“Field-shell” of the self-interacting quantum electron

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ABSTRACT. Self-interacting dynamics of non-local Dirac’s electron has been proposed. Its “shape” was revealed by the projective representation of internal energy-momentum operator corresponding to the spin/charge non-linear dynamics. Energy-momentum field is described by the system of quasi-linear “field-shell” PDE’s following from the conservation law expressed by the affine parallel transport in $CP(3)$. We discuss here solutions of these equations in the connection with the following problems: curvature of $CP(3)$ as a potential source of electromagnetic-like fields and the self-consistent problem of the electron mass.

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1 Introduction

Statistical analysis of the energy distribution is the basis of the black body radiation [1] and the Einstein’s hypothesis of the light emission and absorption [2]. Success of Einstein hypothesis of photons, de Broglie wave concept of particles [3] and the Schrödinger equation for hydrogen atom [4] established so-called the corpuscular-wave duality of matter. This conceptual line was logically finished by Dirac in his method of the second quantization [5]. Such approach perfectly fits to many-body weakly interacting quantum systems and it was assumed that the “corpuscule-wave duality” is universal. The corpuscular-wave duality may be, however, broken in strong interacting quantum systems and in a single quantum particle. Physically it is clear why: the quantum particle is self-interacting system and this interaction is at least of the order of its rest mass. Since the nature of the mass is an open problem we do not know the energy distribution in quantum particles up to now. Here we try to show a possible approach to this problem in the framework of simple model of self-interacting quantum electron with possible "unparticle" excitations. The "unparticle" sector of quantum excitations is intensively discussed now in the framework of effective QFT [6, 7, 8].

Blochintzev 60 years ago discussed the unparticle sector in the framework of universality of wave - particle "duality" for interacting quantum fields [9, 10]. For such fields the universality is generally broken. Namely the attempt to represent two interacting boson fields as the set of free quantum oscillators leads to two types of oscillators: quantized and non-quantized. The second gives rise to the simple relation $g > \frac{m_1 m_2 c^2}{h^2}$ between coupling the constant g and masses m_1 and m_2 of two scalar fields. For such intensity of coupling we obtain a field with excitation states in two sectors: particle and "unparticle". Furthermore, excitations in "unparticle" sector have imaginary mass and they propagate with group velocity larger than c. For a self-interacting scalar field of mass m the intensity of self-interaction g leads to breakdown of the universality of the wave - particle "duality" if it is larger than the inverse square of the Compton wavelength: $g > \frac{m^2 c^2}{h^2} = \frac{1}{\lambda_C^2}$.

Blochintzev's examples were oversimplified for clarity. We discuss here the self-interacting electron in the spirit of reaction $e^- \to \mathcal{U} \to e^-$. In other words we study the particle/unparticle sectors of matter in a wide range of momenta in order to solve the localization problem of the foundations of quantum physics. In order to formulate a robust theory of self-interacting quantum "particles", say, electron, one should analyse the quantum invariants and their relations to space-time symmetries.

Extended relativistic electron is represented in the present model by closed geodesics in complex projective Hilbert space $CP(3)$ [11, 12]. Namely, cyclic motions of quantum spin/charge degrees of freedom are generated by the coset transformations from $G/H = SU(N)/S[U(1) \times U(3)] = CP(3)$ and state-dependent gauge transformations from $H = U(1) \times U(3)$ rotate geodesics line as whole. We pose the following question: could the curvature of the $CP(3)$ play the role of the source of actual electromagnetic potentials surrounding the self-interacting non-local electron? Thus, one may assume *that geodesic variation of electron motion have a character of electromagnetic-like field.* Thereby hypothesis of de Broglie about internal periodic process in electron may get natural support. It is almost literally coincides with de Broglie striking clarifying questions and statements: "Must we suppose that this periodic phenomenon occurs in the interior of energy packets? This is not at all necessary; the results of §1.3 will show that it is spread out over an extended space. Moreover, what must we understand by the interior of a parcel of energy? An electron is for us the archetype of isolated parcel of energy, which we believe, perhaps incorrectly, to know well; but, by received wisdom, the energy of an electron is spread over all space with a strong concentration in a very small region, but otherwise whose properties are very poorly known" [13].

Two original mathematical formulations of quantum theory of Hiesenberg and Schrödinger are equivalent in the framework of so-called optics-mechanics analogy [14]. This analogy, however, is limited by itself in very clear reasons: mechanics is merely a coarse approximation (even being generalized to many-dimension dynamics of Hertz) and the "optics" of the action waves is too tiny for description of complicated structure of "elementary" quantum particles. It was realized already under the first attempts to synthesize relativistic and quantum principles.

Analysis of the foundations of quantum theory and relativity shows too that it is impossible to use macroscopic primordial elements like particles, material points, etc., trying to build consistent theory. Even space-time cannot conserve its independent and a priori structure. Therefore the unification of relativity and quantum principles may be formalized if one uses new primordial elements: pure quantum degrees of freedom and the classification of their motions. So, the rays of quantum states will be used instead of material points (particles) and the complex projective Hilbert state space $CP(N-1)$ where these states move under the action of unitary group $SU(N)$ will be used instead of space-time, i.e. *distance between bodies will be replaced by the distance between quantum states* [12, 15]. The physical reason for such decisive steps will be discussed below.

2 External and internal (quantum) formulations of the inertia law

Success of Newton's conception of physical force influencing on a separated body may be explained by the fact that the *geometric counterpart to the force - acceleration in some inertial frame* was found with the simplest relation to the mass of a body. The consistent formulation of mechanical laws has been realized in Galilean inertial systems. The class of the inertial systems contains (by a convention) the one unique inertial system - the system of remote stars. Then, on the abstract mathematical level arose “space” - the linear space with appropriate vector operations on forces, momenta, velocities, etc. General relativity and new astronomical observations concerning accelerated expansion of Universe show that all these constructions are only a good approximation, at best.

The line of Galileo-Newton-Mach-Einstein argumentations made accent on some absolute global reference frame associated with the system of remote stars. This point of view looks as absolutely necessary for the classical formulation of the inertial principle itself. This fundamental principle has been formulated, say, “externally”, i.e. as if one looks on some massive body perfectly isolated from Universe. In such approach only “mechanical” state of relative motion of the body has been taken into account. Nevertheless, Newton clearly saw some weakness of such approach. His famous example of rotating bucket with water shows that there is an absolute motion since the water takes on a concave shape in any reference frame. Here we are very close to different - “internal” formulation of the inertia principle and, probably, to understanding of the quantum nature of inertial mass. Namely, the “absolute motion” should be turned towards not outward, to distant stars, but inward – to the body deformation. In fact one should take into account that external force not only change the inertial character of its motion: motion with the constant velocity transforms to accelerated motion; moreover – the body deforms.

Two aspects of a force action - acceleration relative inertial reference frame and deformation of the body are very important already on the classical level as it has been shown by Newton’s bucket rotation. The second aspect is especially important for quantum “particles” since the acceleration requires a good localization of moving body in space-time (the second derivative of coordinate $\vec{a} = \frac{d^2\vec{x}}{dt^2}$ should have a sense) but just such a localization of quantum particles is very problematic in quantum theory [18, 20, 19]. In such a situation one should make accent on the second aspect of the force action – body deformation, i.e. microscopically the state of body is changed. In fact it is already a *different body*, with different temperature, etc., [21]. In the inertial motion one has opposite situation – the internal state of the body does not change, i.e. body is self-identical during inertial space-time motion. In fact this is the basis of all classical physics. Generally, space-time localization being treated as ability of coordinate description of an object in classical relativity closely connected with operational identification of “events” [23].

It is tacitly assumed that all classical objects (frequently represented by material points) are self-identical and they can not disappear because of the energy-momentum conservation law. The inertia law of Galileo-Newton ascertains this self-conservation "externally". This means that objectively *physical state of body (temporary in somewhat indefinite sense) does not depend on the choice of the inertial reference frame.* One may accept this statement as an "internal" formulation of the inertial law that should be of course formulated mathematically. We put here some plausible reasonings leading to such a formulation.

Generally interaction leads to the absolute change (deformation) of the quantum state [21] (remember: quantum state is the state of motion [24]). Such motion takes the place in state space modeled frequently by some Hilbert space. But there is no geometric counterpart to H_{int} in such functional space. A force action being discussed from the quantum point of view gives the alternative way for the connection of H_{int} action with a *new geometric counterpart of interaction - coset transformations of the quantum states*. It means that instead of absolute external reference frame of remote stars or "space" one may use "internal", in fact – a quantum reference frame [11, 12, 15, 16, 17, 21, 22]. Thus, instead of choosing, say, the system of distant stars as an "outer" absolute reference frame [25] the deformation of quantum state may be used. It means that the deformation of quantum motion in quantum state space serves as an "internal detector" for accelerated space-time motion. The mathematical formulation of the quantum inertia law requires intrinsic unification of relativity and quantum principles.

3 Intrinsic unification of relativity and quantum principles

Pure quantum states represented by rays (points of complex projective Hilbert space $CP(N-1)$) will be used thereafter as fundamental physical concept instead of "material point". It is assumed that quantum lumps obtained as solutions of some fundamental quantum non-linear relativistic wave equations will describe observable quantum particles. Two simple observations may serve as the basis of the intrinsic unification of relativity and quantum principles.

The first observation concerns interference of quantum amplitudes in fixed quantum setup.

A. The linear interference of quantum amplitudes shows the symmetries relative space-time transformations of whole setup. This interference has been studied in "standard" quantum theory. Such symmetries reflects, say, the *first order of relativity*: the physics is same if any *complete setup* subject (kinematical, not dynamical!) shifts, rotations, boosts as whole in single Minkowski space-time. According to our notes given above one should add to this list a freely falling quantum setup in gravitation field of a star.

The second observation concerns a dynamical "deformation" of some quantum setup.

B. If one dynamically changes the setup configuration or its "environment", then the amplitude of an event will be generally changed [28] . Nevertheless there is a different type of tacitly assumed symmetry that may be formulated on the intuitive level as the invariance of physical properties of "quantum particles", i.e. the invariance of their quantum numbers like mass, spin, charge, etc., relative variation of quantum amplitudes. Such symmetry requires more general kind of state-dependent gauge transformations than typically used in gauge theories. Physically it means that properties of, say, electrons are the same in two different setups S_1 and S_2.

One may postulate that the invariant content of this physical properties may be kept if one makes the infinitesimal variation of some "flexible quantum setup" reached by small variation of some fields by adjustment of tuning devices. This principle being applied to quantum system in pure quantum state was called the "super-relativity principle". One may suspect that what we call "physical fields" are in fact state-dependent unitary transformations of "flexible quantum setup" acting in projective Hilbert space of pure quantum states [11, 12, 15, 17].

There is an essential technical approach capable to connect super-relativity and the quantum inertia law. Namely, a new concept of the local dynamical variable (LDV) [17] should be introduced for the realization of infinitesimal variation of a "flexible quantum setup". Our construction is naturally connected with the geometric phases [28] but we seek the local conservation laws for LDV's in the state space $CP(N-1)$.

4 Eigen-dynamics and local dynamical variables

The standard approach in QM or QFT assumes the priority of Hamiltonian or Lagrangian given by some classical model which henceforth should be "quantized". It is known that this procedure is ambiguous. In order to avoid the ambiguity, we intend to use a *quantum state* itself and the invariant conditions of its conservation and perturbation [15]. These invariant conditions are rooted into the global geometry of the dynamical group manifold. Namely, the geometry of $G = SU(N)$, the isotropy group $H = U(1)\times U(N-1)$ of the pure quantum state, and the coset $G/H = SU(N)/S[U(1) \times U(N-1)]$ geometry, play an essential role in the quantum state evolution [15]. The stationary states i.e. the states of motion with the least action may be treated as *initial conditions* for generalized coherent state (GCS) evolution. Particulary, they may represent a local minimum of energy (local vacuum) in the local coordinates

$$\pi^i_{(j)} = \begin{cases} \frac{\psi^i}{\psi^j}, & if \quad 1 \le i < j \\ \frac{\psi^{i+1}}{\psi^j} & if \quad j \le i < N \end{cases}. \tag{1}$$

Notice, the single-value ray solution of the eigen-problem may be used instead of the vector solution with additional freedom of a complex scale multiplication [16].

Now we will introduce the local dynamical variables (LDV's) correspond to the internal $SU(N)$ group symmetry and its breakdown. They should be expressed in terms of the local coordinates π^k. Thereby they will live in geometry of $CP(N-1)$ with the Fubini-Study metric

$$G_{ik^*} = [(1+\sum|\pi^s|^2)\delta_{ik} - \pi^{i^*}\pi^k](1+\sum|\pi^s|^2)^{-2} \tag{2}$$

and the affine connection

$$\Gamma^i_{mn} = \frac{1}{2}G^{ip^*}(\frac{\partial G_{mp^*}}{\partial \pi^n} + \frac{\partial G_{p^*n}}{\partial \pi^m}) = -\frac{\delta^i_m\pi^{n^*} + \delta^i_n\pi^{m^*}}{1+\sum|\pi^s|^2}. \tag{3}$$

Hence the internal dynamical variables and their norms should be state-dependent, i.e. local in the state space [15]. These local dynamical variables realize a non-linear representation of the unitary global $SU(N)$ group in the Hilbert state space C^N. Namely, N^2-1 generators of $G = SU(N)$ may be divided in accordance with the Cartan decomposition: $[B,B] \in H, [B,H] \in B, [B,B] \in H$. The $(N-1)^2$ generators

$$\Phi^i_h \frac{\partial}{\partial \pi^i} + c.c. \in H, \quad 1 \le h \le (N-1)^2 \tag{4}$$

of the isotropy group $H = U(1) \times U(N-1)$ of the ray (Cartan subalgebra) and $2(N-1)$ generators

$$\Phi^i_b \frac{\partial}{\partial \pi^i} + c.c. \in B, \quad 1 \le b \le 2(N-1) \tag{5}$$

are the coset $G/H = SU(N)/S[U(1) \times U(N-1)]$ generators realizing the breakdown of the $G = SU(N)$ symmetry of the GCS. Notice, the partial derivatives are defined here as usual: $\frac{\partial}{\partial \pi^i} = \frac{1}{2}(\frac{\partial}{\partial \Re \pi^i} - i\frac{\partial}{\partial \Im \pi^i})$ and $\frac{\partial}{\partial \pi^{*i}} = \frac{1}{2}(\frac{\partial}{\partial \Re \pi^i} + i\frac{\partial}{\partial \Im \pi^i})$.

Here $\Phi^i_\sigma, \quad 1 \leq \sigma \leq N^2-1$ are the coefficient functions of the generators of the non-linear $SU(N)$ realization. They give the infinitesimal shift of the i-component of the coherent state driven by the σ-component of the unitary field $\exp(i\epsilon\lambda_\sigma)$ rotating by the generators of $AlgSU(N)$ and they are defined as follows:

$$\Phi^i_\sigma = \lim_{\epsilon \to 0} \epsilon^{-1} \left\{ \frac{[\exp(i\epsilon\lambda_\sigma)]^i_m \psi^m}{[\exp(i\epsilon\lambda_\sigma)]^j_m \psi^m} - \frac{\psi^i}{\psi^j} \right\} = \lim_{\epsilon \to 0} \epsilon^{-1} \{\pi^i(\epsilon\lambda_\sigma) - \pi^i\}, \quad (6)$$

[15]. Such definition being applied to Dirac's matrices γ_μ [11] gives us twelve coefficient functions $\Phi^i_\mu(\gamma_\mu)$ in the map $U_1 : \{\psi_1 \neq 0\}$:

$$\begin{aligned}
&\Phi^1_0(\gamma_0)=0, \quad \Phi^2_0(\gamma_0) = -2i\pi^2, \quad \Phi^3_0(\gamma_0) = -2i\pi^3;\\
&\Phi^1_1(\gamma_1)=\pi^2 - \pi^1\pi^3, \quad \Phi^2_1(\gamma_1) = -\pi^1 - \pi^2\pi^3, \quad \Phi^3_1(\gamma_1) = -1 - (\pi^3)^2;\\
&\Phi^1_2(\gamma_2)=i(\pi^2 + \pi^1\pi^3), \quad \Phi^2_2(\gamma_2) = i(\pi^1 + \pi^2\pi^3), \quad \Phi^3_2(\gamma_2) = i(-1 + (\pi^3)^2);\\
&\Phi^1_3(\gamma_3)=-\pi^3 - \pi^1\pi^2, \quad \Phi^2_3(\gamma_3) = -1 - (\pi^2)^2, \Phi^3_3(\gamma_3) = \pi^1 - \pi^2\pi^3. \qquad (7)
\end{aligned}$$

Then the operator of internal energy-momentum in $CP(3)$

$$i\Phi^i_\mu(\gamma)P^\mu \frac{\partial}{\partial \pi^i} + c.c. \quad (8)$$

will be used instead of the Dirac's operator of energy-momentum

$$\hat{\gamma}^\mu p_\mu = i\hbar\hat{\gamma}^\mu \frac{\partial}{\partial x^\mu} \quad (9)$$

in space-time. The last combined operator acts in the direct product $S = C^4 \times H_D$, where H_D means a Hilbert space of differentiable functions. Such splitting seems to be artificial and we try to find more flexible construction of energy-momentum operator. Namely, more reasonable to work in the fibre bundle and only in a section to have locally the splitting into "external" and "internal" degrees of freedom.

Scale-invariant dimensionless local projective coordinates (π^1, π^2, π^3) of free electron in $CP(3)$, i.e. relative components of the bi-spinos of stationary state may be derived from ordinary homogeneous system of eigen-problem

$$mc^2\psi_1 + c(p_x - ip_y)\psi_4 + cp_z\psi_3 = E\psi_1$$

$$
\begin{aligned}
mc^2\psi_2 + c(p_x + ip_y)\psi_3 - cp_z\psi_4 &= E\psi_2 \\
-mc^2\psi_3 + c(p_x - ip_y)\psi_2 + cp_z\psi_1 &= E\psi_3 \\
-mc^2\psi_4 + c(p_x + ip_y)\psi_1 - cp_z\psi_2 &= E\psi_4.
\end{aligned} \tag{10}
$$

It is easy to see [15] that transition from the system of homogeneous equations to reduced system of non-homogeneous equations for the rays has single-value solution in each map of the local coordinates (π^1, π^2, π^3). Say, in the map $U_1 : \{\psi_1 \neq 0\}$, for $E = \sqrt{m^2c^4 + c^2p^2}$ one has

$$
\pi^1 = 0, \quad \pi^2 = \frac{cp_z}{mc^2 + E} \quad \pi^3 = \frac{c(p_x + ip_y)}{mc^2 + E}, \tag{11}
$$

if $D = (E^2 - m^2c^4 - c^2p^2)^2 \neq 0$. This inequality shows that classical dispersion law cannot be correct and off-shell condition opens the way for self-interacting of spin-charge degrees of freedom and the energy-momentum distribution of electron in dynamical space-time (DST). Corrected dispersion law will be established due to formulation of the conservation law of internal energy-momentum in $CP(3)$.

5 The affine gauge fields in $CP(3)$

Pseudo-electric and pseudo-magnetic fields arose as gauge fields with singular potentials at the degeneration points of Hamiltonian spectrum [28]. However, it is well known that the spectrum structure of degenerated linear operator is unstable relative small perturbation. Besides this it is known that re-parametrization of the monopole problem leads to its singularity-free Lagrangian form [29]. Therefore such degenerated Hamiltonians cannot serve as a source of real electromagnetic potentials.

In order to understand the true source of the electromagnetic fields we would like to study the affine unitary gauge fields arose under breakdown (reconstruction) of global $G = SU(4)$ symmetry of degenerated bi-spinors states of quantum electron to the local gauge group $H = S[U(1) \times U(3)]$ acting by state-dependent generators on "phase space" $CP(3)$. Deformation of quantum state under the action of the geodesic flow in $CP(3)$ is treated here as process of the internal motion "in" quantum electron.

The local eigen-dynamics of quantum system naturally related to geometric phase problem. The anholonomy of the wave function arose due to slowly variable environment was widely discussed by Berry and many other authors [28, 29]. It is clear that now we deal with a different problem: *Berry made accent on the global variation of wave function* *generated by an "external" influence whereas for us interesting the quantum invariants of infinitesimal internal variation of the setup.*

The geometric phase is an intrinsic property of the family of eigen-states. There are in fact a set of LDV's that like the geometric phase intrinsically depends on eigen-states. For us will be interesting only the set comprising vector field $\xi^k(\pi^1, ..., \pi^{N-1}) : CP(N-1) \to \mathcal{C}$ in local coordinates $\pi^i_{(j)}$. In view of future discussion of "instant" transformations of LDV's it is useful to compare *velocity* of variation of the Berry's phase

$$\dot{\gamma}_n(t) = -\mathbf{A}_n(\mathbf{R})\dot{\mathbf{R}}, \tag{12}$$

where $\mathbf{A}_n(\mathbf{R}) = \Im < n(\mathbf{R})|\nabla_{\mathbf{R}} n(\mathbf{R}) >$ with the affine parallel transport of the vector field $\xi^k(\pi^1, ..., \pi^{N-1})$ given by the equations

$$\frac{d\xi^i}{d\tau} = -\Gamma^i_{kl}\xi^k \frac{d\pi^l}{d\tau}. \tag{13}$$

The parallel transport of Berry is similar to the affine parallel transport but the last one is fundamental in respect of gauge and scale-invariant conservation law agrees with Fubini-Study "quantum metric tensor" G_{ik^*} in the base manifold $CP(N-1)$. The affine gauge field given by the connection (3) is of course more close to the Wilczek-Zee non-Abelian gauge fields [30] where the Higgs potential has been replaced by the affine gauge potential (3) whose shape is depicted in Fig. 1. It is involved in the affine parallel transport of LDV's [12, 22] which agrees with the Fubini-Study metric (2).

The transformation law of the connection form $\Gamma^i_k = \Gamma^i_{kl} d\pi^l$ in $CP(N-1)$ under the differentiable transformations of local coordinates $\Lambda^i_m = \frac{\partial \pi^i}{\partial \pi'^m}$ is as follows:

$$\Gamma'^i_k = \Lambda^i_m \Gamma^m_j \Lambda^{-1j}_k + d\Lambda^i_s \Lambda^{-1s}_k. \tag{14}$$

It is similar but not identical to well known transformations of non-Abelian fields. The affine Cartan's moving reference frame takes here the place of "flexible quantum setup", whose motion refers to itself with infinitesimally close coordinates. Thus we will be rid of necessity in "second particle" [31, 12, 15, 16] as an external reference frame. Such construction perfectly fits for the quantum formulation of the quantum inertia principle [11] since the affine parallel transport of energy-momentum vector field in $CP(N-1)$ expresses the self-conservation of, say, electron.

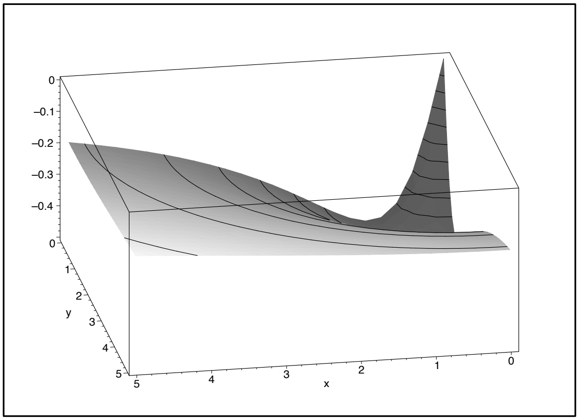

Figure 1: The shape of the gauge potential associated with the affine connection in CP(1): $\Gamma = -2\frac{|\pi|}{1+|\pi|^2}, \pi = x + iy$.

The Fubini-Study "quantum metric tensor" defines not only Riemannian metric and affine connection in $CP(N-1)$ but the symplectic structure too [35]. It is interesting that Schrödinger state vector related to a flux throughout compact projective Hilbert space $CP(N-1)$ that follows from the Berry's formula for 2-form [28]. This formula being applied to state vector

$$|\Psi(\pi, x)>= \sum_a \psi^a(\pi)|a, x> . \tag{15}$$

whose Fourier components $\psi^a(\pi)$ are expressed in local coordinates [16] gives the antisymmetric second-rank tensor

$$\begin{aligned} V_{ik*}(\pi^i) &= \Im \sum_{a=1}^{4} \{ \frac{\partial \psi^{a*}}{\partial \pi^i} \frac{\partial \psi^a}{\partial \pi^{k*}} - \frac{\partial \psi^{a*}}{\partial \pi^{k*}} \frac{\partial \psi^a}{\partial \pi^i} \} \\ &= -\Im[(1 + \sum |\pi^s|^2)\delta_{ik} - \pi^{i^*}\pi^k](1 + \sum |\pi^s|^2)^{-2} \\ &= -\Im G_{ik^*} \end{aligned} \tag{16}$$

that is simply the imaginary part of the Fubini-Study quantum metric tensor. There are following important differences between original Berry's formula referring to arbitrary parameters and this 2-form in local coordinates inherently related to eigen-problem.

1. The $V_{ik*}(\pi^i) = iG_{ik^*}$ is the singular-free expression.

2. It does not contain two eigen-values, say, E_n, E_m explicitly, but implicitly $V_{ik*} = iG_{ik^*}$ depends locally on the choice of single λ_p through the dependence in local coordinates $\pi^i_{j(p)}$. Even in the case of degenerated eigen-value, the reason of the anholonomy lurks in the curvature of $CP(3)$ and therefore it has intrinsically invariant and stable character.

3. It is impossible of course directly identify $V_{ik*} = iG_{ik^*}$ with electromagnetic tensor $F_{ij} = A_{j,i} - A_{i,j}$. We try to understand how the geometry of $CP(3)$ generates electromagnetic-like potentials in terms of "filed-shell" equations for energy-momentum [11].

Following changes have been done in order to derive these equations:

1. The local projective coordinates (π^1, π^2, π^3) of generalized coherent state (GCS) of electron will be use instead of Berry parameters $\mathbf{X}$ of a Hamiltonian $H(\mathbf{X})$ and therefore the quantum metric tensor is the Fubini-Study metric tensor of in $CP(3)$ [11, 12];

2. The iteration procedure of the Foldy-Wouthuysen transformations was replaced by the infinitesimal action of local dynamical variables (LDV) represented by tangent vector fields on $CP(3)$ diffeomorphic to the coset sub-manifold $G/H = SU(4)/S[U(1) \times U(3)] = CP(3)$;

3. The "covariant derivatives" in space-time like $i\hbar\frac{\partial}{\partial x^\mu} + \frac{e}{c}A_\mu$ usually used in QFT has been replaced by the affine covariant derivative $\frac{\partial P^i}{\partial \pi^m} + \Gamma^i_{mn}P^n$ of energy-momentum vector field on the base manifold of the internal degrees of freedom $CP(3)$. Thereby space-time should be included not as manifold of arguments of functions from Hilbert space H_D in the direct product $S = C^4 \times H_D$ but in fibre bundle over $CP(3)$ as will be described below.

6 Dynamical space-time

How one may mark physically some place and even build space-time itself with help of only quantum tools? In classical physics one may use material point, solid scale and a clock, but in quantum physics it is not appropriate approach.

Let assume that one has, say, a quantum "free" electron with charge and spin and its quantum state is given by the local projective coordinates (π^1, π^2, π^3). Dynamical structure of quantum electron should be accepted seriously since now the inertia principle refers just to internal quantum state without evident reference to space-time coordinates.

Hence one need initially to deal with the dynamics of quantum degrees of freedom.

The distance between two quantum states of electron in $CP(3)$ may be measured by the interval invariant relative the Fubibi-Study metric $dS_{F.-S.} = G_{ik*}d\pi^i d\pi^{k*}$. The speed of the interval variation is given by the equation

$$(\frac{dS_{F.-S.}}{d\tau})^2 = G_{ik*}\frac{d\pi^i}{d\tau}\frac{d\pi^{k*}}{d\tau} = \frac{c^2}{\hbar^2}G_{ik*}(\Phi^i_\mu P^\mu)(\Phi^{k*}_\nu P^{\nu*}) \qquad (17)$$

relative "quantum proper time" τ where energy-momentum vector field $P^\mu(x)$ obeys field equations that will be derived later. This internal dynamics should be expressed in space-time coordinates x^μ assuming that variation of coordinates δx^μ arise due to the transformations of Lorentz reference frame "centered" about covariant derivative $\frac{\delta P^\nu}{\delta\tau} = \frac{\delta x^\mu}{\delta\tau}(\frac{\partial P^\nu}{\partial x^\mu} + \Gamma^\nu_{\mu\lambda}P^\lambda)$ in dynamical space-time (DST). Such procedure may be called "inverse representation" [11, 12] since this intended to represent quantum motions in $CP(3)$ by "quantum Lorentz transformation" in DST as will be described below.

Local Lorentz frame will be built on the basis of the qubit spinor η whose components may be locally (in $CP(3)$) adjusted by "quantum boosts" and "quantum rotations" so that the velocity of the spinor variation coincides with velocity variation of the Jacobi vector field: tangent $\eta^0 = J_{tang}(\pi) = (a_i\tau + b_i)U^i(\pi)$ and the normal Jacobi vector field components $\eta^1 = J_{norm}(\pi) = [c_i \sin(\sqrt{\kappa}\tau) + d_i \cos(\sqrt{\kappa}\tau)]U^i(\pi)$ showing deviation from one geodesic to another [32]. The sectional curvature κ of $CP(3)$ is not specified yet. This "measurement" means that in the DST only deviations from the geodesic motion is "observable" due to electromagnetic-like field surrounding electron. This field may be mapped onto DST if one assumes that transition from one GCS of the electron to another is accompanied by dynamical transition from one Lorentz frame to another. Thereby, infinitesimal Lorentz transformations define small DST due to coordinate variations δx^μ.

It is convenient to take Lorentz transformations in the following form

$$\begin{aligned} ct' &= ct + (\vec{x}\vec{a}_Q)\delta\tau \\ \vec{x'} &= \vec{x} + ct\vec{a}_Q\delta\tau + (\vec{\omega}_Q \times \vec{x})\delta\tau \end{aligned} \qquad (18)$$

where we put for the parameters of quantum acceleration and rotation the definitions $\vec{a}_Q = (a_1/c, a_2/c, a_3/c)$, $\quad \vec{\omega}_Q = (\omega_1, \omega_2, \omega_3)$ [34] in order to have for τ the physical dimension of time. The expression for the "4-velocity" V^μ is as follows

$$V^\mu_Q = \frac{\delta x^\mu}{\delta \tau} = (\vec{x}\vec{a}_Q, ct\vec{a}_Q + \vec{\omega}_Q \times \vec{x}). \tag{19}$$

The coordinates x^μ of imaging point in dynamical space-time serve here merely for the parametrization of the energy-momentum distribution in the "field shell" described by quasi-linear field equations [11, 12] that will be derived below.

Any two infinitesimally close spinors η and $\eta + \delta\eta$ may be formally connected with infinitesimal $SL(2,C)$ transformations represented by "Lorentz spin transformations matrix" [34]

$$\hat{L} = \begin{pmatrix} 1 - \frac{i}{2}\delta\tau(\omega_3 + ia_3) & -\frac{i}{2}\delta\tau(\omega_1 + ia_1 - i(\omega_2 + ia_2)) \\ -\frac{i}{2}\delta\tau(\omega_1 + ia_1 + i(\omega_2 + ia_2)) & 1 - \frac{i}{2}\delta\tau(-\omega_3 - ia_3) \end{pmatrix} \tag{20}$$

Then "quantum accelerations" a_1, a_2, a_3 and "quantum angular velocities" $\omega_1, \omega_2, \omega_3$ may be found in the linear approximation from the equation $\delta\eta = \hat{L}\eta - \eta$ and from the equations for the velocities ξ of η spinor variations expressed in two different forms:

$$\hat{R}\begin{pmatrix}\eta^0 \\ \eta^1\end{pmatrix} = \frac{1}{\delta\tau}(\hat{L} - \hat{1})\begin{pmatrix}\eta^0 \\ \eta^1\end{pmatrix} = \begin{pmatrix}\xi^0 \\ \xi^1\end{pmatrix} \tag{21}$$

and

$$\begin{pmatrix}\xi^0 \\ \xi^1\end{pmatrix} = \begin{pmatrix} \frac{\delta\{(a_i\tau + b_i)U^i(\pi)\}}{\delta\tau} \\ \frac{\delta\{[c_i \sin(\sqrt{\kappa}\tau) + d_i \cos(\sqrt{\kappa}\tau)]U^i(\pi)\}}{\delta\tau} \end{pmatrix} \tag{22}$$

Since $CP(3)$ is totally geodesic manifold [35], each geodesic belongs to some $CP(1)$ parameterized by the single complex variable $\pi = e^{-i\phi}\tan(\theta/2)$. Then the tangent vector field $U(\pi) = \frac{\delta\pi}{\delta\tau} = \frac{\partial\pi}{\partial\theta}\frac{\delta\theta}{\delta\tau} + \frac{\partial\pi}{\partial\phi}\frac{\delta\phi}{\delta\tau}$, where

$$\begin{aligned}\frac{\delta\theta}{\delta\tau} &= -\omega_3\sin(\theta) - ((a_2+\omega_1)\cos(\phi) + (a_1-\omega_2)\sin(\phi))\sin(\theta/2)^2 \\ &\quad -((a_2-\omega_1)\cos(\phi) + (a_1+\omega_2)\sin(\phi))\cos(\theta/2)^2; \\ \frac{\delta\phi}{\delta\tau} &= a_3 + (1/2)(((a_1-\omega_2)\cos(\phi) - (a_2+\omega_1)\sin(\phi))\tan(\theta/2) \\ &\quad -((a_1+\omega_2)\cos(\phi) - (a_2-\omega_1)\sin(\phi))\cot(\theta/2)).\end{aligned} \tag{23}$$

will be parallel transported, i.e. $U^{'}(\pi) = \frac{\delta^2\pi}{\delta\tau^2} = 0$. The linear system of 6 real non-homogeneous equation

$$\begin{aligned}
\Re(\hat{R}_{00}\eta^0 + \hat{R}_{01}\eta^1) &= \Re(\xi^0), \\
\Im(\hat{R}_{00}\eta^0 + \hat{R}_{01}\eta^1) &= \Im(\xi^0), \\
\Re(\hat{R}_{10}\eta^0 + \hat{R}_{11}\eta^1) &= \Re(\xi^1), \\
\Im(\hat{R}_{10}\eta^0 + \hat{R}_{11}\eta^1) &= \Im(\xi^1), \\
\frac{\delta\theta}{\delta\tau} &= F_1, \\
\frac{\delta\phi}{\delta\tau} &= F_2,
\end{aligned} \tag{24}$$

gives the “quantum boost” $\vec{a}_Q(\Re(\eta^0), \Im(\eta^0), \Re(\eta^1), \Im(\eta^1), \theta, \phi)$, and “quantum rotation” $\vec{\omega}_Q(\Re(\eta^0), \Im(\eta^0), \Re(\eta^1), \Im(\eta^1), \theta, \phi)$. The components of the spinor $(\Re(\eta^0), \Im(\eta^0), \Re(\eta^1), \Im(\eta^1))$ should be agreed with the coefficients (a, b, c, d) into (24) by the condition of solvability of the system (24) reads as $Det|...| = 0$ for the extended matrix $|...|$. Two frequencies (F_1, F_2) will be found from the spectrum of excitations discussed below. One frequency gives the coset deformation acting along some geodesic in $CP(3)$ and the second one gives the velocity of rotation of the geodesic under the action of the gauge isotropy group $H = U(1) \times U(3)$.

7 “Field-shell” equations for non-local quantum electron

What the inertial principle means for quantum systems and their states? Formally the inertial principle is tacitly accepted in the package with relativistic invariance. But we already saw that the problem of identification and therefore the localization of quantum particles in classical space-time is problematic and it requires a clarification.

Let formulate the quantum inertia law in the case of non-local quantum electron [11, 12] as follows:

inertial motion of quantum electron may be expressed as a self-conservation of its local dynamical variables like energy-momentum, spin, charge, etc.

Then the conservation law of the energy-momentum vector field in $CP(3)$ during inertial evolution will be expressed by the equation of the affine parallel transport

$$\frac{\delta P^i(\pi, x)}{\delta\tau} = \frac{\delta[\Phi^i_\mu(\pi)P^\mu(x)]}{\delta\tau} = 0. \tag{25}$$

This is equivalent to the following system of the four coupled quasi-linear PDE for dynamical space-time distribution of energy-momentum "field shell" of the quantum state

$$V^\mu_Q(\frac{\partial P^\nu}{\partial x^\mu}+\Gamma^\nu_{\mu\lambda}P^\lambda)=-\frac{c}{\hbar}(\Gamma^m_{mn}\Phi^n_\mu(\gamma)+\frac{\partial\Phi^n_\mu(\gamma)}{\partial\pi^n})P^\nu P^\mu \qquad (26)$$

if the ordinary differential equations for the relative amplitudes

$$\frac{d\pi^k}{d\tau}=\frac{c}{\hbar}\Phi^k_\mu P^\mu, \qquad (27)$$

will be treated as *equations of characteristic* for linear "super-Dirac" equation

$$iP^\mu\Phi^i_\mu(\gamma_\mu)\frac{\partial\Psi}{\partial\pi^i}=mc\Psi, \qquad (28)$$

that supposes ODE for single "total state function"

$$i\hbar\frac{d\Psi}{d\tau}=mc^2\Psi \qquad (29)$$

with the solution for variable mass $m(\tau)$

$$\Psi(T)=\Psi(0)e^{-i\gamma_C}e^{-i\frac{c^2}{\hbar}\int_0^T m(\tau)d\tau}. \qquad (30)$$

We will discuss now the solution of the "field-shell" equations (26). The theory of these equations is well known. Particularly, our system is the system with identical principle part V^μ_Q which is properly discussed in the Application 1 to the Chapter II [33]. Such quasi-linear PDE system with the identical principle part V^μ_Q has ODE's of its characteristics

$$\begin{aligned}
\frac{\delta x^\nu}{\delta\tau}&=V^\nu_Q,\\
\frac{\delta P^\nu}{\delta\tau}&=-V^\mu_Q\Gamma^\nu_{\mu\lambda}P^\lambda-\frac{c}{\hbar}(\Gamma^m_{mn}\Phi^n_\mu(\gamma)+\frac{\partial\Phi^n_\mu(\gamma)}{\partial\pi^n})P^\nu P^\mu\\
\frac{d\pi^k}{d\tau}&=\frac{c}{\hbar}\Phi^k_\mu P^\mu.
\end{aligned} \qquad (31)$$

The result of integration the one of the "cross" combination is as follows

$$\frac{\delta x^0}{V^0_Q}=\frac{\delta P^0}{-V^\mu_Q\Gamma^0_{\mu\lambda}P^\lambda+P^0L_\mu P^\mu}, \qquad (32)$$

where $L_\mu = -\frac{c}{\hbar}(\Gamma^m_{mn}\Phi^n_\mu(\gamma) + \frac{\partial \Phi^n_\mu(\gamma)}{\partial \pi^n})$. If $L_0 L_\alpha < 0$ then one gives implicit solution

$$\frac{x^0}{a_\alpha x^\alpha} + t^0 = -\frac{2}{\sqrt{4L_0 V^\lambda_Q \Gamma^0_{\lambda\alpha} P^\alpha + (V^\lambda_Q \Gamma^0_{\lambda 0} - L_\alpha P^\alpha)^2}} \times \tanh^{-1}\left(\frac{2L_0 P^0 - (V^\lambda_Q \Gamma^0_{\lambda 0} - L_\alpha P^\alpha)}{\sqrt{4L_0 V^\lambda_Q \Gamma^0_{\lambda\alpha} P^\alpha + (V^\lambda_Q \Gamma^0_{\lambda 0} - L_\alpha P^\alpha)^2}}\right), \qquad (33)$$

where t^0 is an integration constant. The explicit solution for the energy kink is as follows

$$P^0 = \frac{1}{2L_0}[V^\lambda_Q \Gamma^0_{\lambda 0} - L_\alpha P^\alpha - \sqrt{4L_0 V^\lambda_Q \Gamma^0_{\lambda\alpha} P^\alpha + (V^\lambda_Q \Gamma^0_{\lambda 0} - L_\alpha P^\alpha)^2} \times \tanh(-\frac{x^0 + x^\alpha a_\alpha t^0}{2x^\alpha a_\alpha}\sqrt{4L_0 V^\lambda_Q \Gamma^0_{\lambda\alpha} P^\alpha + (V^\lambda_Q \Gamma^0_{\lambda 0} - L_\alpha P^\alpha)^2}]. \qquad (34)$$

One of the simplified explicit solution ($\Gamma^0_{\lambda 0} = 0$) is the energy kink

$$P^0 = \frac{L_\alpha P^\alpha}{2L_0}[\tanh(-(\frac{x^0}{a_\alpha x^\alpha} + t^0)\frac{L_\alpha P^\alpha}{2}) - 1], \qquad (35)$$

where $\frac{L_\alpha P^\alpha}{2} = 1, L_0 = 1, V = a_\alpha x^\alpha = 0.6$ that represented by the graphic in Fig. 2. This solution represent the lump of electron self-

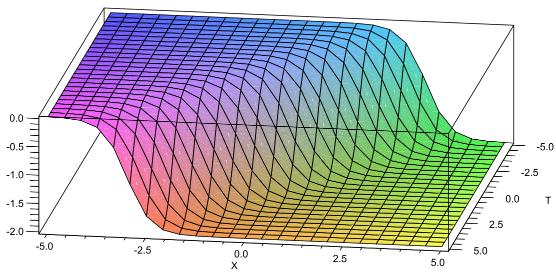

Figure 2: The kink solution of the quasi-linear PDE's in dynamical space-time showing the distribution of energy-momentum "field-shell" of extended quantum electron. It is not solution of a runaway type.

interacting through electromagnetic-like field in co-moving Lorentz reference frame. In the standard QED the self-interacting effects are treated as a polarization of the vacuum. It the present picture the lump is dynamically self-supporting system whose characteristics define by the system of four ODE's

$$\frac{\delta P^{\nu}}{\delta \tau}=-V_Q^{\mu}\Gamma^{\nu}_{\mu\lambda}P^{\lambda}-\frac{c}{\hbar}(\Gamma^m_{mn}\Phi^n_{\mu}(\gamma)+\frac{\partial \Phi^n_{\mu}(\gamma)}{\partial \pi^n})P^{\nu}P^{\mu}. \tag{36}$$

In order to find the off-shell dispersion law one should analyse the equations of characteristics (36) [36]. The standard approach to stability analysis instructs us to find the stationary points. The stationary condition

$$\frac{\delta P^{\lambda}}{\delta \tau}=0 \tag{37}$$

leads to the system of algebraic equations

$$V_Q^{\mu}\Gamma^{\nu}_{\mu\lambda}P^{\lambda}+\frac{c}{\hbar}(\Gamma^m_{mn}\Phi^n_{\mu}(\gamma)+\frac{\partial \Phi^n_{\mu}(\gamma)}{\partial \pi^n})P^{\nu}P^{\mu}=0. \tag{38}$$

One needs to investigate these equations for the stationary points in the non-trivial case $P_0^{\mu}\neq 0$.

The approach to the numerical analysis of these attractors is as follows. Let find initially solution of the non-linear system (7). Its approximate solution in the vicinity of $P^{\mu}_{test}=(mc^2,0,0,0)$ has been found by the method of Newton:

$$P_0^{\mu}=P^{\mu}_{test}+\delta^{\mu}+...,\tag{39}$$

where δ^{μ} is the solution of the Newton's first approximation equations

$$\begin{aligned}
&(2L_0mc+K^0_0)\delta^0+(L_1mc+K^0_1)\delta^1+\\
&\quad(L_2mc+K^0_2)\delta^2+(L_3mc+K^0_3)\delta^3=-\frac{(L_0m^2c^4+K^0_0mc^3)}{c^2}\\
&K^1_0\delta^0+(L_0mc+K^1_1)\delta^1+K^1_2\delta^2+K^1_3\delta^3=-K^1_0mc\\
&K^2_0\delta^0+K^2_1\delta^1+(L_0mc+K^2_2)\delta^2+K^2_3\delta^3=-K^2_0mc\\
&K^3_0\delta^0+K^3_1\delta^1+K^3_2\delta^2+(L_0mc+K^3_3)\delta^3=-K^3_0mc,
\end{aligned}\tag{40}$$

where $L_{\mu}=(\Gamma^m_{mn}\Phi^n_{\mu}(\gamma)+\frac{\partial \Phi^n_{\mu}(\gamma)}{\partial \pi^n})$ are now dimensionless. The solution of this system is as follows:

$$\delta^0=-mc^2\frac{mcL_0^2-K^1_0L_1-K^2_0L_2-K^3_0L_3}{2mcL_0^2-K^1_0L_1-K^2_0L_2-K^3_0L_3}$$

$$\delta^1 = -mc\frac{K^1_0 L_1}{2mcL^2_0 - K^1_0 L_1 - K^2_0 L_2 - K^3_0 L_3}$$
$$\delta^2 = -mc\frac{K^2_0 L_2}{2mcL^2_0 - K^1_0 L_1 - K^2_0 L_2 - K^3_0 L_3}$$
$$\delta^3 = -mc\frac{K^3_0 L_3}{2mcL^2_0 - K^1_0 L_1 - K^2_0 L_2 - K^3_0 L_3}. \quad (41)$$

The probing solution in the vicinity of the stationary points P^μ_0 is as follows

$$P^\mu(\tau) = P^\mu_0 + p^\mu e^{\omega\tau}. \quad (42)$$

The "optical" dispersion law with a mass-gap and state-dependent attractor corresponding to finite mass of the electron follows from the equation (38). The homogeneous linear system

$$\frac{\hbar\omega}{c}p^\nu + \frac{\hbar}{c}V^\mu_Q\Gamma^\nu_{\mu\lambda}p^\lambda + (\Gamma^m_{mn}\Phi^n_\mu(\gamma) + \frac{\partial\Phi^n_\mu(\gamma)}{\partial\pi^n})p^\mu P^\nu_0 = 0. \quad (43)$$

has the determinant

$$D_1 = (\frac{\hbar\omega}{c})^4 + \alpha(\frac{\hbar\omega}{c})^3 + \beta(\frac{\hbar\omega}{c})^2 + \gamma(\frac{\hbar\omega}{c}) + \delta, \quad (44)$$

with complicated coefficients $\alpha, \beta, \gamma, \delta$. If one puts $K^\nu_\lambda = \frac{\hbar}{c}V^\mu_Q\Gamma^\nu_{\mu\lambda}$ and $M^\nu_\mu = (\Gamma^m_{mn}\Phi^n_\mu(\gamma) + \frac{\partial\Phi^n_\mu(\gamma)}{\partial\pi^n})P^\nu_0$ then

$$\alpha = Tr(K^\nu_\lambda) + Tr(M^\nu_\mu) \quad (45)$$

and

$$\begin{aligned}\beta = -[&K^0_0(L_1P^1_0 + L_2P^2_0 + L_3P^3_0) + K^1_1(L_0P^0_0 + L_2P^2_0 + L_3P^3_0)\\ &+K^2_2(L_1P^1_0 + L_0P^0_0 + L_3P^3_0) + K^3_3(L_1P^1_0 + L_0P^0_0 + L_2P^2_0)\\ &-K^0_1L_0P^1_0 - K^1_0L_1P^0_0 - K^0_2L_0P^2_0 - K^2_0L_2P^0_0 - K^0_3L_0P^3_0 - K^3_0L_3P^0_0\\ &-K^1_2L_1P^2_0 - K^2_1L_2P^1_0 - K^1_3L_1P^3_0\\ &-K^3_1L_3P^1_0 - K^2_3L_2P^3_0 - K^3_2L_3P^2_0].\end{aligned} \quad (46)$$

Coefficients γ, δ have higher order in $\frac{\hbar}{c}$ and they will be temporarily discarded in approximate dispersion law. This dispersion law may be written as follows

$$(\frac{\hbar\omega}{c})^2[(\frac{\hbar\omega}{c})^2 + \alpha(\frac{\hbar\omega}{c}) + \beta] = 0. \quad (47)$$

The trivial solution $\omega_{1,2} = 0$ has already been discussed [11]. Two non-trivial solutions when $\alpha^2 \gg \beta$ are given by the equations

$$\hbar\omega_{3,4} = c\alpha \frac{-1 \pm \sqrt{1 - \frac{4\beta}{\alpha^2}}}{2} \approx c\alpha \frac{-1 \pm (1 - \frac{2\beta}{\alpha^2})}{2};$$
$$\hbar\omega_3 = \frac{-c\beta}{\alpha}, \quad \hbar\omega_4 = -c\alpha + \frac{c\beta}{\alpha}. \tag{48}$$

The negative real part of these two roots of ω being substituted in the probing function (42) will define attractors and two finite masses.

If hypothesis about dynamical nature of electron mass defined by self-interacting spin/charge degrees of freedom is correct then it is very natural to assume that

$$F_1 = \frac{\delta\theta}{\delta\tau} = \Re(\omega_3) = \frac{c}{\hbar}\Re(\frac{-\beta}{\alpha}), or$$
$$F_1 = \frac{\delta\theta}{\delta\tau} = \Re(\omega_4) = \frac{c}{\hbar}\Re(-\alpha + \frac{\beta}{\alpha}), and$$
$$F_2 = \frac{\delta\phi}{\delta\tau} = \Im(\omega_3) = \frac{c}{\hbar}\Im(\frac{-\beta}{\alpha}), or$$
$$F_2 = \frac{\delta\phi}{\delta\tau} = \Im(\omega_4) = \frac{c}{\hbar}\Im(-\alpha + \frac{\beta}{\alpha}). \tag{49}$$

Numerical solutions of complicated self-consistent problem (23), (24), (49) is not found yet.

8 Conclusion

The physically correct transition from quantum to classical mechanics arose as a serious problem immediately after the formulation of "wave mechanics" of Schrödinger [26]. The failure to build stable wave packet for single electron from solutions of linear PDE's lead to statistical interpretation of the wave function. The further progress in the theory of non-linear PDE's like sin-Gordon or KdV renewed generally the old belief in possibility to return to deterministic quantum physics of "elementary" particles [27]. The fundamental problem is, however, to find "first" physical principles capable lead to quantum non-linear PDE's.

The deep difficulty of the definition of single body acceleration without reference to the "space" or remote stars had come to us by inheritance from Newton, Mach and Einstein. The ghost of this problem lives for a long time in the castle of QFT, and it vexes increasingly to its happy inhabitants of QED-rooms [37]. Namely, runaway solutions of relativistic and non-relativistic classical equations of motion with radiation reaction and divergences in QED are rooted in the deep inconsistency of relativity and quantum principles. The development of the high energy physics and especially "relativistic optics" [38] put the theoretical fundamental problem of self-interacting extended quantum particles in a practical plane. In the present article we discuss the simple version of the intrinsic unification of quantum and relativity principles as a general framework for the self-interacting quantum electron. Solutions of the quasi-liner PDE's for the energy-momentum "field-shell" distribution is shortly discussed. Analysis of the equations of motion without runaway solution will be reported elsewhere.

References

[1] Planck M., On the Law of Distribution of Energy in the Normal Spectrum, Ann. Phys., **4**, 553-562 (1901).

[2] Einstein A., Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt, Ann. Phys., **17**, 132-148 (1905).

[3] de Broglie L., Recherches sur la Théorie des Quanta, (Ann. de Phys. 10^e série, t.III (Janvier-Février 1925). Translated by A.F. Kracklauer, ©AFK, 2004.

[4] Schrödinger E., Quantisierung als Eigenwertprobleme, Ann. Phys. **79**, 361-376 (1926).

[5] Dirac P.A., The quantum theory of the emission and absorption of radiation, Proc. Royal. Soc. A **114**, 243-265 (1927).

[6] Georgi H., Unparticle Physics, arXiv:hep-th/0703260v3.

[7] Georgi H., Another Odd Thing About Unparticle Physics, arXiv:0704.2457v2.

[8] Gaete P., Spallucci E., Un-particle Effective Action, arXiv:0801.2294v1.

[9] Blochintzev D.I., Whether always the "duality" of waves and particles does exist?, Uspechy Phys. Nauk, **XLIV**, No.1, 104-109 (1951).

[10] Blochintzev D.I., Elementary particles and field, Uspechy Phys. Nauk, **XLII**, No.1, 76-92 (1950).

[11] P. Leifer, arXiv:0904.3695v4 [physics.gen-ph].

[12] P. Leifer, The quantum content of the inertia law and field dynamics, arXiv:1009.5232v1. (Submitted to EJTP)

[13] L. de Broglie, Ann. de Phys., 10^e série, **III**, Janvier-Février (1925).
[14] E. Schrödinger, Ann. Physik, **79**, 361, 489 (1926).
[15] P. Leifer, Found. Phys. **27**, (2) 261 (1997).
[16] P. Leifer, Annales de la Fondation Louis de Broglie, **32**, (1) 25 (2007).
[17] P. Leifer, JETP Letters, **80**, (5) 367 (2004).
[18] Y. Aharonov et al, Mearurement of Time-Arrival in Quantum Mechanics, Phys. Rev., **A57**,4130 (1998).
[19] T.D. Newton and E.P. Wigner, Localized States for Elementary Systems, Rev. Mod. Phys., **21**, No.3, 400-406 (1949).
[20] L.L. Foldy, S.A. Wouthuysen, Phys. Rev., **78**, 29 (1950).
[21] P. Leifer, Inertia as the "threshold of elasticity" of quantum states, Found.Phys.Lett., **11**, (3) 233 (1998).
[22] P. Leifer, An affine gauge theory of elementary particles, Found.Phys.Lett., **18**, (2) 195-204 (2005).
[23] A. Einstein, Ann. Phys. Zur Electrodynamik der bewegter Körper, **17**, 891-921 (1905).
[24] P.A.M. Dirac, The principls of quantum mechanics, Fourth Edition, Oxford, At the Clarebdon Press, 1958.
[25] A. Einstein, Ann. Phys. Die Grunlage der allgemeinen Relativitätstheorie, **49**, 769-822 (1916).
[26] E. Schrödinger, Nanurwissenschaften, **14**, H 28, 664, 1926.
[27] R. Rajaraman, *An Introsuction to Solitons and Instantons in Quantum Field Theory*, Norht-Holland Publishing Company, Amaterdam, New-York, Oxford (1982).
[28] M.V. Berry, "The Quantum Phase, Five Years After" in *Geometric Phases in Physics*, World Scientific. 1989.
[29] I.J.R. Aitchison, "Berry phases, magnetic monopoles, and Wess-Zumino terms or how the skyrmion got its spin", in *Geometric Phases in Physics*, World Scientific. 1989.
[30] F. Wilczek and A. Zee, Phys. Rev. Lett., **52**, (24) 2111-2114 (1984).
[31] J. Anandan, Y. Aharonov, Phys. Rev. D, **38**, (6) 1863-1870 (1988).
[32] Besse A.L., Manifolds all of whose Geodesics are Closed, Springer-Verlag, Berlin, Heidelberg, New Yourk, (1978).
[33] R. Courant, D. Hilbert, *Methods of Mathematical Physics*, Vol. 2, Partial Differential Equations (Wiley, 1989).
[34] Misner C.W., Thorne K.S., Wheeler J.A.,Gravitation, 1279, W.H.Freeman and Company, San Francisco (1973).
[35] Kobayashi S., and Nomizu K., 414, Foundations of Differential Geometry, V. II, Interscience Publishers, New York-London-Sydney, (1969).
[36] M.I. Rabinivich, D.I. Trubetzkov, *Introduction in the theory of oscillations and waves*, (Moscow, 1984).

[37] R.T. Hammond, “Relativistic Particle Motion and Radiation Reaction in Electrodynamics” in EJTP 7, No. 23 (2010) 221 - 258.

[38] G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. 78, 309 (2006).

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