A new electromagnetism based on 4 photons : electric, magnetic, with spin 1 and spin 0

Part I: theory of light

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ABSTRACT. *The Introduction recalls two theories : a) Electrodynamics and Electromagnetism in the presently admitted for. b) The theory of the magnetic leptonic monopole due to the author. Then, the paper is devoted to a generalization of de Broglie's theories of photon. A second part will be devoted to the theory of the graviton. The theory is based on the de Broglie assumption, that a photon and a graviton are not elementary particles but* ***particles with respectively maximum spin 1 and spin 2,*** *resulting from the* ***fusion of two or four spin*** ½ ***Dirac particles****. Starting from the de Broglie equations without adding anything else, it will be proved that not only one but* ***four different photons*** *are automatically defined, which play different physical roles :*

1) At first, two spin 1 photons, defining an electromagnetic field :

a) An ***"electric photon",*** *defined by a* ***polar potential*** *: the classical Einstein photon, which acts on electrically charged particles and is emitted by them.*

b) A ***"magnetic photon",*** *defined by an* ***axial potential****, which acts on the magnetic monopoles and is emitted by them.*

2) Then two spin 0 fieldless photons :

a) A spin 0 photon defined by an ***axial potential without field*** *and associated with an "electric spin 1 photon" : both appear in the Aharonov-Bohm effect.*

b) A spin 0 photon defined by a ***polar potential without field*** *and associated with a "magnetic spin 1 photon" : it must appear in an " inverse " Aharonov-Bohm effect.*

1 Introduction.

Our scientific and industrial civilisation fundamentally rests on two discoveries dating from the beginning of the XX-th century : **electron and photon** that are the founding stones of quantum theory. The first of them is described by the **Dirac equation of spin ½ particles** and the second is linked to the **Mawxell equations**. But **the de Broglie photon equations, which find again the Maxwell equations** are a direct consequence of the Dirac equation.

The first fact (concerning electron) is wellknown, but the last (concerning photon) is less known. We shall come back to it and we intend to show something more, namely that the domain of the presently admitted electromagnetism may be strongly enlarged, especially in the direction of magnetism.

Which means that all our technical activity can be enlarged too in the future, by this new theory.

2 Brief recall of electrodynamics and generalization to magnetism. Electron and lepton magnetic monopole.

a) Dirac free equation in the γ form and equation of the electron :

$$\gamma_\mu \partial_\mu \psi + \frac{m_0 c}{\hbar}\psi = 0 \quad x_\mu = \{x_k, ict\}\,; \qquad \gamma_\mu = \left\{ \gamma_k = i\begin{pmatrix} 0 & s_k \\ -s_k & 0 \end{pmatrix}, \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \right\} \tag{1}$$

where s_k are the Pauli matrices. Further, we shall also use the α representation :

$$\frac{1}{c}\frac{\partial \psi}{\partial t} = \alpha_k \frac{\partial \psi}{\partial x_k} + i\frac{m_0 c}{2\hbar}\alpha_4 \psi \qquad \alpha_k = \begin{pmatrix} 0 & s_k \\ s_k & 0 \end{pmatrix}, \alpha_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{2}$$

The relativistic covariance of (1) is manifest because γ_μ and ∂_μ vary as quadrivectors and $\frac{m_0 c}{\hbar}$ is a space-time scalar. ψ is a spinor. Of course, the equivalent form (2) is relativistic too.

b) Dirac equation of the electron.

If m_0 is the electron mass, we shall introduce electromagnetism substituting ∂_μ by the following **covariant derivative**, where the electric charge

e is a scalar and A_μ is a **polar quadrivector** (the Lorentz potential) :

$$\partial_\mu \rightarrow \partial_\mu + i\frac{e}{\hbar c}A_\mu \tag{3}$$

The equation (1) remains covariant and becomes the **Dirac equation of the electron** :

$$\gamma_\mu \left(\partial_\mu + i\frac{e}{\hbar c}A_\mu\right)\psi + \frac{m_0 c}{\hbar}\psi = 0 \quad (\gamma \text{ representation}) \tag{4}$$

Equivalently, with $\left\{\frac{1}{\sqrt{2}}\left(\gamma_4+\gamma_5\right)\ \psi = \begin{pmatrix}\xi\\ \eta\end{pmatrix}\right\}$ we find :

$$\begin{aligned}\left[\frac{1}{c}\frac{\partial}{\partial t} - \mathbf{s}.\nabla - i\frac{e}{\hbar c}\left(V + \mathbf{s}.\mathbf{A}\right)\right]\xi + i\frac{m_0 c}{\hbar}\eta = 0\\ \left[\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{s}.\nabla - i\frac{e}{\hbar c}\left(V - \mathbf{s}.\mathbf{A}\right)\right]\eta + i\frac{m_0 c}{\hbar}\xi = 0\end{aligned} \tag{5}$$

(Weyl representation).

These equations include the essential properties which are at the basis of the electronics : radio, television, automation, computer science etc.

The potential $A_\mu = (\mathbf{A}, V)$ defines the electromagnetic field :

$$\mathbf{H} = rot\mathbf{A}; \quad \mathbf{E} = -\mathbf{grad}V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \tag{6}$$

c) Equation of a leptonic monopole (see Literature : Lochak)

Let us go back to the problem of **covariant derivative**. The formula (3) is more or less considered as universal, but this is wrong because, even remaining in the linear frame of the free Dirac equation (1), we can look for a more general derivative :

$$\partial_\mu \rightarrow \partial_\mu + q\,\Gamma\,B_\mu \tag{7}$$

where q is a charge, Γ a charge-operator and $B_\mu(x, y, x, z, t)$ a generalized potential, which are unknown at the moment but we know *a priori* some relativistic conditions, so that the following equation including the precedent formula remains relativistically invariant :

$$\gamma_\mu\left(\partial_\mu + q\,\Gamma\,B_\mu\right)\psi + \frac{m_0 c}{\hbar}\psi = 0 \tag{8}$$

If the theory is writtten in Clifford algebra (as is the Dirac one), the Γ operator is developped as :

$$\Gamma = \sum_{N=1}^{16} a_N \Gamma_N \, ; \, \Gamma_N = \left\{ I, \gamma_\mu, \gamma_{[\mu}\gamma_{\nu]}, \gamma_{[\lambda}\gamma_\mu\gamma_{\nu]}, \gamma_5 \right\} \tag{9}$$

Now, **the covariance needs that Γ commute in the same manner with all the** γ_μ, which happens only in two cases : α) If Γ **commutes** with all the γ_μ. There is only one possibility : $\Gamma = I$, which needs that B_μ must be a **polar vector** A_μ**,** thus a Lorentz potential, $q = e$ (electric charge), and we find the Dirac equation. β) The only second possibility is that Γ **anticommutes** with all the γ_μ, and the unique answer is $\Gamma = \gamma_1\gamma_2\gamma_3\gamma_4 = \gamma_5$. Now, $\gamma_\mu\gamma_5$ has not the variance of a polar vector, but of a **pseudovector**, so that the covariance will be saved only if B_μ is a **pseudovector** too, i.e. a **pseudopotential** ; so we find a new covariant derivative and it is the only possible one (the absence of i before B_μ is simply due to the fact that it is a pseudovector) :

$$\partial_\mu \rightarrow \partial_\mu - \frac{g}{\hbar c}\gamma_5 B_\mu \tag{10}$$

Because of the variances of $\gamma_\mu\gamma_5$ and B_μ, the mass term of the Dirac-like equation must disappear because $\gamma_\mu\gamma_5 B_\mu$ is not a scalar but a pseudoscalar. The new relativistic equation is (see *Literature Lochak*) :

$$\gamma_\mu \left(\partial_\mu - \frac{g}{\hbar c}\gamma_5 B_\mu \right) \Psi = 0 \tag{11}$$

It can be shown (see *Literature Lochak*) that this is the **equation of a leptonic**[1] **magnetic monopole**. The fields defined by the pseudo potential $B_\mu$are :

$$\mathbf{H} = \mathbf{grad} W + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \quad \mathbf{E} = rot\mathbf{B} \tag{12}$$

So, while the old electrodynamics found only one covariant derivative (3) and the equation of the electron (4), here, we find a second - **and only one** - covariant derivative (10) corresponding to the equation of the leptonic magnetic monopole (11).

Something must be added. Here, we shall speak only of massless monopoles, but the absence of mass is due to the hypothesis of linearity.

[1] " leptonic " because of the zero mass.

There is a theory with **nonlinear equations** (see Literature *Lochak*), but it is not the subject of the present paper.

Therefore we can see how the incompleteness of classical definitions implied the forgetting of a second side of electrodynamics : the magnetic side. So we have found magnetic monopoles with equations, which are confirmed by many experimental proofs which have led to new ideas and physical consequences, while other theories of monopoles had not any experimental proofs and physical consequences (and often, they do not describe a wave-particle object). We shall see that something analogous happens in electromagnetism.

3 De Broglie's theory of light

a) The equations.

There was a discrepancy at the beginning of wave mechanics : the foundations due to Louis de Broglie started from relativistic reasonings, while the Schrödinger equation was non relativistic. Even more shocking was the fact that the first brilliant success of the Schrödinger theory was in atomic physics while the first relativistic generalization - the Klein-Gordon equation - was largely wrong in atomic physics. And worse, the de Broglie theory was based on the photon hypothesis and thus on Maxwell theory of light, but the latter was inaccessible to the Schrödinger equation because it was non relativistic, and to the Klein-Gordon equation which was scalar and thus unable to describe the polarization of light.

The miracle was the Dirac equation, which was in accordance with the de Broglie results, which included spin and gave, at least for the lightest atom (hydrogen), far better results than the Schrödinger equation. In addition, the Dirac wave had a polarization which was not the one of light, but that gave immediately to de Broglie the hope of a generalization to optics. In addition, de Broglie was impressed by the fact that, among other tensors the Dirac equation defined an antisymmetric tensor of rank two, which was interpreted in an electromagnetic sense and looked like the Maxwell field tensor.

After several attempts, de Broglie came to the conclusion that a photon is not an elementary particle generalizing in some manner the Dirac particle, but a **composite particle** constituted by two Dirac particles, which was in accordance with a property that he had discovered several years before : the fact that a photon must have a spin 1 ($= 1/2 + 1/2$).

So, he started from two Dirac equations that he wrote in the Dirac α-representation (2) :

$$\frac{1}{c}\frac{\partial\psi}{\partial t} = \alpha_k \frac{\partial\psi}{\partial x_k} + i\frac{\mu_0 c}{2\hbar}\alpha_4\psi; \quad \frac{1}{c}\frac{\partial\varphi}{\partial t} = \alpha_k \frac{\partial\varphi}{\partial x_k} + i\frac{\mu_0 c}{2\hbar}\alpha_4\varphi \qquad (13)$$

and he tried to describe the photon as their center of mass. But this is impossible in the ordinary relativistic formalism in which there are no systems of particles. So, he described axiomatically the **fusion** of these two particles, describing a composite particle as a new wave function of 16 components $\phi = \{\psi_n \varphi_m\}$, $(n = 1,2,3,4;\ m = 1,2,3,4)$, obeying a single equation in a system of coordinates (t,x,y,z), identifying the coordinates (t_1,x_1,y_1,z_1) and (t_2,x_2,y_2,z_2) of the initial particles. The equation of the new composite particle is obtained from the equations (7) with a set of **fusion conditions** :

$$\frac{\partial\psi_n}{\partial t}\phi_m = \psi_n\frac{\partial\phi_m}{\partial t} = \frac{1}{2}\frac{\partial\left(\psi_n\phi_m\right)}{\partial t}; \quad \frac{\partial\psi_n}{\partial x_k}\phi_m = \psi_n\frac{\partial\phi_m}{\partial x_k} = \frac{1}{2}\frac{\partial\left(\psi_n\phi_m\right)}{\partial x_k} \qquad (14)$$

These conditions mean, for plane waves, that the composing particles have the same energy-impulse, but de Broglie put the condition (14) as a general **postulate** and he obtained his **photon equations :**

$$\frac{1}{c}\frac{\partial\phi}{\partial t} = a_k\frac{\partial\phi}{\partial x_k} + i\frac{\mu_0 c}{\hbar}a_4\phi; \quad \frac{1}{c}\frac{\partial\phi}{\partial t} = b_k\frac{\partial\phi}{\partial x_k} + i\frac{\mu_0 c}{\hbar}b_4\phi \quad (\phi = \psi\,\varphi) \quad (15)$$

The matrices a and b are defined as:

$$\begin{aligned} a_r &= \alpha_r \times I, \quad (a_r)_{ik,\,lm} = (\alpha_r)_{il}\,\delta_{km} \\ b_r &= I \times \alpha_r, \quad (b_r)_{ik,\,lm} = (-1)^{r+1}(\alpha_r)_{km}\,\delta_{il} \end{aligned} \quad (r = 1,2,3,4) \qquad (16)$$

a and b commute and they separately verify the Dirac anti-commutation relations :

$$a_r a_s + a_s a_r = 2\delta_{rs}; \quad b_r b_s + b_s b_r = 2\delta_{rs}; \quad a_r b_s - b_s a_r = 0 \qquad (17)$$

owing to which, it is easy to prove that in (15) the components of ϕ obey the Klein-Gordon equation.

4 The photon spin.

We give here only a brief account (see : *Broglie 4*). At first a nonsymmetric energy-momentum tensor $T_{\mu\nu}$ is defined, according to the equations

(15), which gives an angular momentum m_{jk}:

$$m_{jk} = -\frac{i}{c}\left(x_j T_{4k} - x_k T_{4j}\right); \quad j,k = 1,2,3 \tag{18}$$

Like in the Dirac theory, the momentum m_{jk} is not conservative. The conservative quantity is :

$$\begin{aligned} &m'_{jk} = m_{jk} + S_{jk}; \\ &\text{with:} \quad S_{jk} = i\hbar\frac{b_4 a_j a_k + a_4 a_j b_k}{2}; \quad j,k = 1,2,3 \end{aligned} \tag{19}$$

$$s_j = \varepsilon_{jlk} S_{lk} \tag{20}$$

is a pseudo-vector in R^3.

The set of matrices s_j is completed by a fourth matrix s_4, equal to :

$$s_4 = c\hbar\frac{b_4 a_1 a_2 a_3 + a_4 b_1 b_2 b_3}{2} \tag{21}$$

The operators s_k verify the commutation rules of Pauli-Dirac with the eigenvalues –1, 0, +1. We have thus a particle with the maximum spin 1.

5 Introduction of a square matrix wave function.

Now, we introduce the relativistic coordinates $x_k = (x, y, z)$, $x_4 = ict$ and $\gamma_\mu$matrices :

$$\begin{aligned} &\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}; \quad \mu,\nu = 1,2,3,4; \\ &\gamma_k = i\alpha_4\alpha_k; \quad \gamma_4 = \alpha_4; \quad \gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 \end{aligned} \tag{22}$$

Owing to (16) and multiplying (15) by $i\gamma_4$, **we find a new system, which is not written in terms of a 16 lines column wave function ϕ but of a square 4×4 matrix wave function** ψ :

$$\partial_\mu\gamma_\mu\psi - \frac{\mu_0 c}{\hbar}\psi = 0; \quad \partial_\mu\psi\tilde{\gamma}_\mu - \frac{\mu_0 c}{\hbar}\psi = 0 \quad (\mu = 1,2,3,4;\ \tilde{\gamma}_\mu = \gamma_\mu\ \mathit{transp.}) \tag{23}$$

The transposed matrices $\tilde{\gamma}$ are easily eliminated because : if two arbitrary sets of Dirac matrices γ_μ and $\tilde{\gamma}_\mu$, satisfy the anti-commutation rules (22), **there are two and only two non singular matrices** Λ **and** Γ**, such that (***Pauli 1*) :

$$\tilde{\gamma}_\mu = \Lambda\gamma_\mu\Lambda^{-1};\ \tilde{\gamma}_\mu = -\Gamma\gamma_\mu\Gamma^{-1};\ \Lambda = \Gamma\gamma_5; \quad \mu = 1,2,3,4 \tag{24}$$

γ_5 is given in (22). Of course (24) is true also if the matrices denoted $\tilde{\gamma}_\mu$ are the transposed matrices of γ_μ. If the γ_μ are as in (22), a solution for Λ and Γ is :

$$\Gamma = -i\gamma_2\gamma_4; \quad \Lambda = \Gamma\gamma_5 = -i\gamma_3\gamma_1 \tag{25}$$

We see on (24) and (25) that, if we have a Λ matrix**, there is one and only one " symmetrically " associated Γ matrix and thus only one possible pair ; we shall prove that it is the correspondence between electricity and magnetism.** We shall chose the case (25), which is not important in itself : the important fact is the existence of a pair.

The Λ matrix was given in (*Pauli 1*), and the Γ matrix was introduced by de Broglie to eliminate $\tilde{\gamma}_\mu$ in (23). Indeed, introducing Γ into (23), we find the system given by de Broglie, Tonnelat and Pétiau *(Broglie 5 Ch. VII):*

$$\partial_\mu\gamma_\mu\left(\psi\Gamma\right) - \frac{\mu_0 c}{\hbar}\left(\psi\Gamma\right) = 0; \quad \partial_\mu\left(\psi\Gamma\right)\gamma_\mu + \frac{\mu_0 c}{\hbar}\left(\psi\Gamma\right) = 0 \tag{26}$$

The equations obtained by substituting Λ with Γ in (23) were given recently (*Lochak 1,3*) :

$$\partial_\mu\gamma_\mu\left(\psi\Lambda\right) - \frac{\mu_0 c}{\hbar}\left(\psi\Lambda\right) = 0; \quad \partial_\mu\left(\psi\Lambda\right)\gamma_\mu - \frac{\mu_0 c}{\hbar}\left(\psi\Lambda\right) = 0 \tag{27}$$

The little difference of the minus sign in the second group of equations has actually a great physical importance, because the solutions of **(26) and (27) are dual in space-time : a multiplication by γ_5 implies an exchange between themselves and between electricity and magnetism.**

In other words, the substitution of Λ to Γ in the representation by square matrices of the initial de Broglie's equations (**??**) gives now two systems : the system (26) was given by de Broglie, and the new system (27) was given recently (*Lochak 1,3*). **In other words the introduction Λ or Γ in the de Broglies equations (15) is not equivalent because they give two different photons. And we shall prove that they correspond respectively to electricity and magnetism. Finally we shall find four different photons, according to the spinvalues 1 or 0, and to the electric or to the magnetic case.**

6 The first two kinds of photons : electric and magnetic.

6.1 General formulae.

The fundamental electromagnetic formulae were given by de Broglie, starting from the column form of the equations (15) *(Broglie 1, 2, 3, 4).* Here we use the square form (26), applying a procedure later suggested by *M.A. Tonnelat* and developped by de Broglie (*Broglie 4*) ; and we shall apply symmetrically the same procedure to the equation (27). Let us start fom the expansion of a 4×4 matrix Θ, on the Clifford algebra, with the corresponding tensorial variances :

$$\Theta = I\phi_0 + \gamma_\mu \phi_\mu + \gamma_{[\mu\nu]}\phi_{[\mu\nu]} + \gamma_\mu\gamma_5\phi_{\mu 5} + \gamma_5\phi_5 \tag{28}$$

φ_0: scalar; φ_μ: polar vector; $\varphi_{[\mu\nu]}$: antisymmetric tensor of rank two; $\varphi_{\mu 5}$: axial vector; φ_5: pseudo-scalar

De Broglie defined the electromagnetic quantities in $\mathbb{R}^3$ here completed by **B**, $W, I_1,\ I_2$:

$$\mathbf{H} = Kk_0\,(\varphi_{23},\varphi_{31},\varphi_{12})\,;\quad \mathbf{E} = Kk_0\,(i\varphi_{14}, i\varphi_{24}, i\varphi_{34})$$
$$\mathbf{A} = K\,(\varphi_1,\varphi_2,\varphi_3)\,;\ \ iV = K\varphi_4;\quad \mathbf{B} = K\,(i\varphi_{15}, i\varphi_{25}, i\varphi_{35})\,;\ W = K\varphi_{45}$$
$$I_1 = K\varphi_0;\ iI_2 = K\varphi_5\quad \left(k_0 = \frac{\mu_0 c}{\hbar};\ K = \frac{\hbar}{2\sqrt{\mu_0}}\right) \tag{29}$$

It must be noticed that in the classical tensor representation of electromagnetism, we had only the Lorentz potentials : a polar vector **A** and an invariant V in $\mathbb{R}^3$, gathered in a relativistic polar vector : $K\varphi_\mu = (\mathbf{A}, V)$. Here, we have a second set of potentials : a pseudo vector **B** and a pseudo invariant W in $\mathbb{R}^3$, gathered in a relativistic pseudo vector $K\varphi_{\mu 5} = (\mathbf{B}, W)$. It must be remembered that **B** is not an induction, but a pseudo potential, and that we must not confuse the relativistic invariant $I_1 = K\varphi_0$ with the pseudo invariant $iI_2 = K\varphi_5$.

*6.2 System (26) : de Broglie's **electric** photon equations in the electromagnetic formalism.*

Introducing $\Psi = \psi\Gamma = \psi\,(-i\gamma_2\gamma_4)$ in (26), we find below **two groups** (M) and (NM) **of differential equations associated with the electric photon** because, as we know, **the system** (26) **is not the equation of a spin 1 particle but of a particle with a <u>maximum</u> spin 1**. The "Maxwellian" system (M) corresponds to the spin 1 and represents the photon equation, based on **polar potentials**. The "Non-Maxwellian" system (NM) corresponds to the spin 0 and it is based on **pseudo scalar potentials** ; but here, there is a problem of physical meaning, which we shall carefully examine later. In the present paragraph, we consider only the system (M).

$$(M)\begin{cases} -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} = rot\mathbf{E}; \quad \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = rot\mathbf{H} + k_0^2\mathbf{A} \\ div\mathbf{H} = 0; \quad div\mathbf{E} = -k_0^2 V \\ \mathbf{H} = rot\mathbf{A}; \quad \mathbf{E} = -\mathbf{grad}V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}; \quad \frac{1}{c}\frac{\partial V}{\partial t} + div\mathbf{A} = 0 \end{cases} \tag{30}$$

$$(NM)\begin{cases} -\frac{1}{c}\frac{\partial I_2}{\partial t} = k_0 W; \quad \mathbf{grad}I_2 = k_0\mathbf{B}; \quad \frac{1}{c}\frac{\partial W}{\partial t} + div\mathbf{B} = k_0 I_2 \\ rot\mathbf{B} = 0; \quad \mathbf{grad}W + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0; \ \{(k_0 I_1 = 0; \ k_0 \neq 0) \Rightarrow I_1 = 0\} \end{cases} \tag{31}$$

The equations (M) are Maxwell's equations with two differences :

a) The **mass term** introduces a link between electromagnetic fields and potentials. The last become physical quantities, and they are not gauge invariant. As the mass is very small, the macroscopic properties are saved, but some microscopic properties are modified.

b) Fields are automatically expressed in terms of Lorentz potentials, but with the Lorentz condition. They are not gauge invariant :

$$\mathbf{H} = rot\mathbf{A}; \quad \mathbf{E} = -\mathbf{grad}V - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}; \quad \frac{1}{c}\frac{\partial V}{\partial t} + div\mathbf{A} = 0 \tag{32}$$

As a consequence of (30), all the physical quantities obey the Klein-Gordon equation :

$$\Box F + k_0^2 F = 0; \quad (F = \mathbf{E}, \ \mathbf{H}, \ \mathbf{A}, \ V, \ \mathbf{B}, \ W, I_1, \ I_2) \tag{33}$$

The spherical potential is not the Coulomb potential $\frac{1}{r}$ but the Yukawa potential: $V = \frac{e^{-k_0 r}}{r}$, which remains nevertheless, a long range potential because of the smallness of the Compton wavenumber : $k_0 = \mu_0 c/\hbar$.

The system (30) describes an electric photon for several reasons :

Fields $(\mathbf{E}, \mathbf{H})$ and **polar potentials** $(V, \mathbf{A})$ - linked to $(\mathbf{E}, \mathbf{H})$ by (30) and (32) - are exactly those which are known in the dynamics of an electric charge. A consequence of $k_0 \neq 0$ is that $div\mathbf{E} \neq \mathbf{0}$, which means that the electric field $\mathbf{E}$ is not transverse : it has a longitudinal component of the order of k_0, contrary to the magnetic field $\mathbf{H}$ which remains orthogonal despite the photon mass.

The equations (30), (31) : ($\mathbf{M}$)and ($\mathbf{NM}$) are gathered in a block associated with the electric photon. The system **(NM)** does not include the invariant I_1 but the **pseudo invariant** I_2, which defines the **axial 4-potential** $(W, \mathbf{B})$ and the fields $(\mathbf{E}', \mathbf{H}')$, all related to magnetism. This is the first part of a cross-symmetry between electricity and magnetism : the counterpart will appear in the magnetic photon.

6.3 The spin 0 particle associated to the electric spin 1 photon

The (NM) equations describe a spin 0 particle, which may be proved by the theory of spin (note, that in these equations : $\mu_0 =$ photon mass, just as in the equations (30)). This spin 0 particle is a **chiral particle** because in (31) I_1 is a true invariant but $I_1 = 0$; conversely $I_2 \neq 0$ but I_2 is a **pseudo-invariant** from which are deduced the **pseudopotentials** $(W, \mathbf{B})$: i.e. **a pseudo − quadrivector dual of a tensor of rank 3.** De Broglie noted that (31) defines new fields, $(\mathbf{H}', \mathbf{E}')$ which he called “ anti-fields ” (*Broglie 1. . . 4*), and which are related to the pseudo-potentials $(W, \mathbf{B})$:

$$\mathbf{H}' = \mathbf{grad} W + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \quad \mathbf{E}' = rot\mathbf{B} \tag{34}$$

Until now we have $\mathbf{H}' = \mathbf{B}' = 0$ in virtue of (31). Later, the anti-fields will be different from zero and they will be linked to magnetism. So appears another electromagnetism based on the potentials $(W, \mathbf{B})$, just as the first was based on the Lorentz potentials $(V, \mathbf{A})$ in the theory of the electron. The definitions (30) of $(\mathbf{H}', \mathbf{E}')$ in terms of $(W, \mathbf{B})$ are a direct consequence of the geometrical optics approximation of the quantum monopole equations (see : *Lochak 4,5,6*). The most important problem of the spin 0 photon is its experimental properties, a problem that will be the subject of **Part 2** of this paper.

6.4 The ***magnetic*** *spin 1 photon and its associated spin 0 particle (Lochak 1,2 ; Spehler, Marques)*

Symmetrically to § **5.2** we introduce $\Psi = \psi\Lambda = \psi\Gamma\gamma_5 = \psi\,(-i\gamma_3\gamma_1)$ in (27) and we find two separate systems again. Contrary to the preceding case, when the first system was a spin 1 electric photon, and the second a spin 0 magnetic photon, **we shall find a spin 1 magnetic photon and a spin 0 electric photon**.

$$(M)\begin{cases} -\frac{1}{c}\frac{\partial \mathbf{H}'}{\partial t} = rot\mathbf{E}' + k_0^2\mathbf{B}'; \quad \frac{1}{c}\frac{\partial \mathbf{E}'}{\partial t} = rot\mathbf{H}' \\ div\mathbf{H}' = k_0^2 W'; \quad div\mathbf{E}' = 0 \\ \mathbf{H}' = \mathbf{grad}W' + \frac{1}{c}\frac{\partial \mathbf{B}'}{\partial t}; \quad \mathbf{E}' = rot\mathbf{B}'; \quad \frac{1}{c}\frac{\partial W'}{\partial t} + div\mathbf{B}' = 0 \end{cases} \tag{35}$$

$$(NM)\begin{cases} -\frac{1}{c}\frac{\partial I_1}{\partial t} = k_0 V'; \quad \mathbf{grad}I_1 = k_0\mathbf{A}'; \quad \frac{1}{c}\frac{\partial V'}{\partial t} + div\mathbf{A}' = k_0 I_1 \\ rot\mathbf{A}' = 0; \quad \mathbf{grad}V' + \frac{1}{c}\frac{\partial \mathbf{A}'}{\partial t} = 0; \\ (k_0 I_2 = 0; \quad k_0 \neq 0) \Rightarrow I_2 = 0 \end{cases} \tag{36}$$

We find a couple of fields again but the situation is inversed :

1) The spin 1 field $(\mathbf{E}', \mathbf{H}')$ and the **axial 4 potential** $(W', \mathbf{B}')$ obey the **maxwellian system** (M) (35).

2) The a priori definition (34) of the "anti-fields" appears automatically in the equations (35), and $(\mathbf{E}', \mathbf{H}')$ **are no more equal to zero**.

3) The field spin 1 : $(\mathbf{E}', \mathbf{H}')$ defined by $(W', \mathbf{B}')$, is exactly the one which was known in the dynamics of a magnetic monopole (*Lochak, 4,5,6 and Chap. 1, 2, 3*).

4) Contrary to the electric case, we have : $div\mathbf{H}' \neq 0$, so that in a plane wave, the magnetic field **H'** (and not the electric one) has a longitudinal component of the order k_0, while $\mathbf{E}'$ is now transversal : we have a **magnetic spin 1 photon**.

The (NM) equations describe, like precedingly, a spin 0 particle.

But now the polar potential $(V', \mathbf{A}')$ appears instead of $(W, \mathbf{B})$ in the non maxwellian system (NM). The invariant I_1 and the pseudo-invariant I_2 exhange their roles. We have : $I_1 \neq 0$ and $I_2 = 0$.The elecromagnetic field $(\mathbf{E}, \mathbf{H})$ defined by the classical Lorentz formulae (32) give now $\mathbf{E} = \mathbf{H} = 0$ just as we had $\mathbf{E}' = \mathbf{H}' = 0$ in the electric case. Conversely the field $(\mathbf{E}', \mathbf{H}')$and the 4 axial potential $(W', \mathbf{B}')$ now obey the maxwellian system (M) and, of course, they are not equal to zero .

So, we have shown that the de Broglie fusion of two Dirac equations gives rise to Maxwell equations of two classes of photons : electric photons and magnetic photons. And both (electric or magnetic) can have either spin 1 or spin 0.

In addition **the spin 1 photons are linked to the spin 0 photons by a cross symmetry** : the polar "electric photon" of spin 1 is linked with an axial magnetic spin 0 photon, while the axial "magnetic photon" of spin 1 is linked with a polar spin 0 "electric photon".

It must be added that the spin 1 photons are already known in quantum mechanics for electrons and leptonic monopoles. They appear in the minimal electric and magnetic interactions between matter and electromagnetic field, in the Dirac equation and in the monopole equation (see : *Lochak*) :

$$\gamma_\mu\left(\partial_\mu - i\frac{e}{\hbar c}\mathbf{A}_\mu\right)\psi + \frac{m_0 c}{\hbar}\psi = 0; \qquad \gamma_\mu\left(\partial_\mu - \frac{g}{\hbar c}\gamma_5\mathbf{B}_\mu\right)\psi = 0 \quad (37)$$

And both equations are experimentally confirmed : the electron more than the monopole, of course, but the last already appeared in hundreds of experiments done by Urutskoiev and others.

Nevertheless, until now a question remains : are the spin 0 photons already known ? The answer is yes : in the Aharonov-Bohm effect.

Part 2 : The Aharonov-Bohm effect.

Consider the equations of (NM) potentials **:** (31) and (36) :

1) Spin 0 magnetic photon :

$$-\frac{1}{c}\frac{\partial I_2}{\partial t} = k_0 W; \ \mathbf{grad} I_2 = k_0\mathbf{B}; \ \frac{1}{c}\frac{\partial W}{\partial t} + div\mathbf{B} = k_0 I_2$$

2) Spin 0 electric photon :

$$-\frac{1}{c}\frac{\partial I_1}{\partial t} = k_0 V'; \ \mathbf{grad} I_1 = k_0\mathbf{A}'; \ \frac{1}{c}\frac{\partial V'}{\partial t} + div\mathbf{A}' = k_0 I_1$$

We must remember that the spin 1 electric photon is associated with a magnetic spin 0 photon through the pseudo-invariant I_2, while the spin 1 magnetic photon is associated with an electric spin 0 photon through the true invariant I_1. The preceding relations immediately imply that the spin 0 potentials are the **gradients of relativistic invariants**, which verifiy the Klein-Gordon equation :

$$\partial_\mu I_1 = k_0 \mathbf{A}_\mu; \ \ \Box I_1 + k_0^2 I_1 = 0; \qquad \partial_\mu I_2 = k_0 \mathbf{B}_\mu; \ \ \Box I_2 + k_0^2 I_2 = 0 \quad (38)$$

We know that in virtue of (31) and (36) the corresponding electromagnetic fields equal zero. So we must answer the question : **how the spin 0 photon may be detected** ? More precisely : since these fieldless potentials are unable to generate a force, what does remain which could be observed ? Of course : the **phase** first characteristic of a wave. The Aharonov-Bohm effect was precisely imagined to answer the question, and to prove that contrary to a common idea, the electromagnetic potentials are not mathematical intermediates (even if they can play this role) : they are observable physical quantities.

7 The effect.

The idea of the effect (see : *Aharonov-Bohm, Tonomura, Olariu-Popescu, Lochak 3*) was to modify electron interferences by a fieldless magnetic potential created by a magnetic string or by a thin solenoid orthogonal to the plane of interfering electron trajectories, as is shown on Fig. 1. In the experiment described by the figure, the Young slits are obtained owing to the Fresnel – Möllenstedt biprism.

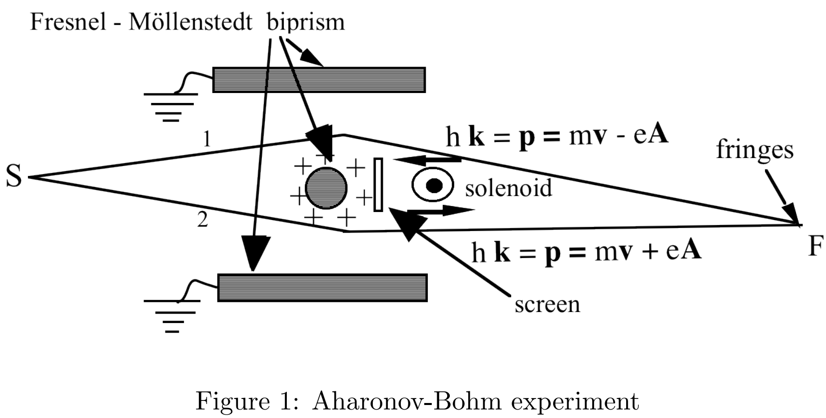

Figure 1: Aharonov-Bohm experiment

The solenoid must be in principle infinitely long, so that the magnetic field emanating from the extremities cannot perturb the experiment : it is assumed in the calculations, but actually a few millimeters are sufficient because the transverse dimensions of the device are of the order of microns. This arrangement of the solenoid has led to the idea that the magnetic flux through the trajectories quadrilateral plays an essential role. Many disagree with that idea (see : *Lochak 3*).

The problem of eliminating this hypothesis was elegantly solved by Tonomura (see : *Tonomura*) by substituting the rectilinear string by a microscopic torus ($10\mu m$) : one of the electron beams passes through the torus and the other outside, the magnetic lines being trapped in the torus.

Let us give an intuitive interpretation of the Aharonov-Bohm experiment. The principle is that the wave vector of an electron in a magnetic potential is given by the de Broglie wave (*Broglie,5*) which is a direct consequence of the identification of the principles of Fermat and of least action (**p** is the Lagrange momentum) :

$$\frac{h}{\lambda}\mathbf{n} = h\mathbf{k} = \mathbf{p} = m\mathbf{v} + e\mathbf{A} \tag{39}$$

It is obvious on the preceeding formula, that interference and diffraction phenomena are influenced by the presence of a magnetic potential independently of the presence of the field because the interferences only depend on the phase. It is well known in optics : an interference figure maybe shifted, in a Michelson interferometer by introducing a plate of glass in one of the virtual beams, which causes a phase shift and thus a change of the optical path.

These phenomena are manifestly **gauge dependent** : if we add something to **A**, would it be a gradient or not in the de Broglie wave (39) λ is modified. This is evident even on the classical de Broglie formula : $\lambda = \frac{h}{mv}$ when $\mathbf{A} = 0$, which is gauge dependent too, a fact often emphasized by de Broglie himself who said : **"If gauge invariance would be general in quantum mechanics, the electron interferences could not exist".**

In the case of the Aharonov-Bohm experiment there is an additive phase on both interfering waves in opposite directions, which doubles the shift of te interference fringes. Let us recall a proof of the effect, independent from the fact that a potential generates forces or not (*Lochak 3*).

8 The magnetic potential of an infinitely thin and infinitely long solenoid.

We consider the case corresponding to the realized Aharonov-Bohm experiment : electrons diffracted on Young slits and falling on a magnetic solenoid orthogonal to the plane of the electron trajectories, according to Fig.1, and thus according to the schematic following Fig.2, the solenoid will be along Oz.

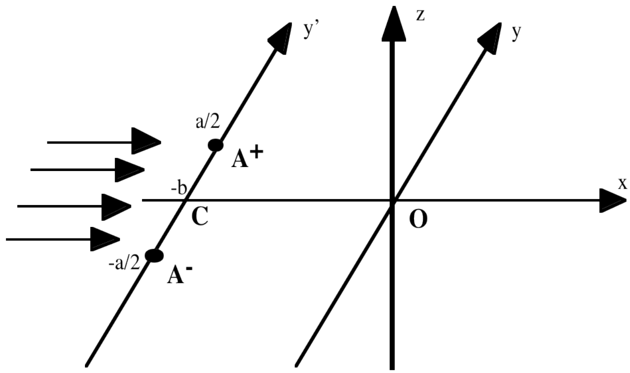

Figure 2: Aharonov-Bohm schematic

To simplify the calculations, we shall neglect the photon mass, because in the present case, its role is only important in the symmetry laws, which are already taken into account in all the formulae ; so that, to omit here the photon mass only means to omit some negligible numerical corrections.

Now, the **electric charge** of the diffracted electrons implies that they "see" the electromagnetism through the Lorentz potentials $(V, \mathbf{A})$ and thus through the equations (M) : (30). And thus, the effect of the fieldless potential of the solenoid will be the one described by the **associated equations** (NM) : (31).

These equations derive from the pseudo – invariant I_2. Now, there is an obvious invariant in the Aharonov-Bohm effect : the rotation angle $\varphi = \arctan y/x$ around the axis Oz. So we shall write :

$$I_2 = \varepsilon k_0 \arctan y/x \tag{40}$$

Where k_0 is the quantum wave number of the photon and ε a convenient dimensional constant, the value of which is not important for our calculation. Something seems wrong because (y/x) is P-invariant and with the definition (40), I_2 seems to be an invariant and not a pseudo invariant, as in (31). But it is not so because (y/x) is P-invariant only in the space $\mathbb{R}^2$: (x, y) ; but not in the space $\mathbb{R}^3$: (x, y, z), because the inversion, the P- transformation $(x, y, z) \rightarrow (-x, -y, -z)$ implies the inversion of the Oz and thus of the rotation angle φ. So that (y/x) is a pseudo – invariant in $\mathbb{R}^3$.

Thus we have, in virtue of (31) :

$$\textbf{grad}\, I_2 = k_0 \mathbf{B} \tag{41}$$

Or :

$$\mathbf{B}_x = -\varepsilon \frac{y}{x^2+y^2} \; ; \quad \mathbf{B}_y = \varepsilon \frac{x}{x^2+y^2} \; ; \quad \mathbf{B}_z = 0 \tag{42}$$

9 Theory of the effect.

The commonly admitted theories are uselessly complicated (*Olariu S., Iovitsu Popescu*). For the physical bases of the effect, the best source is the brillant book of *Tonomura*. Here, to find the formula of fringes it is sufficient to take the geometrical optics approximation with the phase $\varphi = S/\hbar$ of de Broglie's wave and the principal Hamilton function S obeying the Hamilton-Jacobi equation with the potential (42) :

$$2m\frac{\partial S}{\partial t} = \left(\frac{\partial S}{\partial x} + \varepsilon \frac{y}{x^2+y^2}\right)^2 + \left(\frac{\partial S}{\partial y} - \varepsilon \frac{x}{x^2+y^2}\right)^2 \tag{43}$$

The electronic wave propagates from $x = -\infty$ to $x = +\infty$ and the Young slits A^+ and A^- (Fig. 2) are on a parallel to Oy, at a distance $\pm\frac{a}{2}$ from the point C located at $x = -b$.

The potential $\mathbf{B}$ appearing in (42) and (43) is the gradient of I_2 and thus $\mathbf{B}$ and I_2 satisfy, up to μ_0, the equations (NM), (31) (taking into account, that they are independent of t and that $W = 0$).

The equation (43) is immediately integrated, defining the phase :

$$\Sigma = S - \varepsilon \arctan y/x \tag{44}$$

Which gives :

$$2m\frac{\partial \Sigma}{\partial t} = \left(\frac{\partial \Sigma}{\partial x}\right)^2 + \left(\frac{\partial \Sigma}{\partial y}\right)^2 \tag{45}$$

Chosing a complete integral of (45) and thus of (43), owing to (44), we have :

$$\Sigma = Et - \sqrt{2mE}\;(x\cos\theta_o + y\sin\theta_o) \tag{46}$$

$$S = Et - \sqrt{2mE}\;(x\cos\theta_o + y\sin\theta_o) + \varepsilon \arctan\frac{y}{x} \tag{47}$$

Or, in polar coordinates $x = r\cos\theta$, $y = r\sin\theta$:

$$S = Et - \sqrt{2mE}\, r\, \cos(\theta - \theta_o) + \varepsilon\,\theta \tag{48}$$

The Jacobi theorem gives the trajectories (**the wave rays**) :

$$\begin{aligned} \frac{\partial S}{\partial \theta_o} &= \sqrt{2mE}\;(x\sin\theta_o - y\cos\theta_o) = \mu\;; \\ \frac{\partial S}{\partial E} &= t - \sqrt{\frac{m}{2E}}\;(x\cos\theta_o + y\sin\theta_o) = t_o \end{aligned} \tag{49}$$

Finally, with[2] $E = \frac{1}{2}m\mathrm{v}^2$ we have the motion :

$$x\cos\theta_o + y\sin\theta_o = \mathrm{v}\,(t - t_o) \tag{50}$$

We see that the *rays* (electron trajectories), defined in (50) are **orthogonal to the moving planes but they are not orthogonal to the equal phase surfaces** (47) - (48) except far from the magnetic string ($x \to \infty$), when the potential term of the order of ε becomes negligible.

Therefore, despite the presence of a potential, the electronic trajectories remain rectilinear and are not deviated, because the magnetic field equals zero in virtue of (31). The velocity $\mathrm{v} = Const$ **remains the one of the incident electrons because of the conservation of energy.**

But the diffraction of waves through the slits A^+ and A^-creates, for the electron trajectories, an interval of possible angles θ_o, equal to the angles of the interference fringes, modified by the magnetic potential :

There is not deviation of the electrons, only a deviation of the angles of phase synchronization between the waves issued from A^+ and A^-. This is the Aharonov-Bohm effect, which is in accordance with the definition of the spin 0 photon (31).

[2]We are obviously far from relativity.

It would be useless to reproduce the end of the theory of Aharonov-Bohm effect (*see for instance Lochak 3*). Let us only recall the total phase-shift :

$$\Delta\varphi = \frac{\Delta S}{h} = \frac{a\theta_o}{\lambda} + \frac{2\varepsilon\xi}{h} \quad ; \quad \xi = \arctan\frac{a}{2b} \tag{51}$$

The first term gives the standard Young fringes (the notations are those of Fig. 2), **the second term is the Aharonov-Bohm effect** : $\xi = \arctan\frac{a}{2b}$ equal to half the angle under which the Young slits are seen from the solenoid, which entails a dependence of the effect on the position of the string : the effect must decrease when the distance b increases.

We see that the theory of the Aharonov-Bohm effect is a simple consequence of the choice of the invariant in the system (36), as the invariant rotation angle around the axis of he solenoid.

10 Conclusion.

We suggest a new theory of light based on 4 photons.

1. At first the famous Einstein photon well known in optics from 1905, and identified by de Broglie (1922) as a **vectorial spin 1** particle, which we call now the **electric photon,** because it interacts with the electric charges (principally with electrons)

2. A **pseudovectorial spin 1 magnetic photon**, which is symmetric to the electric Einstein photon and which appeared in the theory of **leptonic magnetic monopole**s (*see Lochak*). The magnetic photon plays in the physics of monopoles a role exactly similar to the role played in the theory of electrons by the preceding electric photon.

3. Two **spin 0** photons (one **electric** and the other **magnetic**), related to 2 classes of respectively electric and magnetic fieldless phenomena, an example of which is the Aharonov-Bohm effect.

All the present paper is no more than a generalization of the **Broglie's Theory of light.** Perhaps it is useful to recall that this theory was highly considered by Pauli and Firtz, who developped similar ideas, and by Heisenberg who wrote (see : *Heisenberg*) :

> *" ... the idea expressed by L. de Broglie in 1936 that light quanta must be considered as composite structures, leads to principle problems of the same importance as those which were raised by the celebrated discovery of matter waves[3]."*

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[3]More precisely, it was 1934.

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[4]The book of Akira Tonomura, written in non-technical terms, is in principle, a popular book but it is so clear and so profound that it must be included in a bibliography on the Aharonov-Bohm effect.