Comment on "A New Proof of Bell's Theorem Based on Fourier Series Analysis"[1]

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ABSTRACT. A new variant of Bell's nonlocality proof was recently presented by H. Razmi [4]. We point out that Razmi's proof does not establish the same result as Bell's.

In his recent paper "A new proof of Bell's theorem based on Fourier series analysis" [4], H. Razmi proposes a new variant of Bell's [1] proof of quantum nonlocality. However, Razmi's proof has a crucial flaw. It uses an additional hypothesis (H) that Bell did not use. Razmi creates the impression that (H) is a consequence of the physical setting considered in Bell's proof, but it is not. Thus, Razmi's proof fails to establish the same result as Bell's.

The relevant hypothesis is expressed in Eq. (9) of [4]. It asserts, in Bell's [2] notation,

$$P(A = +1|a = \theta, \lambda) = P(B = +1|b = \theta, \lambda)\,. \tag{H}$$

In words, for any given specification λ of the complete state (possibly involving hidden variables) and any direction θ in space, (H) asserts that the probability of outcome $+1$ in Alice's experiment when she sets her free parameter a equal to θ equals that in Bob's experiment when he sets his free parameter b equal to θ.

Razmi claims that (H) follows from the symmetry of the physical setting under interchange of Alice and Bob, a symmetry that interchanges

[1] N.D.L.R. As this paper discusses some results of a work recently published in this journal, we felt it best to let Dr Razmi give his own answer. It is given after this article and, as far as we are concerned, closes the discussion.

in particular the quantities A and B, as well as a and b. However, this symmetry may well act on λ by means of a nontrivial mapping $\lambda \mapsto T(\lambda)$. Thus, a more reasonable version of (H) would read

$$P(A = +1|a = \theta, \lambda) = P(B = +1|b = \theta, T(\lambda)) . \tag{2}$$

For example, in Bohmian mechanics [3] the complete state is given by $\lambda = (Q, \psi)$ with Q a point in configuration space and ψ a wave function. For an EPR pair, $Q = (Q_A, Q_B)$ consists of the two particle positions $Q_A, Q_B \in \mathbb{R}^3$, and the wave function $\psi = \psi_{r,s}(q_A, q_B)$ is a function of the two position variables $q_A, q_B \in \mathbb{R}^3$ and has two spin indices $r, s \in \{-1, +1\}$. Exchanging Alice and Bob may correspond to replacing $\lambda = (Q, \psi)$ with

$$T(\lambda) = T(Q, \psi) = \Big((Q_B, Q_A), \psi_{s,r}(q_B, q_A) \Big) . \tag{3}$$

A more realistic theoretical description may involve a reflection across the plane in the middle between Alice and Bob, to ensure that the two particles always move away from that plane. What is important is that $T(\lambda)$ can be different from λ. Note that it is essential for Bell's theorem that no assumption is made on the nature of λ. Here is another, simpler but more artificial, example, a (local) hidden-variables model motivated by the fact that it reproduces the quantum predictions whenever $a = b$: Let $0 \le \lambda \le 1$ with uniform probability distribution, and let A and B be deterministic functions of λ, namely $A = +1$ and $B = -1$ for $0 \le \lambda \le 1/2$, while $A = -1$ and $B = +1$ for $1/2 < \lambda \le 1$. While this model makes predictions that are in general different from quantum mechanics, what is relevant is that its predictions are symmetric against the exchange $A \leftrightarrow B$, namely probability 1/2 for $A = +1, B = -1$ and probability 1/2 for $A = -1, B = +1$, while (H) is violated. Indeed, (2) holds with $T(\lambda) = 1 - \lambda$.

More generally, there is no reason why λ should transform in any simple way at all under exchange of Alice's and Bob's sides. After all, Bell proves his theorem for *any* choice of λ, *any* probability distribution $\rho(\lambda)$ and *any* probabilities $P(A, B|a, b, \lambda)$, as long as they entail the same observable statistics $P(A, B|a, b)$ as predicted by quantum mechanics, that is,

$$P(A, B|a, b) = \int d\lambda\, \rho(\lambda)\, P(A, B|a, b, \lambda) . \tag{4}$$

Note that $P(A, B|a, b)$ (for the singlet state) is symmetric under the simultaneous exchange $A \leftrightarrow B$ and $a \leftrightarrow b$, but (4) does not imply that $P(A, B|a, b, \lambda)$ shares this symmetry. Thus, contrary to (H), one cannot expect any simple relation between the functions $\lambda \mapsto P(A = +1|a = \theta, \lambda)$ and $\lambda \mapsto P(B = +1|b = \theta, \lambda)$.

Razmi’s proof, however, depends crucially on (H) and would not even work with (2).

As a last illustration of how unreasonable a hypothesis (H) is, note that if it were granted, then a much simpler proof than Razmi’s would show the incompatibility between locality and the quantum prediction: Consider $a = b$, recall that quantum mechanics predicts perfect anti-correlation $A = -B$ in that case, so

$$P(A = +1, B = +1|a = b = \theta, \lambda) = 0 \tag{5}$$

for almost every λ. By locality, this probability equals

$$P(A = +1|a = \theta, \lambda)\, P(B = +1|b = \theta, \lambda)\,, \tag{6}$$

so one of the factors must be zero. But by (H) the factors are equal, so they must both be zero, for almost every λ. But by (4) this implies $P(A = +1|a = \theta) = 0$, which however is known to be 1/2.

References

[1] Bell, J. S.: On the Einstein–Podolsky–Rosen paradox. *Physics* **1**: 195–200 (1964). Reprinted as chapter 2 of [2].

[2] Bell, J. S.: *Speakable and Unspeakable in Quantum Mechanics.* Cambridge: Cambridge University Press (1987).

[3] Berndl, K., Daumer, M., Dürr, D., Goldstein, S., Zanghì, N.: A Survey of Bohmian Mechanics. *Il Nuovo Cimento* **110B**: 737–750 (1995).

[4] Razmi, H.: A New Proof of Bell’s Theorem Based on Fourier Series Analysis. *Annales de la Fondation de Broglie* **32(1)**: 69–76 (2007).

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