Quantum computer feasibility and quantum mechanics interpretation

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RÉSUMÉ. On montre que la faisabilité de l'ordinateur quantique dépend de l'interprétation de la mécanique quantique: faisable dans l'interprétation de Copenhague où la mécanique quantique est complète et où le qubit suffit à décrire complètement le système quantique; infaisable dans l'interprétation de Broglie-Bohm où la mécanique quantique est incomplète et où le qubit ne décrit pas complètement le système quantique. La réalisation d'un ordinateur quantique serait donc une réfutation de l'interprétation de Broglie-Bohm. L'échec actuel de l'ordinateur quantique RMN s'explique, par contre, très simplement dans l'interprétation de Broglie-Bohm.

ABSTRACT. We show that the feasibility of a quantum computer depends on quantum mechanics interpretation: feasible in the Copenhagen interpretation where quantum mechanics is complete and where the qubit is sufficient to completely describe the quantum system; unfeasible in the Broglie-Bohm interpretation where quantum mechanics is incomplete and where the qubit does not completely describe the quantum system. Thus, the realization of a quantum computer would be a refutation of the Broglie-Bohm interpretation. However, the current failure of quantum computer NMR is straightforwardly explained in the Broglie-Bohm interpretation.

1 Introduction

Based on the principle of the superposition of quantum mechanical states, the feasibility of creating quantum computers that would permit massive parallel computation has been conceivable since 1985 when

David Deutsch's work was published [1]. Subsequently, industrial interest towards such a computer was spurred by Peter Shor's (1994) polynomial quantum algorithm for factoring numbers [2] and Grover's algorithm (1996) for searching within a non-indexed database [3].

The feasibility of such a computer seems to have been confirmed by Chang et al's creation of a 2 qubit quantum computer [4] in 1998 using the nuclear magnetic resonance (NMR) technique, and a 7 qubit quantum computer [5] in 2001 using Shor's algorithm to factor the number 15.

However since 2001, the subject seems to stagnate, in spite of a large amount of research. Chuang and Gershenfeld gave up their approach. They did not consider solutions beyond 15 or 20 qubits because the magnetic signal which measure the spins orientation and determine the quantum states becomes excessively weak as the number of qubits increases, weakening of a factor close to 2 for each additional qubit.

In the orthodox interpretation of quantum mechanics, the source of these difficulties comes from **the decoherence** which destroys the superposed states as soon as the quantum system interacts with the environment. The solution is thus to overcome this decoherence problem and the quantum computer still remains feasible.

We propose here another possible explanation of the failure of the quantum computer based on NMR technique.

We show indeed that the feasibility of the quantum computer depends on the accurateness of current interpretation of quantum mechanics. This supposes that quantum mechanics is complete and that the wave function, and thus the qubit, completely represents the quantum system. It is an assumption that Einstein has always fought[6].

The quantum parallelism consists in carrying out several operations in a simultaneous way thanks to the qubit, linear quantum superposition of two basic states, noted $|0\rangle$ and $|1\rangle$. We will consider the case where the two-level quantum system is represented by the spin of a particle as in NMR technique.

The detailed study of the spin in the experiment of Stern and Gerlach will show that the qubit quantum computer does not have the same reality (section 2) and the same behaviour (section 3 and 4) in two principal interpretations of the wave function and measurement in quantum mechanics: Copenhagen interpretation and measurement by decoherence on one hand and deterministic Broglie-Bohm interpretation on the other hand. We will then deduce that the quantum computer feasibility depends on quantum mechanics interpretation.

Finally, we show (section 5) that the failure of the quantum computer NMR is explained very simply in the Broglie-Bohm interpretation.

Let us note that our involvement only relates to the parallelism of the quantum computer. It is not related to the problems of secrecy in telecommunications which are well solved thanks to the phenomenon of entanglement in quantum transmission [8].

2 The qubit interpretation

In Copenhagen interpretation, wave function and particle are identified, and the wave function thus completely describes the quantum system. A qubit is represented, at a given moment, in a Hilbert space H, whose a base is made up of two vectors $|0\rangle$ and $|1\rangle$. This qubit is described by an unit vector $|\psi\rangle$:

$$|\psi\rangle = e^{-i\frac{\varphi}{2}}\cos\frac{\theta}{2}|0\rangle + e^{i\frac{\varphi}{2}}\sin\frac{\theta}{2}|1\rangle = \begin{pmatrix}\cos\frac{\theta_0}{2}e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2}e^{i\frac{\varphi_0}{2}}\end{pmatrix}. \tag{1}$$

It corresponds to a particle whose spin is given, compared to a direction given (0z for example) by the spinor $|\psi\rangle$ where θ and φ can be considered as polar and azimuth angles which parameterize a point on the Bloch sphere.

The temporal evolution of this qubit then allows us to understand how one can handle it. Although this evolution of $|\psi\rangle$ is deterministic, the result of measurement is not: we find the state $|0\rangle$ with the probability $\cos^2\frac{\theta}{2}$ or the state $|1\rangle$ with the probability $\sin^2\frac{\theta}{2}$. The result of a measurement thus gives a random result, except if its state is degenerated into $|0\rangle$ or $|1\rangle$ ($\theta = 0$ or π).

In the Broglie-Bohm interpretation , the wave function and the physical particle are not identified, and thus the wave function does not completely describe the quantum system. It is necessary to add the initial position of the physical particle. Consequently with one moment given, the definition of the qubit (1) is not sufficient to completely describe the quantum system. Many particles may have the same wave function. In order to identify the rightone, it is necessary to know the initial position of the particle, as the square of the wave function is only providing the density of the initial positions.

Moreover the representation of the spinor (1) is misleading because a real spinor must indeed have a space extension such as the Gaussian spinor in x and z below:

$$\Psi^0(x,z) = (2\pi\sigma_0^2)^{-\frac{1}{2}} e^{-\frac{(z^2+x^2)}{4\sigma_0^2}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix}. \tag{2}$$

It is such a spinor that it is necessary for the Broglie-Bohm interpretation. Space extension of the spinor (2) then enables us to take into account the initial position (x_0, z_0) of the particle. The evolution of the quantum system (wave function + position) will now be deterministic as we will see below.

In the Copenhagen interpretation, we only consider the qubit (1) for the quantum system, and thus we lose the singularity property induced by the position (x_0,z_0) in the spinor (2). In fact we only keeps the statistical point of view.

3 Qubit evolution and measurement

The detailed study of the spin measurement in the Stern and Gerlach experiment, will show how much different can be the two preceding interpretations of evolution and measurement of a qubit.

In the Stern-Gerlach experiment, cf. figure 1, silver atoms contained in the oven E are heated to a high temperature and escape through a narrow opening. A second aperture, T, selects those atoms whose velocity, $\mathbf{v}_0$, is parallel to the y-axis. The atomic beam crosses the gap of the electromagnet A_1 before condensing on the screen, P_1 . The magnetic moments of the silver atoms before crossing the electromagnet are oriented randomly (isotropically).

On the outlet side of the electromagnet, instead of being scattered, the beam splits into two symetric beams, which produce two spots of equal intensity on a screen, at equal distances from the axis of the original beam.

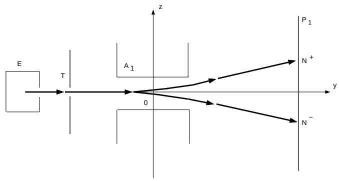

Figure 1: Stern-Gerlach experiment scheme.

Following Dewdney et al [14], we revisit this experiment starting from a complete resolution of the equation of Pauli **over time and space** [15].

The initial spinor of each atom is form (2) with (θ_0, φ_0) randomly chosen [15].

Figure 2 shows the probability density of the silver atoms as a function of z at several values of t (the plots are labelled with $y = vt$). The beam separation does not appear at the end of the magnetic field (1 cm), but 10 cm further along.

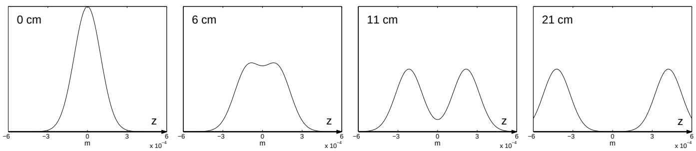

Figure 2: Evolution of the probability density of silver atoms.

This density is the one that gets actually observed in an experimental situation. But this density is not directly measured; it is obtained in experiments from the density of the impacts of the silver atoms on the screen. The atoms of the spot N^+ have a positive spin (state $|0\rangle$), those of the task N^- have a negative spin (state $|1\rangle$).

Although the decoherence theory explains the diagonalisation of the matrix density, it does not explain the impacts of the atoms on the screen [7]. In the Broglie-Bohm interpretation, the atoms have positions and the impacts correspond to the positions of the atoms when they collide on the screen. The dispersion of the atoms on the screen corresponds to the initial dispersion of the silver atoms in the initial beam

$\rho_0(x,z) = |\Psi^0(x,z)|^2 = (2\pi\sigma_0^2)^{-1} e^{-\frac{(z^2+x^2)}{2\sigma_0^2}}$. To visualize this relation, we realize in the Broglie-Bohm interpretation a Monte Carlo simulation [15] of the experiment where the initial positions of the atoms have been randomly chosen in the initial beam.

Let us consider first a spinor(pure state) whose initial orientation (θ_0, φ_0) is given: it is the spinor (2). Figure 3 presents, for $(\theta_0 = \frac{\pi}{3}, \varphi_0 = 0)$, a simulation of ten quantum trajectories of silver atoms whose initial position z_0 has been randomly chosen. The spin orientations $\theta(z,t)$ are represented along the trajectories by arrows.

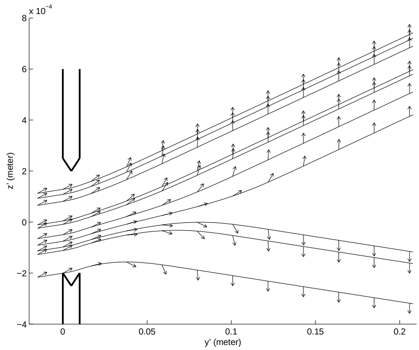

Figure 3: Ten silver atom trajectories with initial polarization $(\theta_0 = \frac{\pi}{3})$ and whose initial position z_0 has been randomly chosen; Arrows represent the spin orientation $\theta(z,t)$ along the trajectories.

These trajectories not only provide a natural explanation for the impact of particles, but also describe the spin quantization along the z-axis. From the initial value $\theta(z,0) = \theta_0$, we notice that there is a z_{θ_0} such that according to the initial position z_0 of the particle in the wave function, the vector of polarization (spin) is gradually directed towards $\frac{\pi}{2}$ if $z_0 > z_{\theta_0}$ (probability $\cos^2 \frac{\theta_0}{2}$) or towards $-\frac{\pi}{2}$ if $z_0 < z_{\theta_0}$ (probability $\sin^2 \frac{\theta_0}{2}$).

Thus the initial wave function $\Psi^0(x,z)$ data by (2) is not sufficient to explain the deterministic evolution; it is necessary to add the initial position z_0.

Let us notice that, at the initial state, spinor (2) is proportional to qubit (1). Nevertheless, this proportionality is no longer verified when the spinor evolves during the measurement process. After the magnetic field, at time t, the spinor becomes [15]

$$\Psi(x,z,t) = \begin{pmatrix} \cos\frac{\theta_0}{2}(2\pi\sigma_0^2)^{-\frac{1}{2}} e^{-\frac{(z-z_\Delta-ut)^2+x^2}{4\sigma_0^2}} e^{i\frac{muz+\hbar\varphi_+(t)}{\hbar}} \\ \sin\frac{\theta_0}{2}(2\pi\sigma_0^2)^{-\frac{1}{2}} e^{-\frac{(z+z_\Delta+ut)^2+x^2}{4\sigma_0^2}} e^{i\frac{-muz+\hbar\varphi_-(t)}{\hbar}} \end{pmatrix} \tag{3}$$

with $z_\Delta = 10^{-5}m$ and $u = 1m/s$. Thus, (3) is no longer proportional to the qubit (1).

Now let us consider a mixture of pure states where the initial orientation (θ_0, φ_0) from the spinor has been randomly chosen. Figure 5 represents a Monte Carlo simulation of 10 quantum trajectories of silver atoms from which the initial positions z_0 are also randomly chosen. We thus obtain the two experimental spots of Stern and Gerlach.

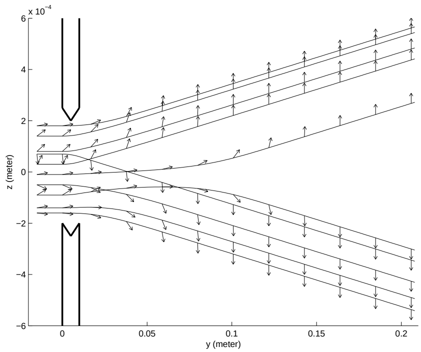

Figure 4: Ten silver atom trajectories where the initial orientation (θ_0, φ_0) has been randomly chosen; Arrows represent the spin orientation $\theta(z,t)$ along the trajectories.

4 The qubit and its measurement into the Broglie-Bohm interpretation

In the Broglie-Bohm interpretation, we permanently have a wave and a particle: the wave is nonlocal and, as a field, spreads on the whole space, while the particle is localised.

In the measurement process, it is the measuring apparatus which gives the orientation of the spin either in the direction of the magnetic field gradient, or in the opposite direction. It depends on the position of the particle in the wave packet. We thus obtain a deterministic result for the unit: wave function + particle position.

Consequently, the initial moment (function of wave + particle position) will give a deterministic result. The superposition of the states $|0\rangle$ and $|1\rangle$ in the Broglie-Bohm interpretation is valid for the wave function, not for the particle. To describe the superposition of the two states, one needs a wave function and at least **two particles**, one with $z_0 < z_{\theta_0}$ and the other with $z_0 > z_{\theta_0}$.

The measurement duration is then the time necessary for the particle to point its spin to the final direction. In this experiment [15], spinor $\Psi(x, z, t)$ being described by equation (3), the duration turns out to be about $T_m = 3.10^{-4}$s, which is of several order of magnitude larger than time related to the decoherence.

There is thus a large difference with Copenhagen interpretation where the qubit (1) is constant during all measurement, then passes at the time of the impact to $|0\rangle$ with one probability $\cos^2\frac{\theta_0}{2}$ and with $|1\rangle$ with a probability $\sin^2\frac{\theta_0}{2}$.

5 Realization of NMR quantum computer

NMR technique, developed by Chuang et al [4], does not use quantum objects individually, but uses instead a set of more than 10^8 active molecules diluted in a solvent. Measurement is thus a measurable and reliable collective signal. Let us recall that the qubits are represented by the spins of the atomic nuclei contained in the molecule and that the construction of entangled states is done by sequences of radio frequency impulses.

However, the fact that in this technique the signal decreases of a factor 2 for each additional qubit implies that the computer must double its size for each additional qubit. It is what it would be necessary to do for a traditional computer. This implementation of a quantum computer thus falls down in a traditional behavior and thus is no longer interesting.

It seems [**?**, 9] that the principal difficulty comes from the operations of initialization and measurement of the final states.

However, in the Broglie-Bohm interpretation, each wave function, and thus each qubit, must be duplicated physically because one needs at least two particles to represent the quantum system as we saw in the Stern-Gerlach experiment. For n qubits, one thus needs 2^n particles. The results of the NMR being statistical, there is then a direct explanation for the division by 2 of the intensity of the signal for each additional qubit, and thus for the failure of the computer of Chuang.

The same impossibility reasoning applies, in the Broglie-Bohm interpretation, to the other quantum computers. For the quantum computers with trapped ions [16], the superposed states of the trapped ions are similar to the superimposed states of the spin. Let us consider for example the case of trapped ions represented by Rydberg atoms. The qubit is non-deterministic and theoretically takes into account the two states, while the quantum system is, in the Broglie-Bohm interpretation, deterministic. The physical result will depend on the position of the electron in its wave function and will take into account only one of the two states.

This impossibility will be seen in Copenhagen interpretation like an effect of the decoherence.

Let us note that entanglement does not play a part in our refutation of the feasibility of the quantum computer. It is not the entanglement which is involved in the Broglie-Bohm interpretation, but the indeterminism of the wave function.

6 Conclusion

In the Broglie-Bohm interpretation, the failure of the quantum computer of Chuang et al. is explained in a very simple way: one needs two particles to represent the two physical states of the qubit. It seems that the same conclusion applies to the other quantum computers.

However, unlike what is often said, the Broglie-Bohm interpretation is not contradicted today by any experiment. In particular, it is a nonlocal interpretation and thus is not invalidated, as it is often believed, by the Bell results [17] and the Aspect experiments [18]. Moreover, about this Broglie-Bohm interpretation with the pilot wave, John Bell wonders [19]:

" Why the image of the pilot wave is ignored in textbooks? Shouldn't it be taught, not as the single solution, but like an antidote with the dominant self-satisfaction? To show that blur, subjectivity, and the indeterminism aren't imposed to us by the experimental facts, but come from a deliberated theoretical choice? "

The current failure of NMR quantum computer seems to be an argument against the Copenhagen interpretation. The realization of a quantum computer would be, on the other hand, a refutation of the interpretation of Broglie-Bohm.

Finally, the realization of a quantum computer gives today an industrial interest to the interpretation of quantum mechanics.

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