A new approach to a unified theory
Electric charges of fermions

G.C. Marques[a], D. Spehler[b]

[a]Instituto de Física, Universidade de São Paulo,
C.P. 66318, 05315-970 São Paulo, SP, BRAZIL
[b]Institut de Recherches Subatomiques, in2p3/CNRS
and Université de Strasbourg, UdS, France

ABSTRACT. We propose an unified model of the electroweak and strong interactions that is based on underlying $SU(3)$ and $SU(2)$ symmetries of the fundamental interactions. Breakdown of $SU(2)$ and parity symmetries is acomplished by means of a proper choice of bosonic field variables. In this paper we describe fermionic interactions. This framework allows us to understand the differences and common features of the interactions of leptons and quarks based on the symmetries of the Lagrangian. This model gives an account of the observed pattern of electric charges of fermions.

The model of S. Weinberg, S. Glashow and A. Salam[1,2] is the most successful theory in providing the unification of the electromagnetic and weak interactions. The underlying symmetries of the electroweak interactions, within the GWS theory, are the $SU(2)\times U(1)$ gauge symmetries. Differences in the electric charges of fermions and in the masses of the particles indicate that this symmetry is broken.

Besides breaking internal symmetries, the weak interactions break also discrete symmetries. Parity violation is taken into account, in the Standard Model, by assigning chiral components of fermionic fields to different representations of the $SU(2)$ group.

As far as the interaction of quarks is concerned, the idea of treating chiral components of a spinor field as different dynamical variables (under $SU(2)$ transformations) leads, however, to a not entirely satisfactory situation from the symmetry point of view. This is so because in the theory of the strong interactions (QCD), the dynamical variables are the quark spinor fields themselves. The spinor fields are written, however, as a sum over two variables having, in the GWS theory, different transformation properties under the $SU(2)$ group. Since the strong interactions do not violate $SU(2)$ symmetry the theory should, somehow, incorporate this feature. That is, the Lagrangian of the strong interactions should be also invariant under $SU(2)$ symmetry.

To treat chiral components as independent field variables requires two different interactions (and coupling constants). By using the coupling constants g and g' and the mixings of fields, one gets the charges of leptons. In order to take into account differences in the interactions of quarks and leptons one makes use of couplings that depend on quantum numbers assigned to fermions (the weak hypercharges). Since we need two different types of hypercharges (hypercharges left, for left-handed components, and hypercharges right), one increases the number of parameters of the model.

In spite of the great successes of theories based on the $SU(3) \times SU(2) \times U(1)$ gauge group(1−3), there are several aspects of the physics of elementary particles that call for another formulation of the Standard Model. We would like to mention, among others, the problem of neutrino masses(4), the problem of generation mixing(5), the possible existence of Majorana neutrinos(6) and the still missing Higgs particles(7,8).

In this paper we propose a different approach to the description of the strong, electromagnetic and weak interactions. In our approach one uses only one type of fermionic variables. Fermions belong to doublet representations of $SU(2)$. No use is made of singlet representations. This ensures that QCD is also $SU(2)$ symmetric. We shall see that one can make this choice of variables by changing also the field variables describing the intermediate bosons. For this reason we shall employ, in our framework, a set of unusual variables in the description of bosons and fermions.

The use of new variables has important consequences. As we shall see, this allows us to introduce a new formulation of an unified theory of the three interactions. In this formulation we do not treat the strong and electroweak interactions as distinctive interactions from the very beginning. We start by writing the most general Lagrangian that is invariant under $SU(3)$ and $SU(2)$ symmetries. As a result, one needs only two interactions. One interaction for each symmetry. Since quarks have electroweak and strong interactions, this aspect of unification is relevant when one is dealing with quarks.

The reduction in the number or interactions implies a small numbers of fundamental constants. As will be shown here, one needs only one parameter, besides the electroweak coupling constant, for the description of the electroweak fermionic interactions. The electric charges of leptons and quarks (and other coupling constants) will be expressed in terms of this parameter.

Since, in our approach, there is no assignment of chiral components of fermionic fields to different representations of $SU(2)$, one needs to find a different way to break parity. The use of a rank 2 spinor field allows us to do so. We break parity through a proper choice of the components of this rank 2 spinor field.

Another consequence of using rank 2 spinor fields is that we cannot introduce, for these variables, the concept of gauge invariance. However, we can formulate the theory of the three interactions by using global internal symmetries, besides Lorentz and discrete (C,P,T and CPT) symmetries.

In this paper we shall define the variables, introduce new mechanisms for breakdown of SU(2) and parity symmetries, and propose an unified theory. As a specific application of our approach we shall deal with the understanding of differences in the interactions of leptons and quarks and the problem of charge assignment to fermions.

In the description of fermions we use, for each family F, two sets of fields. These fields will be denoted by $\ell^{(F)}_{i,\,a_1}$ and $q^{(F)}_{i,\,k,\,a_1}$ (the F-family doublet fields associated to leptons and quarks). We use no singlet fields for fermions. In our notation the index i is an SU(2) index running from 1 to 2; k is an $SU(3)$ index, so that k runs from 1 to 3 and a_1 is a spinor index running from 1 to 4. We define also fermionic variables $L^{(F)}$ and $Q^{(F)}$ whose components are written in terms of the components of $\ell^{(F)}_{i,\,a_1}$ and $q^{(F)}_{i,\,k,\,a_1}$ as:

$$L^{(F)}_1\left(Q^{(F)}_1\right) = \frac{(1-\gamma_5)}{2}\,\ell^{(F)}_1\left(\frac{(1-\gamma_5)}{2}\,q^{(F)}_1\right)\ ,\ L^{(F)}_2\left(Q^{(F)}_2\right) = \ell^{(F)}_2\left(q^{(F)}_2\right) \quad . \tag{1}$$

These field variables are useful when one is dealing with left-handed neutrinos.

The intermediate bosons will be described by two rank 2 spinor fields: $\psi^{(2)}_{a_1a_2,\,i_1i_2}$ (for $SU(2)$ symmetry) and $\psi^{(3)}_{a_1a_2,\,k_1k_2}$ (for $SU(3)$ symmetry). The use of rank 2 spinor fields introduces, however, a large number of field components (in the spinor indices). This requires that, in dealing with spinor fields, we treat the chiral components of these fields as independent dynamical variables of the theory[9–11].

By definition, the bosonic fields transform as direct products of fundamental representations of the groups. By using a basis of orthogonal matrices we can decompose any such a product as a sum over fields transforming as irreducible representations. For the Lorentz group, the basis is formed by a set of 16 γ-matrices[4]. By using this basis we decompose a rank 2 spinor field in terms of a sum over rank 0, rank 1 and rank 2 tensor fields. The chiral components are expressed in terms of these tensors as[10]:

$$\psi^{(2)}_{RR\,a_1a_2,\,i_1i_2}=\left(\frac{(1+\gamma^5)}{2}\,\sigma^{\mu\nu}C\right)_{a_1a_2}(\mathcal{F}_{\mu\nu})_{i_1i_2}+\left(\frac{(1+\gamma_5)}{2}\,C\right)_{a_1a_2}\left(\phi+\phi^A\right)_{i_1i_2}$$

$$\psi^{(2)}_{LL\,a_1a_2,\,i_1i_2}=\left(\frac{(1-\gamma^5)}{2}\,\sigma^{\mu\nu}C\right)_{a_1a_2}(\mathcal{F}_{\mu\nu})_{i_1i_2}+\left(\frac{(1-\gamma_5)}{2}\,C\right)_{a_1a_2}\left(\phi-\phi^A\right)_{i_1i_2}$$

$$\psi^{(2)}_{RL\,a_1a_2,\,i_1i_2}=\left(\frac{(1+\gamma^5)}{2}\,\gamma^{\mu}\,C\right)_{a_1a_2}\left(G^V_\mu+G^A_\mu\right)_{i_1i_2}$$

$$\psi^{(2)}_{LR\,a_1a_2,\,i_1i_2}=\left(\frac{(1-\gamma^5)}{2}\,\gamma^{\mu}\,C\right)_{a_1a_2}\left(G^V_\mu-G^A_\mu\right)_{i_1i_2}\quad . \tag{2}$$

In our approach all tensor fields are independent variables. The equations of motion satisfied by the chiral components lead to the usual relations between $\mathcal{F}_{\mu\nu}$ and derivatives of vector fields and, for internal symmetries, the fields themselves[11,12]. In this sense, gauge invariance can be derived[10,11]. The scalar fields give rise to gauge fixing terms in the Lagrangian[10].

One decomposes the fields G^V_μ and G^A_μ in terms of singlet fields ($a^{(0)}_\mu$ and $a^{(0)}_{5\mu}$) and triplet fields ($a^{(\ell)}_\mu$ and $a^{(\ell)}_{5\mu}$) by using the basis formed by the Pauli matrices $\sigma^{(\ell)}$ ($\ell=1,2,3$) and the unit matrix as:

$$\begin{aligned}\left(G^V_\mu\right)_{i_1i_2} &= -\tfrac{1}{2}a^{(0)}_\mu\,\delta_{i_1i_2}+a^{(\ell)}_\mu\,\sigma^{(\ell)}_{i_1i_2}\\ \left(G^A_\mu\right)_{i_1i_2} &= -\tfrac{1}{2}a^{(0)}_{5\mu}\,\delta_{i_1i_2}+a^{(\ell)}_{5\mu}\,\sigma^{(\ell)}_{i_1i_2}\end{aligned} \tag{3}$$

Since the strong interactions do not break parity, we shall use, in the case of $SU(3)$ symmetry, only a vector field matrix $G^{(3)}_{\mu\,k_1k_2}$. The analogue of decomposition (3) for the $SU(3)$ symmetry group is:

$$\left(G^{3}_{\mu}\right)_{k_1 k_2} = \frac{1}{3}\, A^{(0)}_{\mu}\, \delta_{k_1 k_2} + \sum_{j=1}^{8} \lambda^{(j)}_{k_1 k_2}\, A^{(j)}_{\mu} \,. \tag{4}$$

We call the attention to the fact that the trace of $G^{(3)}_{\mu\, k_1 k_2}$ is, by definition, a singlet field. The 1/3 factor in (4) ensures that $A^{(0)}_{\mu}$ is such a singlet field.

The most general, renormalizable and invariant Lagrangian (invariant under both $SU(3)$ and $SU(2)$ group transformations) describing the interaction of fermions with the intermediate bosons is:

$$\begin{aligned}\mathcal{L}^{\text{fermions}}_{I} = & g\, \overline{L}^{(F)} \left(\psi^{(2)}_{RL} + \psi^{(2)}_{LR}\right) \left(C^{-1}\right) L^{(F)} \\ & + g\, \overline{Q}^{(F)} \left(\psi^{(2)}_{RL} + \psi^{(2)}_{LR}\right) \left(C^{-1}\right) Q^{(F)} \\ & + g_3\, \overline{q}^{(F)} \left(\psi^{(3)}_{RL} + \psi^{(3)}_{LR}\right) \left(C^{-1}\right) q^{(F)},\end{aligned} \tag{5}$$

where g is the electroweak $SU(2)$ coupling constant and g_3 is the $SU(3)$ interaction coupling constant.

From (5) it follows that there are only two fundamental fermionic interactions. The electromagnetic and weak interactions of leptons will emerge from Lagrangian (5) after the breakdown of symmetries. Differences in the interactions of leptons and quarks will be described by using the last term of the rigth hand side of (5).

By using expression (2) for the ψ_{RL} and ψ_{LR} fields, and by using decompositions (3) and (4), we write the fermionic Lagrangian defined in (5) as:

$$\begin{aligned}\mathcal{L}^{\text{fermions}}_{I} = & g\, \overline{L}^{(F)} \gamma^{\mu} \left(G^{V}_{\mu} - \gamma^{5}\, G^{A}_{\mu}\right) L^{(F)} + g\, \overline{Q}^{(F)} \gamma^{\mu} \left(G^{V}_{\mu} - \gamma^{5}\, G^{A}_{\mu}\right) Q^{(F)} \\ & + g_3\, \overline{q}^{(F)} \gamma^{\mu} \left(\frac{A^{(0)}_{\mu}}{3} + A^{(\ell)}_{\mu}\, \lambda^{\ell} \right) q^{(F)}.\end{aligned} \tag{6}$$

By writting the Lagrangian interaction (6) in terms of the fields $L^{(F)}$ and $Q^{(F)}$, we break parity. This is due to the fact that, from expression (1), we are coupling chiral components of some fermionic fields.

Depending on some fermions being left-handed or Dirac particles the Lagrangian (6) will assume different forms. This allows us to predict a difference in the interactions of quarks and leptons that is associated to the handedness of the neutrinos. In fact, if G_μ is the 2×2 matrix associated to the interaction of left-handed neutrinos, then the matrix associated to Dirac particles is a $\mathcal{G}_\mu$ matrix such that the following identity holds true:

$$\overline{Q}^{(F)} \gamma^\mu G_\mu Q^{(F)} = \overline{q}^{(F)} \gamma^\mu \mathcal{G}_\mu q^{(F)} . \tag{7}$$

From (1) and (7), it follows that the matrix $\mathcal{G}_\mu$ can be written as:

$$\mathcal{G}_\mu = \mathcal{P} G_\mu \mathcal{P} . \tag{8}$$

where the matrix $\mathcal{P}_{i_1 i_2, a_1 a_2}$ in (8) is , from (1), given by:

$$\mathcal{P}_{i_1 i_2, a_1 a_2} = \begin{pmatrix} \frac{1}{2} \left(1 - \gamma^5\right)_{a_1 a_2} & 0 \\ 0 & 1_{a_1 a_2} \end{pmatrix} . \tag{9}$$

One can use identity (7) in order to write the Lagrangian exclusively in terms of the $\ell^{(F)}_{i, a_1}$ and $q^{(F)}$ fields.

Since the $A^{(0)}_\mu$ field is necessarily a singlet under both ($SU(2)$ and $SU(3)$) transformations, we write

$$A^{(0)}_\mu = a^{(0)}_\mu + a^{(0)}_{5\mu} . \tag{10}$$

The above identity implies, from (6), that the electromagnetic and weak interactions of quarks is a result of two contributions. One contribution comes from the $SU(3)$ symmetric term whereas the other contribution comes from the broken $SU(2)$ symmetry term of the interaction Lagrangian .

Considering the identity of fields given by (10), our unified model can be made simpler and more predictive by noting that at the classical (zero loop) level one can write the following simple relation:

$$g_3 = 2 g . \tag{11}$$

This relation do not holds true, however, when one takes into account radiative corrections since, in this case, the coupling constants become energy scale dependent. The way the running coupling constants varies with the energy scale depends on how the various fields are coupled. Classical coupling constants (like the fine structure constant of QED) correspond to low energy limits of running coupling constants. In order to check if expression (11) is valid at the tree level we have to check if such a result holds true for the running couplings at some low energy scale. The experimental value[(13)] for g is 0.6529. The running coupling constant of QCD at the M_Z mass scale is[(13)] 1.22. This implies $\frac{g}{g_3(M_Z)} = 0.535$. From this result it follows that, due to the assymptotic freedom property of QCD, the tree level relation (11) is indeed valid at a low energy scale (in fact, lower than the M_Z mass scale).

Since the last term of (6) describes the strong interactions of quarks and since quarks are Dirac particles, the Lagrangian for the electroweak interactions is:

$$\mathcal{L}^{\text{electroweak}} = g\left(\overline{L}^{(F)}\left(\gamma^\mu G_\mu\right) L^{(F)} + \overline{q}^{(F)}\left(\gamma^\mu \mathcal{G}_\mu\right) q^{(F)}\right) + \frac{2}{3}\, g\, \overline{q}^{(F)} \gamma^\mu \left(a_\mu^{(0)} + a_{5\mu}^{(0)}\right) q^{(F)}. \tag{12}$$

The above electroweak Lagrangian is, until this point, $SU(2)$ symmetric. We shall break this symmetry by expressing a sum of singlet fields and the third component of the triplet field as mixings of the neutral electroweak fields A_μ and Z_μ . Independently of the details of such relations one can predict that the electrowek interations of quarks and leptons are different. Symmetry can be evoked, however, in order to explain the differences. Quarks differ from leptons because they interact exhibiting an extra $SU(3)$ symmetry. As a result of (12), one can predict that quarks and leptons have different electric charges. Furthermore, it follows also from expression (12) that electric charge differences of fermions belonging to a doublet will be the same, for any doublet.

Let us turn now to breakdown of parity and $SU(2)$ symmetries in the electroweak interactions. We break parity in the charged sector by choosing:

$$a_{5\mu}^{(1)} = a_\mu^{(1)}, \; a_{5\mu}^{(2)} = a_\mu^{(2)} \; . \tag{13}$$

The above choices are needed in order to reproduce the V-A theory of the weak interactions[(14)]. In order to break parity in the neutral sector we choose:

$$a^{(0)}_{5\mu}=\frac{1}{2}\,\sqrt{1+\xi 2}\;Z_{\mu}. \tag{14}$$

Where ξ is a fundamental parameter of the electroweak model ($\xi = \tan\theta_w$). This is the only free parameter to be introduced in our model. This parameter is an asymmetry parameter. It gives a measure of an asymmetry between the neutral and charged sectors resulting from different parity breaking interaction strengths. For $\xi = 1$ the neutral and charged sectors break parity with equal strengths.

Let us consider now the breakdown of $SU(2)$ symmetry. In the GWS theory this symmetry is broken through the use of two distinct mechanisms(2): spontaneous symmetry breakdown and the electroweak mixings of fields. Weinberg introduced, in his pioneering work(2), two mixings involving linear combinations of the electromagnetic and the neutral weak bosons. Since these mixings imply that a singlet field and the third component of a triplet field transform in a way that is different from the one we would expect if the symmetries were realized in nature, these mixings break global $SU(2)$ symmetry. This is a very subtle aspect of symmetry breakdown of the electroweak interactions.

In this paper we shall discuss only the electroweak mixings of fields leaving the discussion of spontaneous breakdown of symmetry, via the Higgs mechanism, for a future publication. We break $SU(2)$ symmetry, in close analogy with Weinberg's approach(2), by introducing the following ξ dependent electroweak mixings:

$$\begin{aligned} a^{(0)}_{\mu}+a^{(0)}_{5\mu}&=\frac{\xi}{\sqrt{1+\xi 2}}\;(A_{\mu}-\xi\,Z_{\mu}) \\ a^{(3)}_{\mu}+a^{(3)}_{5\mu}&=\frac{1}{\sqrt{1+\xi 2}}\;(\xi\,A_{\mu}+Z_{\mu}) \end{aligned} \tag{15}$$

It follows from the first definition in eq. (15), that the sum over the singlet fields introduced by us play the role of the $U(1)$ vector field in the GWS model(2).

One writes the W_{μ} and W^{+}_{μ} fields as the linear combinations:

$$W_{\mu}=\sqrt{2}\,\left\{a^{(1)}_{\mu}+i\;a^{(2)}_{\mu}\right\},\;W^{+}_{\mu}=\sqrt{2}\,\left\{a^{(1)}_{\mu}-i\;a^{(2)}_{\mu}\right\}\;. \tag{16}$$

By taking $\xi = \tan\theta_w$, one can see, from (15) and (16), that the choice:

$$a^{(3)}_{5\mu}=0\;, \tag{17}$$

permit us to identify our vector triplet fields with the triplet fields of Weinberg[(2)].

By substituting expressions (13)-(17) in (12), we get the electroweak interaction Lagrangian for fermions belonging to the F-family. For the first family one writes this Lagrangian as:

$$\mathcal{L}^{\text{electroweak}} = g\,(\,\overline{\nu}_{eL}\,,\,\overline{e}\,)\,\gamma^{\mu}\left\{\begin{pmatrix}\sqrt{1+\xi^2}\,Z_\mu & 0\\ 0 & \dfrac{-\,\xi\,A_\mu}{\sqrt{1+\xi^2}}+\dfrac{\xi^2}{\sqrt{1+\xi^2}}\,Z_\mu\end{pmatrix}\right.$$

$$\left.+\,\frac{1-\gamma^5}{4}\begin{pmatrix}-\sqrt{1+\xi^2}\,Z_\mu & \sqrt{2}\,W^+_\mu\\ \sqrt{2}\,W_\mu & -\sqrt{1+\xi^2}\,Z_\mu\end{pmatrix}\right\}\begin{pmatrix}\nu_{eL}\\ e\end{pmatrix}$$

$$+\,g\,(\,\overline{u}\,,\,\overline{d}\,)\,\gamma^{\mu}\left\{\frac{\xi}{\sqrt{1+\xi^2}}\begin{pmatrix}\dfrac{2}{3}\,(A_\mu-\xi\,Z_\mu) & 0\\ 0 & -\dfrac{1}{3}\,(A_\mu-\xi\,Z_\mu)\end{pmatrix}\right.$$

$$\left.+\,\frac{(1-\gamma_5)}{4}\begin{pmatrix}\sqrt{1+\xi^2}\,Z_\mu & \sqrt{2}\,W^+_\mu\\ \sqrt{2}\,W_\mu & -\sqrt{1+\xi^2}\,Z_\mu\end{pmatrix}\right\}\begin{pmatrix}u\\ d\end{pmatrix}.$$

By substituting $\xi = \tan\theta_w$ in the above Lagrangian we get the usual expressions for the electroweak interaction of fermions in terms of θ_w .

The conclusion is that, by considering $SU(2)$ and $SU(3)$ as the symmetries of the fundamental interactions one can provide a model that accounts for the different charges of leptons and quarks and for the differences in their interactions. In our model quarks are different from leptons due to the $SU(3)$ symmetry that is inherent to quark interactions. As pointed out here, their interactions differ only by a singlet piece (under both symmetry groups) in the interaction Lagrangian. As a result, relying on symmetries of the fundamental interactions we get the observed pattern of electric charges of Leptons and Quarks.

Our approach provides for an unification of the three interactions. That is, the strong, weak and electromagnetic interactions are described by an $SU(3)\times SU(2)$ invariant Lagrangian. Although we have dealt in this paper only with fermionic interactions, it is also possible to treat bosonic interactions by using rank 2 spinor fields[(12)]. A more extended version of this paper and how we deal with the generation mixing problem will be presented elsewhere[15]. Bosonic interactions will be described, by using the set of variables here proposed, in a future publication[16].

We would like to emphasize that we can foresee various scenarios for endowing particles with masses through the use of the Higgs mechanism[8]. For the masses of the intermediate bosons it is enough to couple Higgs fields assigned to a doublet representation of $SU(2)$. Masses of fermions require, however, Higgs fields transforming as a product of two doublet representations of the $SU(2)$ group.

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