but the metric tensor $g_{\mu\nu}$ is still constrained by the equation $\tilde{\nabla}_\alpha g_{\mu\nu} = 0$ (nonmetricity =0), where $\tilde{\nabla}$ being the covariant

derivative with torsion. The contorsion tensor $K_{\alpha\beta}^{\cdot\cdot\mu}$ is introduced by the relation [5]:

$$\Gamma_{\alpha\beta}^{\mu} = \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} - K_{\alpha\beta}^{\cdot\cdot\mu}, \quad K_{\alpha\beta}^{\cdot\cdot\mu} = -S_{\alpha\beta}^{\cdot\cdot\mu} + S_{\beta\cdot\alpha}^{\cdot\mu} - S_{\cdot\alpha\beta}^{\mu} = -K_{\alpha\cdot\beta}^{\cdot\mu}. \quad (6)$$

As usual, indices are raised and lowered with the help of the metric tensor. It should be mentioned that the contorsion tensor $K_{\alpha\beta}^{\cdot\cdot\mu}$ possesses 24 components and depends on metric and torsion tensors [5]. The reader can find the classification of the torsion tensor in [6], [7].

2.2 Action of renormalized quantum gravity with torsion

The action of renormalized quantum gravity with torsion is given by (see [8] and references therein)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\lambda} C_{\mu\nu\alpha\beta}^2 - \frac{\omega}{3\lambda} R^2 - \frac{1}{\kappa^2} R + \Lambda + \alpha_1 S_{\mu\nu}^2 + \alpha_2 (\nabla_{\mu} S^{\mu})^2 + \alpha_3 (S_{\mu} S^{\mu})^2 + \alpha_4 R^{\mu\nu} S_{\mu} S_{\nu}, \right. \\ \left. + \alpha_5 R S_{\mu} S^{\mu} - \frac{1}{\kappa^2} \xi S_{\mu} S^{\mu} \right), \quad (7)$$

where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor that is expressed through the curvature tensor, $R_{\mu\nu}$ is the Ricci tensor, Λ is the cosmological constant, $S_{\mu\nu} = \partial_{\mu} S_{\nu} - \partial_{\nu} S_{\mu}$, and the axial vector S_{μ} is the antisymmetric part of the torsion tensor $S_{\alpha\beta}^{\cdot\cdot\mu}$: $S^{\mu} = (1/3!) \epsilon^{\mu\nu\alpha\beta} S_{\nu\alpha\beta}$. Constants κ , ω , λ , ξ , α_i characterize the gravitational interaction.

We note that in Eq.(7) all terms appear as counterterms within the renormalization procedure. Therefore, one can not ignore some terms in Eq.(7) to have the renormalized quantum theory. But there are still two difficulties in the quantum theory based on the action (7). One problem is connected with the cosmological constant. To quantize the metric, we need to expand the flat background, but the flat metric $\eta_{\mu\nu}$ is not the solution of equations of motion. If one puts $\Lambda = 0$ in the bar action, a divergent counterterm appears from the loop calculations. The second problem is the unitarity problem. This is the main difficulty in the higher derivatives quantum theory. In such theories the ghosts appear with the negative norm that are considered as non-physical states. Nonunitarity of R^2 -gravity shows that the gravity theory with the action (7) may be considered as an approximation (or the effective model) of

the fundamental theory. The interaction of torsion with matter fields is discussed in [8], [9]. It should be mentioned that the axial vector S_μ interacts with all fermions, but other components of torsion fields $S_{\mu\nu\alpha}$ can interact with matter only if one introduces non-minimal couplings [9]. This interaction can not appear at the quantum level if we consider only the minimal interaction with torsion fields.

The generalization of the Einstein-Hilbert action (1) on the case of gravity with torsion leads to the Einstein-Cartan theory. In this case the torsion fields do not propagate because there are no kinetic terms in the Lagrangian. The effect of torsion fields is in contact spin-spin interactions. At low energies this effect is negligible and can be important for physics in the Early Universe. For example, the problem of singularity can be solved in the Einstein-Cartan theory. The main shortcoming of the Einstein-Cartan theory is its non-renormalizability.

In the renormalized quantum gravity kinetic terms of torsion fields appear due to loop calculations, and therefore they have to present in the bare Lagrangian. To investigate the mass spectrum and properties of propagating torsion fields, we consider here, for simplicity, the flat space-time ($g_{\mu\nu} = \eta_{\mu\nu}$) with torsion. So, we neglect the effect of the metric. For this purpose only quadratic terms in the torsion field S_μ in Eq.(7) are important. From Eq.(7), we arrive at the action for the free torsion field

$$S_T = \int d^4x \left(\alpha_1 S_{\mu\nu}^2 + \alpha_2 (\partial_\mu S^\mu)^2 - \frac{1}{\kappa^2} \xi S_\mu S^\mu \right). \quad (8)$$

We have ignored the self-interaction term $\alpha_3 (S_\mu S^\mu)^2$ which is in fourth order in fields. The particular case $\alpha_2 = 0$ was considered in [9], [10], and the case $\alpha_1 = 0$ was discussed in [11]. It should be mentioned that all terms in (8) have to be presented in order to preserve renormalizability of quantum gravity with torsion. The requirement $\alpha_2 = 0$ to have unitarity in the torsion sector is a constraint because even without a torsion the quantum gravity is non-unitary theory. Therefore, we investigate here the general case of non-zero coefficients in Eq.(8).

3 Propagating torsion fields

3.1 First order formulation

By renormalization of the field S_μ , we can make the standard kinetic term in action (8): $(-1/4)S_{\mu\nu}^2$. This means that without loss of generality, one may put $\alpha_1 = -1/4$. The corresponding Lagrangian was already

considered in [12] for complex fields describing charged particles. The torsion fields S_μ are real, and correspond to neutral fields, but it does not matter for the analysis of the mass spectrum. It was shown that fields described by the action (8) possess the state with spin-1 and the square mass $m^2 = 2\xi/\kappa^2$ and the state with spin-0 and the square mass $m_0^2 = -m^2/2\alpha_2$. Then the Lagrangian corresponding to the action (8) becomes

$$\mathcal{L}_T = -\frac{1}{4}S_{\mu\nu}^2 - \frac{m^2}{2m_0^2}(\partial_\mu S^\mu)^2 - \frac{1}{2}m^2 S_\mu S^\mu. \quad (9)$$

From the Lagrangian (9), one can obtain field equations as follows:

$$\partial^\mu \partial_\mu S_\nu + \left(\frac{m^2}{m_0^2} - 1 \right) \partial_\nu (\partial^\alpha S_\alpha) - m^2 S_\nu = 0. \quad (10)$$

At the case $m_0 \rightarrow \infty$, one arrives at the Proca equation describing pure spin-1 states. If $m = m_0$, we come to the case investigated in [13].

It is convenient formally to go to Euclidian space-time ($\eta_{\mu\nu} \rightarrow \delta_{\mu\nu}$) by introducing fourth components of vectors $S_4 = iS_0$. At this convention there is no difference between contra - and covariant components of vectors, and $\partial_\mu = (\partial_m, \partial_4)$ ($m=1,2,3$), $\partial_4 = \partial/\partial(it)$.

Eq.(10) may be represented in the first order formalism as [12]

$$\left(\alpha_\mu \partial_\mu + mP_1 + \frac{m_0^2}{m}P_0 \right) \Psi(x) = 0, \quad (11)$$

where

$$\Psi(x) = \{ \psi_A(x) \} = \begin{pmatrix} (1/m)S(x) \\ S_\mu(x) \\ (1/m)S_{\mu\nu}(x) \end{pmatrix} \quad (A = 0, \mu, [\mu\nu]), \quad (12)$$

$S(x) = -(m^2/m_0^2)\partial_\mu S_\mu(x)$, and matrices are given by

$$\alpha_\mu = \beta_\mu^{(1)} + \beta_\mu^{(0)}, \quad \beta_\nu^{(1)} = \varepsilon^{\mu, [\mu\nu]} + \varepsilon^{[\mu\nu], \mu}, \quad \beta_\nu^{(0)} = \varepsilon^{\nu, 0} + \varepsilon^{0, \nu}, \quad (13)$$

$$P_1 = \varepsilon^{\mu, \mu} + \frac{1}{2}\varepsilon^{[\mu\nu], [\mu\nu]}, \quad P_0 = \varepsilon^{0, 0},$$

expressed through the elements of the entire matrix algebra $\varepsilon^{A,B}$ with the properties: $(\varepsilon^{A,B})_{CD} = \delta_{AC}\delta_{BD}$, $\varepsilon^{A,B}\varepsilon^{C,D} = \delta_{BC}\varepsilon^{A,D}$. Matrices $\beta_\mu^{(1)}$ and $\beta_\mu^{(0)}$ are defined in 10- and 5-dimensional subspaces and obey the

Petiau-Duffin-Kemmer algebra. The projection matrices P_1, P_0 satisfy the relations:

$$P_1^2 = P_1, \quad P_0^2 = P_0, \quad P_1 P_0 = 0, \quad P_1 + P_0 = 1. \quad (14)$$

The relativistic first order 11×11 -matrix equation (11) describes torsion fields possessing two spins 0, 1 with masses, m_0 and m , respectively. The 11-dimensional matrices α_μ obey the algebra as follows:

$$\begin{aligned} \alpha_\mu \alpha_\nu \alpha_\alpha + \alpha_\alpha \alpha_\nu \alpha_\mu + \alpha_\mu \alpha_\alpha \alpha_\nu + \alpha_\nu \alpha_\alpha \alpha_\mu + \alpha_\nu \alpha_\mu \alpha_\alpha + \alpha_\alpha \alpha_\mu \alpha_\nu = \\ = 2(\delta_{\mu\nu} \alpha_\alpha + \delta_{\alpha\nu} \alpha_\mu + \delta_{\mu\alpha} \alpha_\nu). \end{aligned} \quad (15)$$

The projection operators¹

$$M_\varepsilon^{(1)} = \frac{i\widehat{p}^{(1)}(i\widehat{p}^{(1)} - \varepsilon m)}{2m^2}, \quad M_\varepsilon^{(0)} = \frac{i\widehat{p}^{(0)}(i\widehat{p}^{(0)} - \varepsilon m_0)}{2m_0^2}, \quad (16)$$

where $\widehat{p}^{(1)} = \beta_\mu^{(1)} p_\mu$, $\widehat{p}^{(0)} = \beta_\mu^{(0)} p_\mu$, $p_\mu = (\mathbf{p}, ip_0)$ is the momentum of a particle, extract states with the mass m and m_0 . Values $\varepsilon = 1$, $\varepsilon = -1$ correspond to the positive and negative energies of a particle, respectively. The projection matrices $M_\varepsilon^{(1,0)}$ obey the relationships

$$M_\varepsilon^2 = M_\varepsilon, \quad M_\varepsilon^{(1)} M_\varepsilon^{(0)} = M_\varepsilon^{(0)} M_\varepsilon^{(1)} = 0. \quad (17)$$

3.2 Spin operators

With the help of generators of the Lorentz group in the 11-dimensional representation space of the wave function

$$J_{\mu\nu} = \beta_\mu^{(1)} \beta_\nu^{(1)} - \beta_\nu^{(1)} \beta_\mu^{(1)}, \quad (18)$$

we obtain the squared spin operator (the factor 1/2 was missed in [12])

$$\sigma^2 = \left(\frac{1}{2m} \varepsilon_{\mu\nu\alpha\beta} p_\nu J_{\alpha\beta} \right)^2 = \frac{1}{m^2} \left(\frac{1}{2} J_{\mu\nu}^2 p^2 - J_{\mu\sigma} J_{\nu\sigma} p_\mu p_\nu \right), \quad (19)$$

obeying the “minimal” equation

$$\sigma^2 (\sigma^2 - 2) = 0. \quad (20)$$

¹In [12] more complicated operator extracting states with spin-0 was constructed.

The projection operators extracting states with spin-1 and spin-0 are given by [12]

$$S_{(0)}^2 = 1 - \frac{\sigma^2}{2}, \quad S_{(1)}^2 = \frac{\sigma^2}{2} \quad (21)$$

which obey equations $S_{(0)}^2 S_{(1)}^2 = 0$, $(S_{(0)}^2)^2 = S_{(0)}^2$, $(S_{(1)}^2)^2 = S_{(1)}^2$, $S_{(0)}^2 + S_{(1)}^2 = 1$. The operators of the spin projection on the direction of the momentum \mathbf{p} are

$$\sigma_p = -\frac{i}{2|\mathbf{p}|} \epsilon_{abc} p_a J_{bc} = -\frac{i}{|\mathbf{p}|} \epsilon_{abc} p_a \beta_b^{(1)} \beta_c^{(1)}, \quad (22)$$

and satisfy the matrix equation:

$$\sigma_p (\sigma_p - 1) (\sigma_p + 1) = 0. \quad (23)$$

Using the standard procedure, we find the projection operators corresponding to spin projection one and zero [12]:

$$\widehat{S}_{(\pm 1)} = \frac{1}{2} \sigma_p (\sigma_p \pm 1), \quad \widehat{S}_{(0)} = 1 - \sigma_p^2. \quad (24)$$

The projection matrices extracting states with pure spin, spin projection and energy are

$$\begin{aligned} \Delta_{\varepsilon, \pm 1} &= M_\varepsilon^{(1)} S_{(1)}^2 \widehat{S}_{(\pm 1)} = \frac{i \widehat{p}^{(1)} (i \widehat{p}^{(1)} - \varepsilon m)}{2m^2} \frac{1}{2} \sigma_p (\sigma_p \pm 1), \\ \Delta_\varepsilon^{(1)} &= M_\varepsilon^{(1)} S_{(1)}^2 \widehat{S}_{(0)} = \frac{i \widehat{p}^{(1)} (i \widehat{p}^{(1)} - \varepsilon m)}{2m^2} \frac{\sigma^2}{2} (1 - \sigma_p^2), \\ \Delta_\varepsilon^{(0)} &= M_\varepsilon^{(0)} S_{(0)}^2 \widehat{S}_{(0)} = \frac{i \widehat{p}^{(0)} (i \widehat{p}^{(0)} - \varepsilon m_0)}{2m_0^2} \left(1 - \frac{\sigma^2}{2}\right) (1 - \sigma_p^2). \end{aligned} \quad (25)$$

Operators (25) also represent the density matrices for pure spin states. More information about spin operators the reader may find in [12].

3.3 Energy-momentum tensor

The Hermitianizing matrix is given by [12]

$$\eta = -\varepsilon^{0,0} + \varepsilon^{m,m} - \varepsilon^{4,4} + \varepsilon^{[m4],[m4]} - \frac{1}{2} \varepsilon^{[mn],[mn]}, \quad (26)$$

and obey equations: $\eta\alpha_i = -\alpha_i^+\eta^+$, $\eta\alpha_4 = \alpha_4^+\eta^+$ ($i = 1, 2, 3$). This matrix allows us to obtain the relativistically invariant bilinear form $\bar{\Psi}\Psi = \Psi^+\eta\Psi$. Applying the Hermitian conjugation to Eq.(11), and using equalities $[P_1, \eta] = [P_0, \eta] = 0$, one obtains the equation as follows:

$$\bar{\Psi}(x) \left(\alpha_\mu \overleftarrow{\partial}_\mu - mP_1 - \frac{m_0^2}{m} P_0 \right) = 0. \quad (27)$$

Now the Lagrangian, in the framework of the first order formalism, reads

$$\mathcal{L} = -\bar{\Psi}(x) \left(\alpha_\mu \partial_\mu + mP_1 + \frac{m_0^2}{m} P_0 \right) \Psi(x). \quad (28)$$

It should be noted that the functions $\bar{\Psi}(x)$ and $\Psi(x)$ are dependent because the fields (\mathbf{S}, S_0) are real. Therefore, one has to make the variation of the action corresponding to the Lagrangian (28) on $\bar{\Psi}(x)$ or $\Psi(x)$. With the aid of the general expression for the canonical energy-momentum tensor

$$T_{\mu\nu}^c = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Psi(x))} \partial_\nu \Psi(x) - \delta_{\mu\nu} \mathcal{L}, \quad (29)$$

one obtains

$$T_{\mu\nu}^c = -\bar{\Psi}(x) \alpha_\mu \partial_\nu \Psi(x). \quad (30)$$

We took into consideration that $\mathcal{L} = 0$ for fields obeying equations of motion (11). With the help of equations (11), (27), it easy to verify that the canonical energy-momentum tensor is conserved: $\partial_\mu T_{\mu\nu}^c = 0$. Using Eq.(12), (26), one finds $\bar{\Psi}(x) = (-(1/m)S(x), S_\mu(x), -(1/m)S_{\mu\nu}(x))$, and taking into account Eq.(13), we obtain

$$T_{\mu\nu}^c = \frac{1}{m} (S\partial_\nu S_\mu - S_\mu \partial_\nu S + S_\rho \partial_\nu S_{\mu\rho} - S_{\mu\rho} \partial_\nu S_\rho). \quad (31)$$

The canonical energy-momentum tensor is not symmetric but can be symmetrized by the standard procedure. For example, the symmetric energy-momentum tensor may be obtained by varying the action on the metric tensor. See also [14] for the comparison. We note that the identity $j_\mu = \bar{\Psi}(x) \alpha_\mu \Psi(x) = 0$ is valid because the torsion fields (\mathbf{S}, S_0) describe neutral fields and the electric current is equal to zero, $j_\mu = 0$.

4 Electromagnetic interactions of torsion fields

4.1 Non-minimal interactions

Torsion fields can interact with electromagnetic fields only non-minimally because they do not carry the electric charge. In [8] the interaction of the kind $a(\partial_\mu S_\nu)\tilde{\mathcal{F}}^{\mu\nu}$ in the Lagrangian was suggested, where $\tilde{\mathcal{F}}^{\mu\nu}$ is the dual tensor of electromagnetic fields. However, this interaction results in the appearance of the additional term $a\partial_\mu\tilde{\mathcal{F}}_{\mu\nu}$ in Eq.(10) which is non-zero only in the presence of magnetic monopoles.

Here, we consider non-trivial electromagnetic interaction of torsion fields which can be obtained by putting $e = 0$ in Eq.(55) of [12]. As a result, we obtain the matrix equation for interacting torsion fields

$$\left[\alpha_\mu \partial_\mu + \frac{1}{2} \left(\sigma_0 P_0 + \sigma_1 \bar{P} + \sigma_2 \bar{\bar{P}} \right) \alpha_{\mu\nu} \mathcal{F}_{\mu\nu} + m P_1 + \frac{m_0^2}{m} P_0 \right] \Psi(x) = 0, \quad (32)$$

where the projection operators \bar{P} , $\bar{\bar{P}}$, and $\alpha_{\mu\nu}$ being

$$\bar{P} = \varepsilon^{\mu,\mu}, \quad \bar{\bar{P}} = \frac{1}{2} \varepsilon^{[\mu\nu],[\mu\nu]}, \quad \alpha_{\mu\nu} = \alpha_\mu \alpha_\nu - \alpha_\nu \alpha_\mu. \quad (33)$$

The matrix equation (32) is equivalent to the system of tensor equations

$$\begin{aligned} \partial_\mu S_\mu + \frac{m_0^2}{m^2} S + \frac{\sigma_0}{m} \mathcal{F}_{\mu\nu} S_{\mu\nu} &= 0, \\ S_{\mu\nu} - \partial_\mu S_\nu + \partial_\nu S_\mu + \frac{\sigma_2}{m} (\mathcal{F}_{\nu\rho} S_{\mu\rho} - \mathcal{F}_{\mu\rho} S_{\nu\rho}) &= 0, \\ \partial_\nu S_{\mu\nu} + \partial_\mu S + m^2 S_\mu + 2\sigma_1 m \mathcal{F}_{\mu\nu} S_\nu &= 0. \end{aligned} \quad (34)$$

The interaction introduced may be considered as an effective interaction which follows from the fundamental theory. The constants σ_0 , σ_1 , σ_2 are connected with the internal characteristics of torsion particles [12]. It means that if the interaction considered exists, torsion particles may be treated as composite particles. Possible torsion interaction with electromagnetic fields was discussed in [15].

4.2 Hamiltonian form

Let us consider the Hamiltonian form of Eq.(32) for torsion particles in the external electromagnetic field. Introducing dynamical and auxiliary

components of the wave function $\alpha_4^2\Psi(x) = \varphi(x)$, $(1 - \alpha_4^2)\Psi(x) = \chi(x)$ [12], we arrive, after exclusion $\chi(x)$, at the Hamiltonian form:

$$\begin{aligned}
 i\partial_t\varphi(x) = & \alpha_4 \left\{ \frac{m_0^2 - m^2}{m} P_0 \right. \\
 & + \left[(1 - \gamma)(m + \alpha_a \partial_a) + \frac{\sigma_0}{2} P_0 \alpha_{\mu\nu} \mathcal{F}_{\mu\nu} \right. \\
 & \left. \left. + \frac{\sigma_1}{2} \bar{P} \alpha_{\mu\nu} \mathcal{F}_{\mu\nu} + \frac{\sigma_2}{2} \bar{\bar{P}} \alpha_{\mu\nu} \mathcal{F}_{\mu\nu} \right] \left(1 - \frac{1}{m} \Pi \alpha_a \partial_a \right) \right\} \varphi(x),
 \end{aligned} \tag{35}$$

where $\Pi = 1 - \alpha_4^2$, $\gamma = (\sigma_2/2m)\bar{\bar{P}}\alpha_{\mu\nu}\mathcal{F}_{\mu\nu}$. We use here the smallness of σ_2 , σ_0 (see [12]).

The wave function $\varphi(x)$ possesses eight non-zero components corresponding to states with spin-1 and spin-0 with two values of the energy. It should be mentioned that states with spin-1 and spin-0 can not be treated as separate particles, but they are states of one field with multi-spin 1,0 [16].

5 Conclusion

The theory of torsion fields having two spin states, one and zero, may be treated as the effective theory. The formulation of wave equations in the first order formalism allows us to separate states with spin-1 and spin-0 in the covariant manner with help of projection operators. In the framework of the first order formalism the Lagrangian and the energy-momentum tensor are found by the standard procedure. The possible effective interaction of torsion fields with external electromagnetic fields is discussed and the Hamiltonian form of an equation is given.

The main difficulty, however, is that the spin-0 state breaks unitarity. This is because the spin-0 state gives negative contribution to the Hamiltonian [13]. But even without torsion, the higher derivative quantum gravity suffers this shortcoming. We can consider the spin-0 state as a ghost and to remove it, one has to make the limit $m_0 \rightarrow \infty$.

References

- [1] K. S. Stelle, Phys. Rev. **D16**, 953 (1977).
- [2] S. I. Kruglov, Ann. Fond. Broglie **31**, 343, (2006).
- [3] V. V. Dvoeglazov, Int. J. Theor. Phys. **37**, 1009 (1998); Ann. Fond. Broglie **25**, 81 (2000); Hadronic J. **26**, 299 (2003).

- [4] S. I. Kruglov, Ann. Fond. Broglie **29**, 1005 (2004), Erratum-ibid **31**, 489, (2006); Electron. J. Theor. Phys. **10**, 11, (2006); Hadronic J. **29**, 637, (2006); Can. J. Phys. **85**, 887, (2007).
- [5] F. W. Hehl, P. von der Heyde, and G. D. Kerlick, Rev. Mod. Phys. **48**, 393 (1976).
- [6] F. W. Hehl, J. D. McCrea, E. W. Mielke, and Y. Ne'eman, Phys. Rep. **258**, 1 (1995).
- [7] S. Capozziello, G. Lambiase, and C. Stornaiolo, Ann. Phys. (Leipzig) **10**, 713 (2001).
- [8] I. L. Buchbinder, S. D. Odintsov, I. L. Shapiro, Effective action in quantum gravity (IOP Publishing Ltd., Btistol, UK, 1992).
- [9] I. L. Shapiro, Phys. Rep. **357**, 113 (2002).
- [10] A. S. Belyaev, I. L. Shapiro, and M. A. B. do Vale, Phys. Rev. **D75**, 034014 (2007).
- [11] S. M. Carroll, G. B. Field, Phys. Rev. **D50**, 3867 (1994).
- [12] S. I. Kruglov, Int. J. Mod. Phys. **A16**, 4925 (2001).
- [13] S. I. Kruglov, Hadronic J. **24**, 167 (2001).
- [14] V. V. Dvoeglazov, Int. J. Mod. Phys. **B20**, 1317 (2006).
- [15] Y. Itin and F. W. Hehl, Phys. Rev. **D68**, 127701 (2003).
- [16] S. I. Kruglov, Symmetry and Electromagnetic Interaction of Fields with Multi-Spin (Nova Science Publishers, Huntington, New York, 2001).

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