Revisiting Quantum Mechanics in the Light of Gravity

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ABSTRACT. The author's nonrelativistic model On the Gravitational Coupling Constant of Elementary Particles [7] is revisited, especially with regard to the broken scaling-invariance and the breakdown of the superposition principle in the ensuing nonlinear Schrödinger equation. In the light of Weinberg's paper on Testing Quantum Mechanics [9], standard homogeneity and linearity of quantum mechanics are secured with sufficient precision at the present cosmological epoch. The meaning of the so-called Planck-scale is discussed in various contexts. Some computational work on this subject is briefly addressed.

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1 Introduction

1. Quantum Theory is based on complex numbers in a linear vector space of at most infinite dimensions.

2. The Theory of Gravitation, formulated within the framework of General Relativity, is based on real numbers in a nonlinear 3+1-dimensional NonEuclidean space-time.

Ad 1) Complex numbers are necessary for the probabilistic interpretation of that theory. In conjunction with linearity one then understands interference of superposed quantum states.
Ad 2) Spacial distances and time-intervals are expressed in terms of real

numbers. Since relativistic gravity acts back on itself (equivalence of mass and energy), the correct theory is nonlinear: Starting off with Special Relativity, measuring rods and clocks become distorted to the extent that space-time becomes effectively NonEuclidean [1].
Ad 1) and Ad 2) In quantum theory the fundamental linear Schrödinger equation has a nonrelativistic limit. There is also a nonrelativistic limit in the theory of gravitation, called the Newtonian approximation which happens to be also linear. However, in what follows in this essay, there is no way to generate a unified linear scheme!

One reason for unifying quantum physics with gravity is to understand the interrelationship between fundamental constants: for example, there are

$$\begin{aligned} \hbar &= \text{ Planck's constant divided by } 2\pi \\ G_0 &= \text{ Newton's constant of gravitation} \\ c &= \text{ speed of light} \end{aligned}$$

From these constants we derive a fundamental mass-unit (called the Planck-mass):

$$\sqrt{\frac{\hbar c}{G_0}} \approx 10^{-5} \text{ gram },$$

which weighs somewhat less than a grownup flea [2].

Penrose has put forward a proposal as to the meaning of the Planck-mass, with G_0 fixed: This fundamental mass might come into play when superpositions of quantum states undergo a gravitationally induced collapse towards one single state [2], [3], [16].

In recent years string-theory has come up with intriguing ideas about the the Planck-mass: In models with large (more than 3+1) extra spacial dimensions it is believed that the corresponding Planckian energy-scale ($\approx 10^{16}$ TeV) can effectively be lowered to the TeV-regime, which is now in close reach for accelerators in collision experiments: The actual reason for this is, so the claim, that gravity then freely propagates outside so-called three-branes which are produced in these experiments, along with mini black holes (see [4]). In this case standard General Relativity is supposedly reduced to the low energy-limit of a high-dimensional quantum gravity theory.

On the other hand, in 3+1-dimensional quantum gravity schemes (perhaps along the lines of Loop gravity and Twistor theory, or a combination of both) it is hoped for that the Planck-scale offers a cut-off for divergent sums or integrals: as a credo, this should ultimately lead to a finite theory with a realistic mass-scale (see [16] and the references therein).

I have a different conjecture to this monstrous mass-unit (in reference to the actual mass-scale of elementary particles, roughly 10^{-24} gram): The gravitational constant is not fundamental, it varies with the epoch [5], [6]. So, the Planck-mass has no universal meaning!

By comparison, my own ideas are rather simple and straightforward, although there is a certain clash of ordinary linear quantum physics with gravity (already seen in the nonrelativistic sector, first developed in [7]): Let ϵ be a stationary energy state of an isolated quantum object, localized within a three-dimensional region of radius a, then one naively expects

$$\epsilon \approx \frac{\hbar^2}{2Ma^2}, \quad \frac{\partial \epsilon}{\partial a} = 0, \tag{1}$$

where M is the inertial mass; however, there is no such stationary value, unless $a \to \infty$.

Help comes from universal gravity: Since the mass M is also gravitational, there will be a self-energy contribution within that region, s.t.

$$\epsilon \approx \frac{\hbar^2}{2Ma^2} - \frac{M^2 G_0}{a}, \quad \frac{\partial \epsilon}{\partial a} = 0. \tag{2}$$

From this we see at once that for stable stationary states, $\epsilon < 0$:

$$\frac{M^3 G_0}{\hbar^2} \approx \frac{1}{a}. \tag{3}$$

For isolated elementary particle masses the region of localization is pretty large, so cosmology comes into play: For example, if one puts for $\max(a) \approx 10^{28}$ cm (roughly the Hubble-radius at the present epoch), a tenth of the proton/neutron-mass, possibly a critical fundamental mass, emerges, see [5], [8]; at any rate, this mass is of a desirable order of magnitude for elementary particles (close to the Pion-mass, compare with [17]), and G_0 would thus depend on the epoch!

I am now convinced, contrary to what was said in the original paper [7], that low mass-particles, electrons or neutrinos, etc., do not fit into this nonrelativistic scheme. Incidentally, if the region of localization a for free objects were also bounded from below, s.t. possibly $\min(a) \approx \hbar/Mc$, then, in nonrelativistic approximation, the Planck-mass would be an upper bound for elementary particles! Note, this kind of argument here is tantamount with stating that in Newtonian approximation the absolute value of the gravitational self-energy must be less than Mc^2.

2 A Nonlinear Quantum Action

In nonrelativistic classical mechanics, a self-interacting system with gravitational mass $\int_\Omega \rho(x)\, d^3x$ in a 3-dimensional region Ω has the self-energy proportional to

$$-\frac{1}{2}\int_\Omega \int_\Omega \rho(x)\,\rho(y)\,K(x-y)\,d^3x d^3y, \tag{4}$$

where $K(x-y)$ is a positive symmetric twopoint-interaction kernel, and ρ is the mass density. For example, in Einstein-Newtonian gravity:

$$K(x) = \frac{1}{4\pi\,|x|}, \tag{5}$$

i.e. strictly Coulomb.

Within an extended quantum theory, which should encompass gravity in nonrelativistic approxi-mation, we then postulate a nonlinear functional over a complex vector field $|\phi\rangle$ with a Lagrange-parameter e:

$$I_\phi = \left\langle \phi \left| \left(T - \frac{1}{2}\left(\phi\,|K|\,\phi\right)\right) \right| \phi \right\rangle - e\left(\langle\phi\,|\phi\rangle - \lambda^2\right) \tag{6}$$

where T denotes the kinetic energy. By minimizing we thus get:

$$\frac{\partial I_\phi}{\partial\langle\phi|} = (H_\phi - e)\,|\phi\rangle = 0, \tag{7}$$

$$\langle\phi|\phi\rangle = \lambda^2, \tag{8}$$

where the Hamiltonian, depending on the state ϕ, is given by $H_\phi = T - \langle\phi|K\,|\phi\rangle$, with λ^2 denoting the gravitational coupling constant.

Note: Strictly speaking, if λ^2 was zero (no gravity, see remark 2), then $\forall\phi : |\phi\rangle \equiv 0$!

Remark 1: Note that, (compare with [8], [9])

1. this model is not homogeneous, i.e. there is no scaling-invariance under the transformation $|\phi\rangle \mapsto \alpha\,|\phi\rangle$, where a is some arbitrary complex number;

2. there is no linear superposition of quantum states.

3 A Nonlinear Schrödinger Representation

For the Hamiltonian in the extended Schrödinger picture we should have

$$H_\phi = -\Delta - \int_{\mathbb{R}^3} |\phi\,(y)|^2 K\,(x-y)\,d^3y \tag{9}$$

where Δ is the Laplacian, and $\phi\,(x)$ being the Schrödinger amplitude, s.t. the overall gravitational self-energy in $\mathbb{R}^3$ is given by:

$$-\frac{1}{2}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} |\phi\,(x)|^2 |\phi\,(y)|^2 K\,(x-y)\,d^3x d^3y, \tag{10}$$

in correspondence to the above classical expression, eq (4). The nonlinear Schrödinger eigenvalue equation then reads [10]:

$$[-\Delta - V_\phi\,(x)]\,\phi\,(x) = e\phi\,(x)\,, \tag{11}$$

$$\int_{\mathbb{R}^3} |\phi\,(y)|^2 K\,(x-y)\,d^3y = V_\phi\,(x)\,, \tag{12}$$

$$\int_{\mathbb{R}^3} |\phi\,(x)|^2 d^3x = \lambda^2. \tag{13}$$

Note that the Lagrange-parameter e is synonymous with the eigenvalue-parameter of the ensuing Schrödinger equation.

Remark 2: After little thought, the above elementary estimate on $\epsilon < 0$ leads to

$$\lambda^2 = \frac{8\pi G_0 M^3}{\hbar^2}. \tag{14}$$

Remark 3: Eqs (11) and (12) are designed for stationary neutral matter fields with nonlocal self-interaction in the one particle Schrödinger-picture, as is the case with pure gravity due to the gravitational self-coupling given by eqs (13) and (14), discarding external influences. Incidentally, these equations would also follow from Einstein's General Theory of Relativity in the nonrelativistic sector, with the interaction kernel given by eq (5). Although there is no probabilistic foundation for such equations in the absence of both linearity and homogeneity, one may still assume a statistical state function interpretation in nonrelativistic approximation.

Note that computational work has been done, for example, some time ago by Synge [11], and more recently by Adomian [12], with the simplified assumption that K behaves like a Yukawa-kernel. For K strictly Couloumb, see [18]; for general methods of integration, also see [13].

A fundamental theorem: Let the kernel K be a bounded operator, with $e < 0$: then λ^2 is bounded from below; this was proved rigorously for certain finite norms of K in $L^p\left(\mathbb{R}^3\right)$ [10]. There is thus a lower bound on the mass of gravitating particles, in agreement with the original conjecture [7].

Interpretation: If this mass-bound is fundamental, and of the right order of magnitude in the introductory sense, with the Hubble-radius being the effective range of K: then the gravitational constant depends on the cosmological epoch, compare with [5]; thus G_0 drops off as the universe expands (also see the Appendix)!

Note, with G_0 strictly decreasing, the evolution of the universe could not be cyclical, which might have some bearing on the second law of thermodynamics [14]. At any rate, the standard Einstein-Friedmann cosmology is at stake!

4 Transition to Ordinary Linear Quantum Physics

One may wonder why ordinary quantum mechanics is extremly applicable, with an unprecedented degree of accuracy [9]:
Since $G_0 \neq 0$, we simply transform $\phi \mapsto \lambda\phi$, then by remark 2 we get the following set of equations, also introducing an external potential $U\left(x\right)$:

$$\left[-\frac{\hbar^2}{2M}\Delta + U\left(x\right) - V_\phi\left(x\right)\right]\phi\left(x\right) = \epsilon\phi\left(x\right), \tag{15}$$

$$4\pi M^2 G_0 \int_{\mathbb{R}^3} \left|\phi\left(y\right)\right|^2 K\left(x-y\right) d^3y = V_\phi\left(x\right), \tag{16}$$

$$\int_{\mathbb{R}^3} \left|\phi\left(x\right)\right|^2 d^3x = 1. \tag{17}$$

Note: For the time dependent scheme, referring to free objects, see the Appendix.

Now, scaling-invariance and linearity can be secured for all practical purposes, since in all tested applications we usually have:

$$|\langle\phi\,|U|\,\phi\rangle| \gg \langle\phi\,|V_\phi|\,\phi\rangle\,; \tag{18}$$

thus, at this stage the clash of quantum physics with gravity could not be overcome if this inequality should turn out to be wrong! Note that, in this (seemingly) valid linear approximation, statistical averaging can be done with sufficient precision in the usual manner, compare with [9], where the modulus of the wave function squared has the usual meaning of a probability density in the sense of Borns postulates.

However, given the hypothesis that gravity changes with the epoch, we cannnot be certain as to the validity of this inequality in the cosmological past. Here one is reminded of certain critical remarks by the late R.P. Feynman on the law of gravity [15].

Concluding Remark

The Clash of Quantum Physics with Gravity, i.e.

1. loss of homogeneity for the Schrödinger equation,
2. loss of linear superposition for the same scheme

is the prize one pays for, when one establishes a mass-standard of the right order of magnitude from the equivalence of the inertial and the gravitational mass of quantum objects. At the present cosmological epoch it is quite reassuring that for all practical purposes linearity holds sway because various external disturbances are likely to suppress the nonlinear gravitational self-field.

The present model is certainly a reasonable first step towards generating massive quantum objects within a more unifying relativistic scheme (possibly at the expense of general covariance [5]), but there are also highly esteemed models that claim to generate such objects without the mention of gravity: Note that, from a string-theoretical viewpoint, the production of mini black holes (referring to forthcoming LHC-experiments) is quite an acceptable proposal since gravity is very much the motor behind it; however, looking at the linear Schrödinger equation (obviously an approximate scheme), the mass that goes into it by hand is usually not a black hole. So, one would still not understand the mass of regular quantum objects, e.g. hadrons! At this point in time, regardless of the outcome at CERN, the presented mass-standard in this essay for elementary particles (close to the Pion-mass) is the best estimate from a nonlinear model in terms of universal constants!

Appendix

I present a time-dependent version in the Schrödinger picture, whereby $\phi(x) \mapsto \psi(x,t)$, which also works in the variation of G_0 with the epoch. For free objects we should have:

$$\left(-\frac{\hbar^2}{2M}\Delta - V_\psi\right)\psi = i\hbar\frac{\partial\psi}{\partial t}, \tag{19}$$

$$4\pi M^2 G_0 \int |\psi(y,t)|^2 K(x-y)\, d^3y = V_\psi, \tag{20}$$

$$\int |\psi(x,t)|^2 d^3x = 1. \tag{21}$$

The dimensionless formulation: Let $x \mapsto \mu^{-1}x$, and $t \mapsto \left(\mu^2\hbar/2M\right)^{-1}t$, where μ^{-1} is the effective range of K, bounded in $L^p\left(\mathbb{R}^3\right)$, scaling Coloumb-like: $K \mapsto \mu K$. With μ varying slowly as ψ evolves unitarily as a function of time, we obtain the following dimensionless scheme:

$$(-\Delta - V_\psi)\psi = i\frac{\partial\psi}{\partial t}, \tag{22}$$

$$\int |\psi(y,t)|^2 K(x-y)\, d^3y = V_\psi, \tag{23}$$

$$\int |\psi(x,t)|^2 d^3x = \frac{\lambda^2}{\mu}; \tag{24}$$

thus, according to remark 2, G_0 then varies proportional to μ: Since μ^{-1} is supposedly the Hubble-radius, the gravitational constant G_0 decreases in an expanding universe!

As to the initial value problem and other rather intricate questions associated with eqs (22) and (23), see [19], [20], [21]: The stability of certain solutions in unbounded domains has been discussed in [8], especially the stability of standing-waves for the strict attractive case whose amplitudes obey eq (24). Note that for a certain class of local nonlinear extensions stationary solutions would be unstable as shown in [19] and [21].

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