Structure of the photon and magnetic field induced birefringence and dichroism

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ABSTRACT. In this letter we show that the dichroism and ellipticity induced on a linear polarized light beam by the presence of a magnetic field in vacuum can be described in the framework of the de Broglie's fusion model of a photon. In this model it is assumed that the usual photon is the spin 1 state of a particle-antiparticle bound state of two spin 1/2 fermions. The other $S = 0$ state is referred to as the *second* photon. On the other hand, since no charged particle neither particles having an electric dipole are considered, no effect is predicted in the presence of electric fields and this model is not in contradiction with star cooling data or solar axion search.

1 Introduction

Propagation of light in vacuum in the presence of a transverse magnetic field has been experimentally studied since 1929 [1]. The first motivation was to look for a magnetic moment of the photon. Only around 1970, thanks to the effective Lagrangian established in 1935 and 1936 by Kochel, Euler and Heisenberg [2] [3], it has been shown that in a vacuum a Cotton-Mouton effect should also exist [4] [5] similar to the one in ordinary matter. This magnetic birefringence effect in matter has been studied in detail by A. Cotton and H. Mouton [6] at the beginning of the 20th century. The velocity of light propagating in the presence of a transverse magnetic field B depends on light polarization, i.e. the index of refraction $n_\parallel$ for light polarized parallel to the magnetic field is different from the index of refraction $n_\perp$ for light polarized perpendicular to the magnetic field. For symmetry reasons, the difference $\Delta n = (n_\parallel - n_\perp)$ is proportional to B^2. Thus, an incident linearly polarized light exits elliptically polarized from the magnetic field region. The ellipticity to be measured ϵ can be written as

$$\varepsilon = \pi \frac{L}{\lambda} \Delta n \sin 2\theta \tag{1}$$

where L is the optical path in the magnetic field region, λ the light wavelength, and θ the angle between light polarization and the magnetic field.

In dilute matter like gases, such an effect is usually very small and it needs very sensitive ellipsometers to be measured [7]. From the theoretical point of view, Cotton-Mouton effect is related to the electromagnetic properties of matter like magnetic hyperpolarizability. *Ab initio* calculations can be performed using the most advanced computational techniques and they still remain very challenging [7].

On the other hand, in vacuum, quantum ElectroDynamics (QED) predicts that a field of 1 T should induce an anisotropy of the index of refraction of about 4×10^{-24}. This very fundamental prediction has not yet been experimentally verified.

Some of the earlier experiments were based on the use of an interferometer of the Michelson-Morley type. One of the two arms passed through a region where a transverse magnetic field was present inducing a difference in the light velocity that should have been observed as a phase shift [8] [9]. In 1979 Iacopini and Zavattini [10] proposed to measure the ellipticity induced on a linearly polarized laser beam by the presence of a transverse magnetic field using an optical cavity in order to increase the optical path in the field. The effect to be measured was modulated in order to be able to use an heterodyne technique in order to increase the signal to noise ratio.

In 1986 Maiani, Petronzio, and Zavattini [11] also showed that an hypothetical low mass, neutral, spinless boson, both scalar or pseudoscalar, that couples with two photons could induce an ellipticity signal in the Zavattini apparatus similar to the one predicted by QED. Moreover, an apparent rotation of the polarization vector of the light could be observed because of conversion of photons into real bosons resulting in a vacuum magnetic dichroism which is absent in the framework of standard QED [5]. The measurements of ellipticity and dichroism including their signs can in principle completely characterize the hypothetical boson, its mass m_a, the inverse coupling constant M_a, and the pseudoscalar or scalar nature of the particle. Maiani, Petronzio, Zavattini's paper was essentially motivated by the search for Peccei and Quinn's axions. These are pseudoscalar, neutral, spinless bosons introduced to solve what is called the *strong CP problem* [12]. However, it was soon clear that such an optical apparatus could hardly exclude a range of axion parameters not already excluded by astrophysical bounds [13].

Following Zavattini's proposal, an apparatus has been set up at the Brookhaven National Laboratory, USA [14]. No evidence for dichroism induced by the magnetic field was found nor for ellipticity. The sensitivity being not enough to detect QED effect, limits on the axion parameters has been published in 1993 [14].

In 1991, a new attempt to measure the vacuum magnetic birefringence has been started at the LNL in Legnaro, Italy, by the PVLAS collaboration [15]. This experiment is again based on ref. [10]. A Fabry-Perot cavity is used to increase the effect to be measured, while a superconductive 5 T magnet rotates around its own axis to modulate it. Eventually, the collaboration has published the observation of a magnetically induced dichroism in vacuum [16], and also of magnetically induced ellipticity in excess of what is expected according to QED [17]. These results have triggered a lot of interest in the field, in particular because of the existence of axions could be the explanation of this unexpected signal [18]. This explanation however is in contradiction with other existing experimental data. In particular, the particle needed to justify the PVLAS results should be largely produced in the star core by interaction of photons with plasma electric fields. Such a particle should escape because of its very low coupling with matter, and induce a fast cooling of stars at a level already excluded by astrophysical observations[13]. Moreover, CAST experiment [19] devoted to detect solar axions by conversion in a magnetic field, has already excluded the existence of such a particle in the range of mass and coupling constant necessary to give the PVLAS effect. Very recently PVLAS collaboration have posted on internet a preprint disclaiming their previous observation of magnetically induced dichroism in vacuum. An ellipticity signal is still present at 5.5 T, while no ellipticity signal is observed at 2.3 T [20]. On the other hand, the axion interpretation of the PVLAS optical results has been also independently excluded by a photoregeneration experiment [21]. Such a kind of experiment is based on the idea that once an axion is created by photon conversion in a magnetic field, the particle escapes from the magnet region while the light beam can be easily confined. Now, if the created particle passes through another magnet region, it can be converted back into a photon that one can detect in a region where no photon should exist [22].

In this letter we show that dichroism and ellipticity induced on a linear polarized light beam by the presence of a magnetic field in vacuum can be described in the framework of the de Broglie's fusion model of a photon [23]. In this model it is assumed that the usual photon is the spin 1 state of a particle-antiparticle bound state of two spin 1/2 fermions. The other $S = 0$ state is referred to as the *second* photon. Following de Broglie's original proposal, the constituent particles can be thought as neutrino-like massless, chargeless fermions. The mass and the charge of the usual photon is therefore supposed to be zero or negligible. A relativistic quantum field generalization of the fusion theory as a modification of Heisenberg's approach [24] has been developed recently [25, 26], were the basic ingredients are assumed to be unobservable subfermions.

We consider here an approximate version of this theory taken the spin-spin coupling and the interaction with an external magnetic field proportional to $\boldsymbol{s}_1.\boldsymbol{s}_2$ and $\boldsymbol{B}.(\boldsymbol{s}_1 - \boldsymbol{s}_2)$, respectively. We show that magnetic induced birefringence and dichroism are obtained with a $(\boldsymbol{\epsilon}.\boldsymbol{B})^2$ pseudo-scalar symmetry, where $\boldsymbol{\epsilon}$ is the polarization of the photon (as usual $\boldsymbol{\epsilon}$ is defined by the direction of the electric field). Thus both dephasing and absorption appear for linearly polarized light parallel to the external applied magnetic field. On the other hand, since no charged particle neither particles having an electric dipole are considered, no effect is predicted in the presence of electric fields and this model is not in contradiction with star cooling data or solar axion search.

2 The model

We consider the photon as composed of a spin 1/2 particle and its antiparticle. The spin Hamiltonian is assumed to be approximate by

$$H_0 = -\frac{\eta}{\hbar^2}\,\boldsymbol{s}_1.\boldsymbol{s}_2 \tag{2}$$

with $\eta > 0$. The ground state eigenstates are then given by:

$$\begin{aligned} &|S{=}1, M_z{=}1\rangle = |\uparrow,\uparrow\rangle; \quad |S{=}1, M_z{=}-1\rangle = |\downarrow,\downarrow\rangle \\ &|S{=}1, M_z{=}0\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle\Big) \end{aligned} \tag{3}$$

with energy $E_1 = -\eta/4$, corresponding to the ordinary photon γ_1. The *second* photon γ_0 is then given by the excited singlet state

$$|S{=}0, M_z{=}0\rangle = \frac{1}{\sqrt{2}} \Big(|\uparrow, \downarrow\rangle - |\downarrow, \uparrow\rangle \Big) \tag{4}$$

with energy $E_0 = (3/4)\,\eta$. The energy difference between the two photons γ_1 and γ_0 is then given by η.

We assume the particle/antiparticle have magnetic moments $\boldsymbol{m}_1 = (\beta\,\mu_B/\hbar)\,\boldsymbol{s}_1$ and $\boldsymbol{m}_2 = -\boldsymbol{m}_1$. Thus the total magnetic moment $\boldsymbol{m} = (\beta\,\mu_B/\hbar)\,(\boldsymbol{s}_1 - \boldsymbol{s}_2)$ has zero average value for the γ_1 photon and $\langle m_z \rangle = \beta\,\mu_B$ for the γ_0 photon.

In the presence of a magnetic field $\mathbf{B}$ along Oz we shall have

$$V = (\beta\,\mu_B\,B/\hbar)\,(\boldsymbol{s}_{1z} - \boldsymbol{s}_{2z}) \tag{5}$$

The only non-zero matrix element of V is

$$\langle S{=}1, M_z{=}0|V|S{=}0, M_z{=}0\rangle = \beta\,\mu_B\,B \tag{6}$$

After diagonalization

$$\begin{aligned} |\Psi_{1,0}\rangle &= \cos\theta\,|S{=}1, M_z{=}0\rangle + \sin\theta\,|S{=}0, M_z{=}0\rangle \\ |\Psi_{0,0}\rangle &= -\sin\theta\,|S{=}1, M_z{=}0\rangle + \cos\theta\,|S{=}0, M_z{=}0\rangle \end{aligned} \tag{7}$$

with

$$\tan(2\,\theta) = 2\beta\,\mu_B\,B/\eta \tag{8}$$

and eigenvalues

$$\overline{E}_1 = \frac{E_1 + E_0}{2} - \frac{\eta}{2}\sqrt{1 + \tan^2(2\,\theta)}\ ;\ \overline{E}_0 = \frac{E_1 + E_0}{2} + \frac{\eta}{2}\sqrt{1 + \tan^2(2\,\theta)} \tag{9}$$

with the new energy difference

$$\overline{\eta} = \eta\sqrt{1 + \tan^2(2\,\theta)} \tag{10}$$

3 Magnetic field action on a linearly polarized photon

The ordinary photon γ_1 can be described by a linear combination of the two helicity states $|S{=}1, M_{\boldsymbol{k}}{=}\pm 1\rangle$ where $M_{\boldsymbol{k}}$ is the projection of the spin angular momentum in the direction of propagation of the photon. Let first note that if the γ_1 is propagating along the direction of the magnetic field, only $|S{=}1, M_z{=}\pm 1\rangle$ will be involved and no effect is expected. Consider now a γ_1 propagating along Oy and linearly polarized in the direction of Oz. We shall have

$$|\epsilon_z\rangle = -\frac{1}{\sqrt{2}}\Big(|S{=}1, M_y{=}1\rangle - |S{=}1, M_y{=}-1\rangle\Big) \tag{11}$$

but

$$|S, M_y\rangle = \sum_{M_z=0,\pm1} |S, M_z\rangle\, d^S_{M_z,M_y}(\pi/2) \tag{12}$$

where the $d^S_{M_z,M_y}$ are the Wigner d-functions. Using

$$\begin{aligned} d^1_{1,\pm1}(\Theta) &= \frac{1}{2}(1 \pm \cos\Theta) \\ d^1_{0,\pm1}(\Theta) &= \pm\frac{1}{2}\sqrt{2}\,\sin\Theta \\ d^1_{-1,\pm1}(\Theta) &= \frac{1}{2}(1 \mp \cos\Theta) \end{aligned} \tag{13}$$

we get from (11) and (12)

$$|\epsilon_z\rangle = -|S{=}1, M_z{=}0\rangle \tag{14}$$

and this state will be affected by the magnetic field through its coupling to the $|S{=}0, M_z{=}0\rangle$ state. We note in passing that in the case of a linear polarization along the Ox axis we have

$$\begin{aligned} |\epsilon_x\rangle &= \frac{1}{\sqrt{2}}\Big(|S{=}1, M_y{=}1\rangle + i\,|S{=}1, M_y{=}-1\rangle\Big) \\ &= \frac{1}{\sqrt{2}}\Big(|S{=}1, M_z{=}1\rangle + i\,|S{=}1, M_z{=}-1\rangle\Big) \end{aligned} \tag{15}$$

and this state will not be affected by the magnetic field.

3.1 *Time dependence of the state vector*

We assume that the magnetic field is switch-on between $t = 0$ and $t = \tau = L/c$, where L is the field length. We shall have $|\psi(0)\rangle = -|1, 0\rangle =$

$-\cos\theta\,|\Psi_{1,0}\rangle + \sin\theta\,|\Psi_{0,0}\rangle$ where from now on the kets correspond to $|S, M_z\rangle$. At time τ

$$|\psi(\tau)\rangle = -\cos\theta\, e^{-i\,\overline{E}_1\,\tau/\hbar}\,|\Psi_{1,0}\rangle + \sin\theta\, e^{-i\,\overline{E}_0\,\tau/\hbar}\,|\Psi_{0,0}\rangle \tag{16}$$

which in terms of the non-perturbed kets $|1,0\rangle$ and $|0,0\rangle$, will be given by

$$\begin{aligned} |\psi(\tau)\rangle = &-\left(\cos^2\theta\, e^{-i\,\overline{E}_1\,\tau/\hbar} + \sin^2\theta\, e^{-i\,\overline{E}_0\,\tau/\hbar}\right)\,|1,0\rangle \\ &-\cos\theta\,\sin\theta\,\left(e^{-i\,\overline{E}_1\,t/\hbar} - e^{-i\,\overline{E}_0\,\tau/\hbar}\right)\,|0,0\rangle \end{aligned} \tag{17}$$

This can be written as

$$\begin{aligned} |\psi(\tau)\rangle = -e^{-i\,(E_1+E_0)\,\tau/2\hbar}\,\Bigg[&\Big(\cos(\overline{\eta}\,\tau/2\hbar) + i\cos(2\theta)\,\sin(\overline{\eta}\,\tau/2\hbar)\Big)\,|1,0\rangle \\ &+ i\,\sin(2\theta)\,\sin(\overline{\eta}\,\tau/2\hbar)\,|0,0\rangle\Bigg] \end{aligned}$$

3.2 Magnetic induced dichroism

The probability to produce $|0,0\rangle$ is $P_{\gamma_1\to\gamma_0} = |\langle 0,0|\psi(\tau)\rangle|^2$ which, using (18), gives

$$P_{\gamma_1\to\gamma_0} = \frac{\tan^2(2\theta)}{1+\tan^2(2\theta)}\,\sin^2\left(\eta\,\sqrt{1+\tan^2(2\theta)}\,\tau/2\hbar\right) \tag{18}$$

with $\tan^2(2\theta)$ given by (8).

In the limit where $2\beta\,\mu_B\,B \ll \eta$, $\tan^2(2\theta) \ll 1$, and

$$P_{\gamma_1\to\gamma_0} \simeq \left(\frac{\beta\,\mu_B\,B\,\tau}{\hbar}\right)^2\,\left(\frac{\sin\left(\eta\,\tau/2\hbar\right)}{\eta\,\tau/2\hbar}\right)^2 \tag{19}$$

Thus, when $\eta\,\tau/2\hbar \ll 1$, $P_{\gamma_1\to\gamma_0}$ does not depend on η.

In a typical apparatus [10] where a linearly polarized laser passes through a region where a magnetic field pointing at 45 degrees with respect to light polarization plane is present, such a conversion probability will show as a linear dichroism giving an apparent rotation of the polarization plane $\rho = \frac{1}{2}P_{\gamma_1\to\gamma_0}$.

3.3 Magnetic induced birefringence

As for the phase of the $|1,0\rangle$, this is given by

$$\overline{\phi}_1 = -(E_1 + E_0)\,\tau/2\hbar + \arctan\left[\cos(2\theta)\,\tan(\overline{\eta}\,\tau/2\hbar)\right] \tag{20}$$

On the other hand, for the Ox polarization the phase is $\phi_1 = E_1\,\tau/\hbar$. The phase difference between γ_1 states for polarization along and perpendicular to the magnetic field is then given by

$$\delta\phi \equiv \overline{\phi}_1 - \phi_1 = \arctan\left[\cos(2\theta)\,\tan(\overline{\eta}\,\tau/2\hbar)\right] - \eta\,\tau/2\hbar \tag{21}$$

Expanding this function in powers of θ around zero, we found

$$\delta\phi = \left(\frac{\beta\,\mu_B\,B}{\eta}\right)^2 \left(\frac{\eta\,\tau}{\hbar} - \sin(\eta\,\tau/\hbar)\right) \tag{22}$$

Again, in the case of a typical apparatus [10], this dephasing will show as an ellipticity $\varepsilon = \delta\phi/2$ acquired by the polarized beam passing through the magnetic field region. Ellipticity is associated to the existence of a birefringence by the formula 1.

Thus, in the framework of our model a vacuum will show an apparent magnetic birefringence

$$(n_{\parallel} - n_{\perp}) = \left(\frac{\lambda}{2\pi\,c\,\tau}\right)\left(\frac{\beta\,\mu_B\,B}{\eta}\right)^2 \left(\frac{\eta\,\tau}{\hbar} - \sin(\eta\,\tau/\hbar)\right) \tag{23}$$

that depends on the time the photon stays in the magnetic field region. Standard QED predicts that a vacuum is a magnetic birefringent medium showing a $(n_{\parallel} - n_{\perp}) \simeq 4 \times 10^{-24} B^2$ where B is given in Tesla. That only depends on the value of fundamental constants and the square of the magnetic field intensity [5]. QED also predicts that a corresponding effect exists in the presence of an electric field, such an effect is absent in the framework of our model.

4 Discussion

First of all, we note that our formulas for the conversion probability and the dephasing are equivalent to the ones obtained in the axion case [11] since axion-photon coupling can be also treated as a two level system [13].

Our η corresponds to the ratio m_a^2/ω and β to $g_{a\gamma\gamma}$, where m_a is the axion mass, ω the photon energy, and $g_{a\gamma\gamma}$ the axion-photon coupling constant. The main difference between our model and the axion model is that in our case the optical effects do not depend on the photon energy. Thus, in our case, the oscillations between the two states of the hamiltonian only depend on the time the γ_1 stay in the magnetic field i.e. the length of the magnetic field region. In the axion case the oscillations depend on the length divided by the photon energy ω. Oscillations can therefore be avoided by choosing higher energy photons for longer magnets.

An important feature of our model compared to the axion model is that the mixing between the ordinary photon γ_1 and the second photon γ_0 only appears in a magnetic field. This will not affect the energy balance and star evolution, but should be important in the case of photon emission from neutron stars which show magnetic fields as high as 10^9 T. This is anyway an important issue also for axion search (see e.g. ref.[27], and [28]).

The experiments already performed to look for axions studying the propagation of light in a transverse magnet field, and that have given a clear null result can therefore exclude values of the two parameters of our model β and η. In fig. 1 we show the corresponding graph following equation (19)in which dotted line represents the lower border of the parameters plane forbidden by BRFT dichroism null result at a 2σ level [14]. We have assumed as usual that the measured effect is simply the effect predicted by that formula multiplied by the number of passages in the magnetic field due of the presence of optical cavity.

It is important to stress that existing results just allow to exclude a region of parameters that is not very interesting. In fact, β represents the magnetic moment of the neutrino-like particle which is the constituent of the photon in our model. Since existing experimental limits exclude that the neutrino magnetic moment is bigger than about $10^{-10}\mu_B$ [29], one reasonably expects that the values β should be also smaller than 10^{-10}.

On the other hand, future experiments are expected to have sensitivity to measure the QED vacuum magnetic birefringence, as the BMV (*Biréfringence Magnétique du Vide*) experiment [30], which is currently be mounted at the LNCMP (*Laboratoire National des Champs Magnétiques Pulsés*) in Toulouse, France.

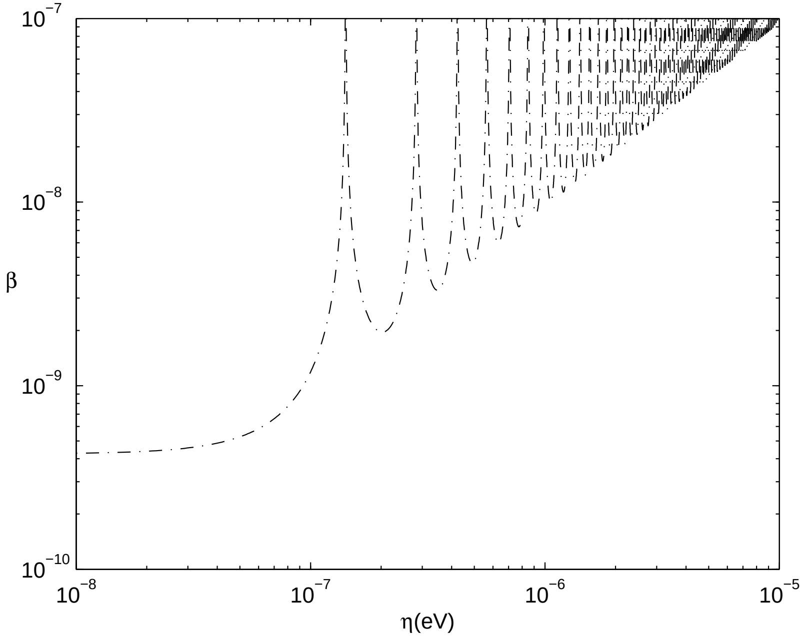

Figure 1: Limits coming from the BRFT dichroism null result at a 2σ level [14]

In fig. 2 we show the expected value of the magnetically induced ellipticity due to the composite photon in the BMV experiment in its final version [30] for a $\beta = 10^{-10}$. In a large range of the η parameter, this ellipticity is bigger than the one predicted by standard QED (i.e. $\simeq 5 \times 10^{-9}$ rad) for the BMV apparatus in the same experimental conditions.

This kind of experiments are therefore also very sensitive to the effect that could be due to the composite nature of the photon.

In any case, as stressed above, the calculation presented in this note is only a low level approximation of the fully relativistic quantum field treatment of bound states in fusion theories [24, 25]. We believe that such fully relativistic quantum field treatment of our model is needed to definitively establish what are the consequences of the de Broglie's

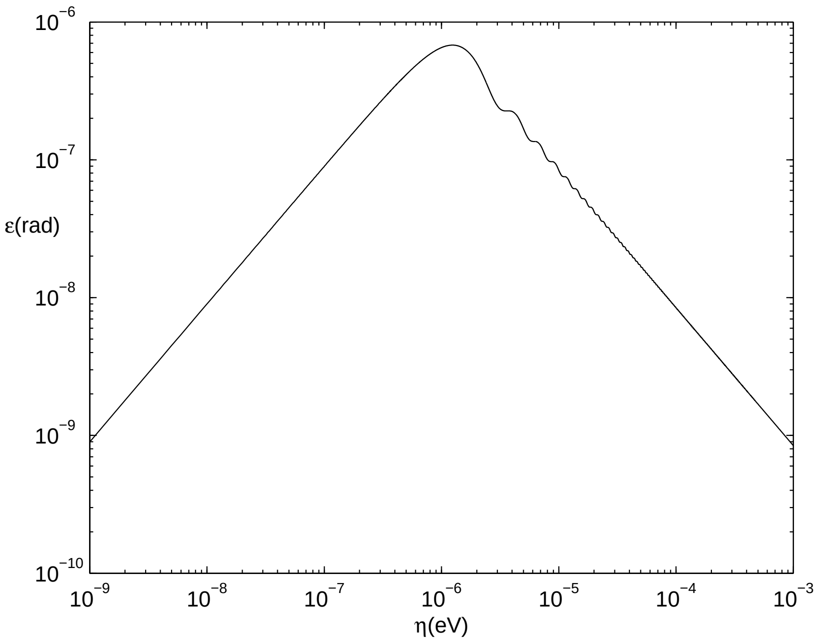

Figure 2: The expected value of the magnetically induced ellipticity due to the composite photon in the BMV experiment in its final version [30] for a $\beta = 10^{-10}$

fusion model on the propagation of light in a magnetic field, and thus test such a model by experiments.

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