Metastable plasma structures in knotted magnetic fields

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ABSTRACT. Recent theoretical and experimental results have proved the existence of certain nontrivial self-organized configurations of electric charges having filamentary or layered structures. The role played by the self-generated or externally imposed magnetic field in these formations is being much discussed. It is argued that the topological properties of the field may cause some metastable plasma configurations, such as current loops with minimum dissipation, that can be persistent under certain conditions if they are coupled to a knotted magnetic field. For such a field, a hypothesis about some sort of alpha-effect, similar to the same one invoked for astrophysical plasmas, is proposed. In some scenarios, very low dissipative effects can occur in the presence of force-free magnetic fields. The magnetic field diffusion can be drastically inhibited due to the reduction on the magnetic diffusivity by the alpha-effect. This also renders a slow decay of both magnetic energy and the magnetic helicity. These quantities provide some constraints to mathematically investigate the global stability of the system configuration. The results can be important to explain some phenomena in both mesoscopic and macroscopic scales, such as plasma filaments and ball lightning. The latter could be modelized as a set of linked current loops of knotted solitons coupled to the magnetic field.

1 Introduction

In recent times, the study of certain surprisingly stable space charge structures in natural and laboratory plasmas has roused great interest. Among these structures special attention has been devoted to several formation in plasmas resembling layered or filamentary forms, as the so-called double layers in cold laboratory plasmas or solar flares in the astrophysical context. Most of these plasma structures arise plasmas under very controlled conditions in a laboratory, but, also, in Nature, these space charge organizations seem to appear in several phenomena such as in Earth's aurora or, perhaps, in ball-lightnings.

A ball-lightning (BL) [1,2,3] consists of flaming balls or fireballs which appear unexpectedly sometimes near the ground, following the discharge of a lightning flash, having a diameter between 10 and 50 cm and an average lifetime between 2 and 5 s. For the moment, there is no accepted scientific explanation accounting for all the main properties of this natural phenomenon and other laboratory ones, but it is sensible to look for plasma models due to the electrical nature of many atmospheric phenomena. The problem of plasma confinement is intrinsically related to the structure of the applied and self-generated electromagnetic fields, implying that the fields lines are also confined, so that, the field topology is essential.

The conducting plasma flow coupled to the field can bend and stretch magnetic lines amplifying the field and providing stability and a high confinement time. However, it is sometimes argued [1] that a plasma model of a plasma structure in atmospheric conditions is unreliable if there are no external sources. Besides this, some researchers interpret the magnetic virial theorem (VT) as a serious obstacle to an electromagnetic model since, according to them, this theorem shows that the magnetic pressure would be so strong that nothing could stand to avert an almost instantaneous explosion [1]. Still, as in dusty plasmas, the existence of microstructures of localized electromagnetic fields may lead to states out of the scope of the classical virial theorem.

In this communication I try to vindicate the role of plasma physics and the topology of confining magnetic field in order to explain the formation of nontrivial space charge structures, which can merge in several natural or experimental phenomena. Thus, the main aim of this communication is to justify the study of the interplay of plasma physics and several complex magnetic field topologies. To do this, I t will bring together some recent theoretical results [4,5,6] which seem to be unrelated but they both may be invoked as a whole to justify the existence of certain stable plasma formations coupled to the topology of the magnetic field. At the same time, some experimental results are brought up to justify the theoretical approaches. Finally, a hypotheses about the stability of a plasma structure in a knotted force-free magnetic field is made, arguing that in such a field a plasma filament could behave as showing a minimum energy dissipation rate and a large effective conductivity, similar to a superconductor material. These arguments are put together to support the topological BL model by Rañada, *et al* [2,3] where a BL is explained as a structure of linked currents and magnetic lines. It is worthy to say here that this model, as well as the arguments presented here can be applied in several plasma scenarios, since BL is dealt here as a practical reference to many other similar phenomena. Therefore, a discussion about the validity of the magnetic virial theorem is presented as a starting point.

2 The magnetic the virial theorem in plasmas.

The virial theorem, formulated by Chandrasekhar and Fermi in 1953 for astrophysical plasmas, was extended by Shafranov [1,5] in dealing with the question of whether a plasmoid can exist in equilibrium self-confined by its magnetic field. Basically, it apparently states that the confinement of a plasma in vacuum is not possible by only self-fields, unless a constant pressure exists outside the system. In this case, the theorem establishes a severe restriction to the maximum amount of stored energy. A very simple formulation of the scalar virial theorem is obtained from the Magnetohydrodynamics (MHD) equation for momentum conservation in a quasi-neutral single fluid plasma (Gaussian units are used) with scalar isotropic pressure p, mass and charge fluid densitiesρ and $\rho_q$and electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\rho\mathbf{v} = \rho_q\mathbf{E} - \nabla p + \mathbf{j}\times\mathbf{B}/c + \rho\mathbf{g}_a. \tag{1}$$

Here $\mathbf{g}_a$ is any additional force per unit of mass, acting on the plasma moving with fluid velocity $\mathbf{v}(\mathbf{r},t)$ in the system frame. By multiplying both sides of (1) by $\mathbf{r}$ and integrating over the plasma volume V bounded by a surface S, using the MHD continuity equation $\partial\rho/\partial t \quad +\nabla\cdot(\rho\mathbf{v}) = 0$ if no fluid flow leaves the volume $\mathbf{v}\cdot d\mathbf{S} = 0$, we have

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2T + 2U + U_B + M \tag{2}$$

for $\mathbf{g}_a \;=\; 0$. This is a simple mathematical expression of the time-depending scalar virial theorem. Here

$$M = (1/4\pi)\int_{\int} S/2)(\mathbf{r}\cdot d\mathbf{S})] - \int_{\int} S \tag{3}$$

and

$$I = \int_{\int} V,\ T = \frac{1}{2}\int_{\int} V,\ \ U = \frac{3}{2}\int_{\int} V = \frac{1}{8\pi}\int_{\int} V \tag{4}$$

where I is called the moment of inertia and T, U and U_B are the kinetic, internal and magnetic energies. If the pressure p is zero outside V, by allowing the integration surface to go to infinity (assuming vanishing integrands) the term U_B would produce a very rapid increase of I, *i.e.* an explosion, since nothing would balance the large magnetic pressure $B^2/2\mu_0$. Furthermore, in a steady plasma state in vacuum, the external pressure is $p_e = 0$, with $\mathbf{g}_a = 0$, and the former reads $2T + 2U + U_B = 0$, which cannot hold, proving that a plasma cannot be confined by its own magnetic field in absence of external super-conducting walls. If the pressure at the surface S is $p_e \neq 0$ one has

$$2T + U_B = 3(p_e - \langle p \rangle)V, \tag{5}$$

where $\langle p \rangle$ the average pressure in V, lower than p_e. Thus, the maximum energy storage cannot exceed $3p_eV$.

It is usually argued that these conclusions remain valid even for a more sophisticated formulation of the virial theorem. However, the previous arguments could not have been applied if the system were bounded by a surface where surface effects may be important for certain kind of intense electromagnetic fields and fluid flows, placed in localized region, where the charge neutrality can be locally violated ($\mathbf{g}_a \neq 0$, $\rho_q \neq 0$).

Otherwise, the influence of the field topology on the certain spatial charge distributions coupled to the field structures can make the plasma state to be highly anisotropic and the simple classical static virial theorem becomes badly posed. For a deeper inspection of a more realistic VT, (1) is rewritten in a more general tensorial form as

$$\frac{\partial}{\partial t}(\rho \mathbf{v} + \frac{\mathbf{E} \times \mathbf{B}}{4\pi c}) + \nabla \cdot \mathrm{Q} = \rho \mathbf{g}_a \tag{6}$$

where $\mathrm{Q} \quad = \sum_{\sum} k$ is the sum of several tensors and $\mathbf{g}_a$ gives the contribution of all forces on the system per unit of mass. For the net tensor Q, the following contributions are usual in a realistic plasma scenario

$$\begin{aligned} &\mathrm{Q}_1 = \rho \mathbf{v}\mathbf{v} && \mathrm{Q}_2 = p_\perp I + \tfrac{P_{||} - P_\perp}{B^2}\mathbf{B}\mathbf{B} \\ &\mathrm{Q}_3 = \tfrac{1}{4\pi}(\tfrac{E^2 + B^2}{2} I - \mathbf{E}\mathbf{E} - \mathbf{B}\mathbf{B}) && \mathrm{Q}_4 = \tfrac{2}{3}\mu \nabla \cdot \mathbf{v} I - \mu \nabla \mathbf{v} \end{aligned} \tag{7}$$

Apart from the common dyadic $\mathbf{vv}$, a divergence-free electric can be included although plasma neutrality is held in $\mathrm{Q}_3? = \mathrm{Q}_{em}$. The Maxwell stress electromagnetic tensor satisfies

$$\frac{\partial}{\partial t}\frac{\mathbf{E}\times\mathbf{B}}{4\pi c} + \nabla\cdot \mathrm{Q}_{em} = -\frac{\mathbf{j}\times\mathbf{B}}{c} - \rho_q \mathbf{E} \tag{8}$$

by means of the Maxwell equations, giving the force density $\mathbf{j}\times\mathbf{B}/c+\rho_q\mathbf{E}$ acting on the plasma. The anisotropic effects (perpendicular and parallel pressures are $P_\perp$ and $P_{||}$ with $3p = 2P_\perp + P_{||}$) and the viscosity μ are also included in the stress tensors Q_2 and Q_4 respectively. On the other hand, some possible terms forcing the system are given by

$$\begin{aligned} &\mathbf{g}_1 = -\nabla\Phi(\mathbf{r})\mathbf{g}_2 = -[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}) + 2\mathbf{\Omega}\times\mathbf{v}] \\ &\mathbf{g}_3 = -\nu_n\mathbf{v} \qquad \mathbf{g}_4 = -\tfrac{\nu}{c}\tfrac{\nabla P_e\times\mathbf{B}}{enB^2} \end{aligned} \tag{9}$$

where Φ is a potential, usually taken as zero, P_e is the electron pressure gradient, providing a diamagnetic term that term can be large in localized conducting currents with electron number density n. The plasma rotation is described by the non sequential angular velocity $\mathbf{\Omega}$ (here $\mathbf{\Omega} = 0$). The plasma collision with neutral surrounding gas is modelized by the term with effective frequency $\nu = m_e\nu_{en}/m_i$. It must be pointed out that these terms can modify the mass flow, as well as the magnetic helicity H (term coined by Moffat [7])

$$H = \int \mathbf{A}\cdot\mathbf{B}\, dV = \int \mathbf{A}\cdot\nabla\times\mathbf{A} dV \tag{10}$$

leading to a change in the topology of the fields and flows, specially if the fluid resembles a set of quasi filamentary structures, even though owning sufficiently smoothed boundaries. We may assume diffuse boundaries in the fluid. Therefore, we suppose a sufficient smooth plasma flow in a simply connected domain of volume V, where the identity $\nabla\cdot\{\mathbf{r}\cdot\mathrm{Q}\} = \mathrm{trace}(\mathrm{Q}) + \mathbf{r}\cdot(\nabla\cdot\mathrm{Q})$ holds for a symmetric tensor. Assuming a surface S convected by the flow, surrounding the volume V, the moment of inertia now obeys the equation of motion

$$\frac{1}{2}\frac{d^2I}{dt^2} + \frac{\nu_n}{2}\frac{dI}{dt} + h(I,t) = 2T + U + U_{em} + M_S + M_g \tag{11}$$

where we have defined the time-depending term h as

$$h(I,t) = \frac{d}{dt}\int \frac{\mathbf{N}\cdot\mathbf{r}}{4\pi} dV, \tag{12}$$

and

$$M_S = -\int_S \left[P_\perp \mathbf{r}\cdot d\mathbf{S} + \frac{P_{||}-P_\perp}{B^2}(\mathbf{r}\cdot\mathbf{B})(\mathbf{B}\cdot d\mathbf{S})\right] +$$

$$\mu\int_S \mathbf{v}\cdot d\mathbf{S} - \mu\int_S \left[\tfrac{2}{3}(\nabla\cdot\mathbf{v})d\mathbf{S}\right] - (\mathbf{r}\cdot\nabla)\mathbf{v}\cdot d\mathbf{S} - \tag{13}$$

$$\frac{1}{4\pi}\int_S \left[(\mathbf{r}\cdot\mathbf{E})(\mathbf{E}\cdot d\mathbf{S}) + (\mathbf{r}\cdot\mathbf{B})(\mathbf{B}\cdot d\mathbf{S})\right] - \frac{1}{8\pi}\int_S (E^2+B^2)\mathbf{r}\cdot d\mathbf{S}$$

and the kinetic moment per unit of volume $\mathbf{l} = \rho_\mathbf{r}\times\mathbf{v}.$, $M_g = \int \rho_\mathbf{r}\cdot(\mathbf{g}_1 + \mathbf{g}_4)dV$ and $\mathbf{N} = 4\pi\mathbf{E}\times\mathbf{B}/c$ is the Poynting vector, U and T have the same sense given above, and U_{em} is the electromagnetic energy due to both fields B and E. Here, the term M_s encloses all the surface contributions given in (**??**).

It is easy to check out that the previous arguments about the positivity of the second time derivative of I may fail, in view of the undefined sign of the terms involved in M_g, even for $M_s = 0$. Observe that in M_g both Φ and the diamagnetic term could contribute locally as negative (non-explosive or confining) terms, playing a similar role as the self-gravitational potential in astrophysical plasmas. On the other hand one has to devote attention on the surface effects described by M_S that can also provide negative contributions to the net energy given by the first four (explosive) terms in (**??**). As a partial conclusion, it can be stated that a deeper inspection of the terms involved in the dynamics of Ishould be required before extracting some of the (assumed) general results of the classic VT.

Besides the previous simple inspections, another important case in the preceding discussion is missed. If the magnetic field satisfies $\mathbf{j} \times \mathbf{B} = 0$, the so-called "the force-free condition", neglecting $\mathbf{E}$ and $\mathbf{g}_a$ in an isotropic plasma, there is no pinch effect and all the terms involving $\mathbf{B}$ in (2) vanish. So that, (5) takes the form $2T = 3(p_e - \langle p\rangle)V$, whatever $\mathbf{B}$ is. Observe that this relation cannot be obtained as a particular case of (5), which is frequently assumed to be of general validity. In the limit case of a stationary fluid, such a plasma could exist with no depression, with the stability provided by a force-free (or Beltrami) field with the cohesive confining magnetic forces. Furthermore, if both $\mathbf{E}$ and $\mathbf{B}$ are divergence-free vectors parallel to the current $\mathbf{j}$, all the terms involving these fields disappear form the motion equation of I.

Note that the previous argument means that a huge amount of energy can be stored in the volume V, although the fields are in fact "hidden", but it drives the plasma fluid probably coupling the flow to their geometries. If the force-free condition $\mathbf{B} \times \nabla \times \mathbf{B} = 0$ is satisfied, the magnetic field only occurs in the contribution of the anisotropic stress tensor for $P_{\perp} \neq P_{II}$, whose sign, in general, is not definite. The fields dynamics are also controlled by the electron pressure gradient, ∇P_e, which can be important, since it can give a positive contribution to the magnetic helicity time rate of change, acting as a helicity injection term, see for instance [8]. The variation of the helicity is directly related to the topology of the field since it is a measure of the twisting, linking and knotting of field lines around themselves.

3 Theoretical arguments about the existence of stable localized plasma structures

As stated in the previous section, the possibility of some special plasma configurations in a surrounding neutral or plasmoidal medium cannot be rejected by means of a simple formulation of the virial theorem. Particular attention has to be focused on those plasma structures for which the magnetic field is a force-free one. It is well-known in plasma theory that such a field is the final state of a Taylor relaxation process [1] when a plasma evolves under the constrain of constant magnetic helicity. This minimum energy relaxed state is almost achieved in many experimental driven plasmas under helicity injection. However, the force-free can be reached in a plasma, not only by helicity conservation but also through several mechanisms that arise in a more natural manner due to some realistic conditions such as heat and mass transport, and beyond the restrictive and very theoretical hypotheses of the helicity conservation.

In this section, I show that these force-free fields may appear in many situations, conferring stability to the system, at the same time the global geometry of the structure is dominated by the field lines topology. Thus, the existence of long living localized plasma states, as filamentary currents, plasma ribbons or layers characterized by a self-confining Beltrami filed are excellent candidates to explain the stability of some global plasma configurations. It is well-known that these configurations are very usual due to the ability of a plasma to self-organize into these kinds of structures.

A very important and conclusive theoretical result to support the lack of generality of the classical virial conclusion can be found in the interesting and celebrated papers by Faddeev *et al* in [4,5]. A general plasma relaxation processes cannot be understood without the coupling between currents and fields. It is well-known that magnetic field reconnection produces charged particle acceleration via heat and momentum transport processes, being the final state of the system a consequence of a very strong interrelation between currents and field dymanincs. Faddeev and Niemi proposed a generalized action term in describing the bulk of a plasma evolution in order to explicitly deals with the hidden nonlinearities of the mean field classical MHD theory. Using this formulation they stated that in a neutral plasma may appear topologically stable solitons in the form of ***knotted and linked flux*** of helical magnetic fields with a ***nontrivial Hopf invariants*** [7]. For these authors, simple arguments based on the virial theorem can not preclude the existence of nontrivial, non-dissipative equilibrium configurations, which are "topologically stable solitons that describe knotted and linked flux tubes of helical magnetic fields" with "self-confining plasma filaments". These nontrivial knotted solitons are in fact plasma self-confined filaments and they are, in the authors words, very good candidates to describe filamental and toroidal structures in plasma, as coronal loops. The same authors pointed out that, besides astrophysical applications, the nontrivial electromagnetic interactions described by the action term become relevant in a large number of plasma scenarios and the "might include even an explanation to highly elusive ball lightning". This assertion is clearly related to the ball lightning model previously proposed by Rañada as an electromagnetic knot structure.

Although the Faddeev-Niemi arguments say nothing about current field alignment, giving rise a force-free filamentary plasma, there are several arguments that give a very strong support to the fact that the occurrence of this kind of fields in plasmas can appear in Nature under several conditions as a result of self-organization processes, beyond the so-called Taylor relaxation. In this way, special mention has to be deserved, from my point of view, to the theoretical works of Tang and Boozer in [6] addressing the natural occurrence of small scale structures with extreme anisotropy. From another very different theoretical perspective that the Faddeev-Niemi approach, Tang and Boozer deserve attention to explain small scale anisotropic structures in the evolution of magnetic fields in conducting media. The theoretical study is here fulfilled by paying attention to the chaotic dynamics of the particles in the fluid system. As pointed out by Moffat, one of the more important challenges to be analyzed in this century in the frame of the "Topoplogical Fluid Mechanics" is the role played by the induced chaos in the particle paths in a plasma. To accomplish this task, one must study the dynamical system described by the equations

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}(\mathbf{r}, t) \text{ with } \mathbf{r}(\Im, t = 0) = \Im \tag{14}$$

In terms of the usual Lagrangian coordinates $\Im$ (initial conditions for $\mathbf{r}$) searching for conditions inducing chaos or, more precisely, one has to look for the existence of plasma volume fraction for which the flow has a chaotic behaviour. In the case of plasmas the magnetic field induction equation, by the generalized Ohm's Law, has to be simultaneously considered, since the electromagnetic field self-consistently drives the flows described by $\mathbf{v}(\mathbf{r}, t)$ through the so-called dynamo action. Tang and Boozer go beyond the use of the Lyapunov exponents

$$\lambda_l = \frac{\ln|\Im|}{t} \tag{15}$$

as a simple statistical tool to characterize the transport processes in the limit of large t. They rigorously examine the time evolution of λ_l for a finite time attending to the temporal evolution of ideal and diffusive magnetic field and current distribution. In both ideal (null resistance) and resistive cases, they have found that natural occurrence of anisotropic structures and the evolution of a magnetic field embedded in a conducting media can be interpreted in terms of the local properties of λ_l in the directions of Lagrangian flow trajectories. Although for a realistic plasma the dissipative effects cannot be rejected, the authors state that the magnetic field can grow exponentially in an almost ideal evolution firstly, before the ohmic dissipation becomes effective. They establish that the heating ηj^2 is concentrated in current channels after an ideal evolution of the chaotic flow. The natural evolution of a magnetic field inside a chaotic flow is to "develop small ***spatial structures with extreme spatial anitropy***, which typically ***take the form of filaments, ribbons and sheets***". The analysis of the Lorentz force evolution, compared with the convective force in a dissipative plasma flow, leads to an interesting and important result: the induced ***current tends naturally to align with the magnetic field*** and "the spatial scale of the current distribution has strong anisotropy similar as the magnetic field", in their own words.

Therefore, the existence of localized structures of plasma seems to be supported by several theoretical studies from very different points of view. What makes interesting the Tang-Boozer approach is the fact that the induced current and the embedded magnetic field evolve in a similar manner to finally tend to be aligned, i.e. the force-free plasma state mentioned above.

4 The claim for experimental support for knotted filaments

If both, Faddeev-Niemi and Tang-Boozer arguments are brought together, one can postulate that a natural tendency of a plasma is to develop anisotropic filamentary structures with aligned currents and fields having also the form of knotted stable solitons. Therefore, unusual long-living filamentary high density structures, with high energy storage, have been already theoretically predicted and experimentally observed in the bulk of plasmas. At this stage, it only rests to reference some experimental evidences that these knotted force-free plasma states really exist .

As it happens in ordinary lightnings, where conduction is stated by small current channels (streamers) it can be imagined that plasma, inside certain structures, consists of a self-organized set of meta-stable highly conductive wire-like of filamentary currents. Filamentary states with minimum dissipation in a magnetic field, with twisted force-free interacting current channels, were found by Gekelman and co-workers [9]. Here the authors stress the role played by the ***self-generated magnetic fields*** in a dynamics driven by the aforementioned electron pressure gradients ∇P_e , which ***increase the helicity as the channels twist*** a each other. This important experimental result shows the effects of ∇P_e in leading the system to a state close to force-free one, and in inducing the helicity H modification. It is crucial to recall here that the force-free state is reached without helicity conservation, as the Taylor relaxation processes impose . Filamentary states with minimum dissipation in a magnetic field, with twisted force-free interacting current channels, has been, in fact, presently found in a mesoscopic scale in laboratory

5 Generalized Trkal flows and force-free fields. Effects of knottedness.

As I have pointed out, the fact that certain plasma structures may contain force-free magnetic configurations is something more than a plausible idea. It can be imagined that plasma inside a fireball, for example, consists in a self-organized set of meta-stable highly conductive wire-like of filamentary currents. In this sense, real transport processes seem to be a cause for field alignment currents in plasmas, giving Taylor-like relaxed states without the imposition of the helicity constraint. It is interesting to treat now the existence and characteristics of a force-free magnetic field in a plasma, as well as its evolution. If the tensor relation (11) is applied to the Magnetic Maxwell tensor, one has

$$\int \frac{B^2}{2} dV + \int_{\int} S \, \frac{B^2}{2} \mathbf{r} \cdot d\mathbf{S} = \int [(\nabla \times \mathbf{B}) \times \mathbf{B}] \cdot \mathbf{r} dV, \qquad (16)$$

for a force-free field, the $r.h.s.$ vanishes, and if $B = 0$ on the surface, it has to be zero at any point, meaning that such a magnetic field in a bounded volume cannot exist, unless it is not zero at the surface. The field may be normal or parallel to the fluid surface. Moreover, if the current is proportional to $\mathbf{B}$ and $\mathbf{j} \cdot \mathbf{n} = 0$, the parameter λ has to be zero at S, but a force-free field may exist. The anisotropic effects make the field to occur and redirect pressure effects, but also, in a force-free field, the anisotropic effects seem to be related to the parameter λ in the equation $\nabla \times \mathbf{B} = \lambda \mathbf{B}$. In collisionless plasma, Bulanov has shown in [10] that λ depends on the square root of $|T_\perp - T_{||}|$. If this property holds in an anisotropic plasma, it is expected that the field at large distances form the plasma ball might decay as a potential magnetic field, suggesting the possibility of an internal quasi-force-free knotted field seeded by far currents.

As an application of a system governed by aligned fields and currents, let us mention a particular case of Trkal or aligned flows in MHD as studied in [11], where the authors prove that these flows may be Lyapunov stable, implying that they can survive to any MHD instability as mentioned by Yoshida in [12]. Here, we consider that all vector fields (flow velocity, vorticity and electromagnetic fields) are parallel, as in the topological model of BL proposed by Rañada. All vectors evolve in time following the diffusive behavior of the magnetic field. This can be understood by means of the previous arguments due to Tang *et al* , [6] but another reason can be given. It is well-known that a plasma can suffer a relaxation process to a final state by passing through a sequential cascade different semi-relaxed states.

In view of the theoretical analysis referenced in the preceding sections, it can be conjectured that is a plasma depart from a (quasi) force-free state, its evolution is driven by current (and flow) aligned whit the fields, which leads to a continuous cascade of such states to complete a time relaxation process, probably toward extinction if no energy or helicity sources are present. In other words, if a plasma structure is force-free, it evolves through a cascade of force-free states with unavoidable field diffusion due to resistive effects. The task is to study the reasons for the stability and unusual slow decay of a knotted plasma filament.

To phenomenologically describe the effect of knottedness, we can add to the temporal rate of change of the magnetic field, this is, the induction equation, a term similar to that one describing the alpha-effect in the dynamo theory, although there the phenomenon is a consequence of small scale helical turbulent motions. In our case, this hypotheses can be justified by attending to the effective helical current flows around a current filament, that may twists itself. The knot itself may be treated as a large scale helical current, producing an additional current which is anti-parallel to the field. In spite of the net current being parallel to the field, this effect leads to a practical reduction of the magnetic diffusivity coefficient η. In the force-free case the field diffusion gives rise to an exponential decaying in time, preserving the topology of the system, whose stability also increases with the linking number [10]. Thus, writing the induction equation in the (MHD approximate) form

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [-\eta \nabla \times \mathbf{B} + \mathbf{v} \times \mathbf{B} + \alpha \mathbf{B}]$$

if it holds at any time for a force-free magnetic field, with parallel or slow flow velocity,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [-\eta \lambda \mathbf{B} + \alpha \mathbf{B}] = \nabla \times [-(\eta \lambda - \alpha)\mathbf{B}]$$

Providing an effective magnetic diffusivity in the form

$$\eta' = \eta \lambda - \alpha$$

and $\mathbf{B}$ would decay exponentially in time, as the field $\mathbf{B} = \mathbf{B}_0(\mathbf{r}) \exp(-\lambda \eta' t)$, in a larger time-scale that would do without α, even with viscosity effects on the fluid flow, but preserving the initial topology of the system depicted by $\mathbf{B}_0(\mathbf{r})$. Although this phenomenological hypotheses would require more inspection into the plasma flow structure and topological aspects of the flow, it can be justified in view of the previously referenced theoretical and experimental results. Moreover, in the context of astronomical plasmas similar additional terms are used in the induction equation and several stability analysis have been performed, indicating that such a term allows for the formulation of global Lyapunov functions for the stability analysis. In this sense, the force-free state behaves as an attractor, as proved by Nuñez [13], using a Lyapunov function composed of a linear combination of magnetic energy and cross and magnetic helicity, in a similar way as done by Tasso and Yoshida. Hence, the stability a plasmas strongly coupled to a knotted (quasi) force-free field is an interesting task in plasma theory since this satates are good candidates to explain unusual long-living charge structures in mesoscopic scales. In this sense, it is interesting to observe that under the assumptions made in this section, the magnetic field coupled to a plasma filament (ribbon or sheet) behaves formally in a similar manner as that one observed in a ***superconducting media***. Here, the field is "trapped" by the knotted fluid, instead of being expelled by the medium, a mirror-like mode of the plasma contained between two cross self-surrounded currents seem to be a natural confining form of a plasma streamline coupled to the field. The amount of the passing almost collisionless particles in the narrow zone should be enough to keep the aligned current and flow. In this sense, the plasma itself would be interpreted as a ***metastable*** state (as metastable states dealt in Thermodynamics) immersed in a large volume plasma or in another medium.

Summary

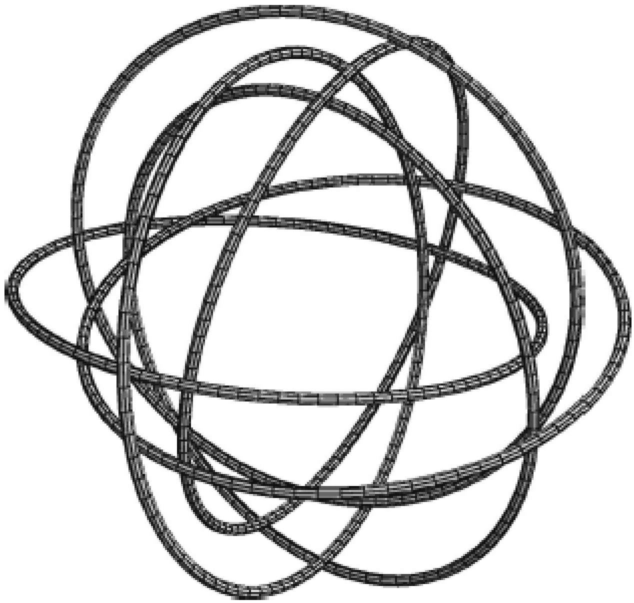
It has been established that some important features of plasma properties are missed when dealing with the classical virial theorem, such as the possible existence of a plasma self-organized into filamentary structures coupled to a knotted **force-free** magnetic field. In this field, there is no pinch effect and the ratio between plasma pressure and magnetic energy can be arbitrary, being the dissipation power due to null mechanical effects. In spite of the results of Fadedeev-Niemi are based on theoretical proposals and numerical solutions, these approaches could be very important to encourage the search of experimental evidence of stable knotted plasma structures as metastable solitons inside a plasma bulk or in a neutral gas medium, as predicted by this approach. The Tang and Boozer statement asserting that in a chaotic plasma flow the magnetic field evolves in such a way, that the induced current tends to align with the field, and the possibility of stable knotted solitons in plasmas have been brought together to suggest the existence of metastable solitons in mesoscopic plasmas.

High energy storages could be justified by means of this formulation if there is a force-free magnetic field aligned with currents and a divergence-free electric field. As an example of this, the topological model of ball lightning has been given. The plasma lifetime can be also increased by the effect of knottedness, which could be simulated as an alpha-effect, similar to the one used in dynamo theory, although it appears here as a result of linking and helical currents twisting around a plasma filament, an effect that slows down the field diffusion. Recently, it is interesting to cite here that the Faddeev-Niemi knotted solitons have been invoked by Y. M. Cho, in the frame of Skyrme theory, a new type of soliton, "the topological knot made of the monopole- antimonopole pair". This suggest the possibility of finding such a knot in a mesoscopic scale if some plasma formations can be obtain in a laboratory.

Finally, I recall that using reference to the Gekelman's experimental work, it has been stressed the role of ***self-generated magnetic fields*** in a dynamics driven by ∇P_e , that increases the helicity "at the same time that the channels twist about each other". This also means that the way to get an experimental evidence of knotted solitons in plasma is to drive it by producing electron pressure gradients to control the magnetic helicity dynamics to increase the linking and twisting till reaching knotted states. The question of whether a knotted magnetic field can be got in the laboratory and if it can drive a coupled knotted charged flow is still opened and it would require many efforts from both theoretical and experimental physicists as well as accurate computations and mathematical efforts which are also required.

Acknowledgement

The authors want to thank to Dr. A. F. Rañada for his stimulating and encouraging comments. This research has been partially supported by the Spanish Ministry of Science and Technology under the project grants ESP2004-01511 and HU2005-0022.



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(Magnetic Monopoles, Physical symmetries, Nodal electric fields. Fondation Louis de Broglie, Peyresq 9-16 août 2007.)