A new electromagnetism based on 4 photons : electric, magnetic, with spin 1 and spin 0 Part II, Graviton

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(The numeration of paragraphs and formulae follows the Part I)

11 General theory of particles with a maximum spin n.

After his *New Theory of Light,* de Broglie developped a *General theory of particles with spin* (*Broglie 4*), which includes a theory of graviton. Unfortunately, we can give here this theory only in a short form[1], but we shall see that the magnetic monopole will play a role, including the correction of some errors.

11.1 The general process of fusion.

Extending the formulae (15), the fusion of n Dirac equations takes the general form :

$$\frac{1}{c}\frac{\partial\phi_{ikl\ldots}}{\partial t} = a_k^{(p)}\frac{\partial\phi_{ikl\ldots}}{\partial x_k} + i\frac{\mu_0 c}{\hbar}a_4^{(p)}\phi_{ikl\ldots} \tag{52}$$

where $p = 1, 2, \ldots n$. Thus we have n equations instead of 2, and a 4^n component wave function (a spinor of *n-th* rank) instead of 16 components for the photon. And there are $4n$ matrices $\left(a_r^{(p)}\right))$ with 4^{2n} elements :

$$\left(a_r^{(p)}\right)_{ik\ldots opq\ldots,i'k',\ldots o'p'q'\ldots} = \delta_{ii'}\delta_{kk'}\ldots\delta_{oo'}\left(\alpha_r\right)_{pp'}\delta_{qq'}\ldots \tag{53}$$

[1] Actually this paragraph 11 gives only some results that constitute a kind of guide for the reading of (*Broglie 4*).

They obey the following relations, generalising (17) :

$$a_r^{(p)}a_s^{(p)} + a_s^{(p)}a_r^{(p)} = 2\delta_{rs} \quad ; \quad a_r^{(p)}a_s^{(q)} - a_s^{(q)}a_r^{(p)} = 0 \qquad (54)$$

The problem is that, there are n times too many equations. Actually it was already the case for the equations (15), the number of which was twice the number of functions. Here, in the general case, we have $n4^n$ equations for 4^ncomponents of the wave function. It is easy to answer the question, including the particular case (15) (*Broglie 4).*

First of all, we put (52) in a maxwellian form, putting :

$$F^{(p)} = a_k^{(p)}\frac{\partial}{\partial x_k} + i\frac{\mu_0 c}{\hbar}a_4^{(p)} \qquad (55)$$

We have the commutation relations :

$$F^{(p)}F^{(q)} = F^{(q)}F^{(p)}, \qquad \forall p, q \qquad (56)$$

(52) takes the following form which means that the wave components obey the Klein-Gordon equation :

$$\frac{1}{c}\frac{\partial \phi}{\partial t} = F^{(p)}\phi, \;\; p = 1, 2, \ldots n, \;\; \left(F^{(p)}\right)^2 = \triangle - k_0^2 \qquad (57)$$

By adding these equations, we find an evolution equation :

$$\frac{1}{c}\frac{\partial \phi}{\partial t} = F\phi \; ; \quad F = \frac{1}{n}\sum_{p=1}^{n} F^{(p)} \qquad (58)$$

Now, substracting the equations (52) one from each other in a convenient way, we can eliminate the time derivatives and find (n-1) "condition equations". It may be done in many ways. For instance we can choose the following system generalising the Maxwell "condition equations" :

$$G^{(p)}\phi = \frac{F^{(1)} - F^{(p)}}{2}\phi = 0 \quad (p = 2, 3, \ldots, n) \qquad (59)$$

It is easy to prove that the new system (58), (59) is equivalent to (52) or (57). Owing to (54)), one can see that F and G commute, but their product doesn't equal zero, contrary to what would have happened in the special case n=2:

$$G^{(p)}F = FG^{(p)} \neq 0 \qquad (60)$$

So, in the special case $n = 2$, we would have : $= 0$ in (60) and the condition (59) would be deducible from the evolution equation (58). Nevertheless, in the general case, the commutations (60) imply that the first members $G^{(p)}(\phi)$ of (59) are solutions of (58), so that, if the conditions (59) are satisfied at an initial time $t = 0$, they are satisfied at any time.

11.2 The quadricurrent density.

De Broglie defines a new set of matrices :

$$B_4^{(p)} = a_4^{(1)} a_4^{(2)} \dots a_4^{(p-1)} a_4^{(p+1)} \dots a_4^{(n)} \tag{61}$$

And the quadricurrent density is:

$$J_k = -c\phi^* \frac{1}{n} \sum_{p=1}^{n} a_k^p B_4^p \phi \quad ; \quad \rho = \phi^* \frac{1}{n} \sum_{p=1}^{n} B_4^p \phi \tag{62}$$

It is easy to verify that this current density is conservative:

$$\frac{\partial \rho}{\partial t} + \partial_k J_k = 0 \tag{63}$$

Following de Broglie, it is interesting to calculate a plane wave for a particle of maximum spin $n/2$ and then the density ρ ; the calculation is rather long (*Broglie 4*), but the result is simple :

$$\rho = \left(\frac{\mu_0 c^2}{W} \right)^{n-1} |\phi|^2 \tag{64}$$

Where : μ_0 = mass of the particle, W = energy, n = number of spin 1/2 particles composing the considered particle. We see that:

- If n is odd, the sign of ρ is positive-definite, as in the case $n = 1$ of a Dirac electron.

- If n is even, ρ has the same sign as energy, it is indefinite: it is the case of a photon (*spin 1*) and it will be the case of a graviton (*spin 2*).

It is interesting to note, with de Broglie, the curious presence in (64), of the $(n-1)$th power of the Lorentz contraction, which means that the density ρ, integrated on a volume ($\int \rho dv$), will be contracted exactly n

times (the number of elementary spin 1/2 particles). The exception is the Dirac particle, for which $n-1=0$, so that the factor disappears and the integral is only contracted by the integration volume itself. De Broglie conjectured that this factor is perhaps an echo of a complicated spatial structure of the composite particle, that we can describe only as a point, in the present state of linear quantum mechanics.

11.3 Energy density.

We begin with an elementary calculation of the *energy density,* using the preceding density ρ for a plane wave. By definition of the density ρ, all the mean values will be obtained by integration of a physical quantity multiplied by ρ. So, the energy density is obtained (for a plane wave) owing to (64) :

$$\rho W = \left(\frac{\mu_0 c^2}{W}\right)^{n-1} W\,|\phi|^2 \tag{65}$$

So that the power of W is not $(n-1)$ but $(n-2)$ and we find a result opposite to the result for ρ:

- If n is odd, ρW has the same indefinite sign as energy : it was the case for $n=1$, for a Dirac electron.

- If n is even, the sign of ρW is positive-definite, which is the case for a photon and the graviton. This is confirmed by more sophisticated calculations using the energy tensor density (*Broglie 4*).

Here, de Broglie introduces two classes of tensors named "corpuscular" and of "type M" (M for Maxwell), inspired by electromagnetism, but we do not need them in the present paper.

11.4 Spin

Just as it happens for the Dirac equation, it may be shown that the orbital moment operator is not conservative for a particle of maximum spin n. To find a conservative operator, we must add spin operators :

$$S_i = \hbar \sum_{p=1}^{n} s_i^{(p)}; \quad (i=1,2,3)\,; \quad S_4 = \hbar \sum_{p=1}^{n} s_4^{(p)} \tag{66}$$

With:

$$s_1^{(p)} = \frac{-i}{2} a_2^{(p)} a_3^{(p)}; \quad s_2^{(p)} = \frac{-i}{2} a_3^{(p)} a_1^{(p)};$$
$$s_3^{(p)} = \frac{-i}{2} a_1^{(p)} a_2^{(p)}; \quad s_4^{(p)} = \frac{-i}{2} a_1^{(p)} a_2^{(p)} a_3^{(p)} \tag{67}$$

One can find in (*Broglie 4*), the general nomenclature of spin states and (in the case of an *even number* of spin 1/2 particles) the decomposition of wave functions in terms of *tensor components*. This nomenclature is based on the Clebsch-Gordan theorem, for a product of irreducible representations, completed by de Broglie, who defines a set of independent constants in the case of a plane wave and the symmetry of tensors defined by an even number of particles.

These problems were treated in a different way, by (*Fierz 1*), whose work is based not on the fusion theory, but on the conditions added to the Klein-Gordon equation, to describe a spin $n/2$ particle. This point of view was developed by (*Fierz* & *Pauli 2, 3*) and on the basis of a previous work of Dirac on the generalization of the equation of the electron, for higher spin values (*Dirac 2*). But the de Broglie theory of spin particles, based on the fusion principle with mass terms (even very small), gives a far better harmony to the whole description, even from the point of view of of the group theory.

Now, the most interesting case of spin particles with $n > 1$ is $n = 4$, i.e. the case of maximum spin 2.

12 The particles with maximum spin 2. Graviton.

The first who found the analogy between the equation of a *spin 2 particle* and the linear approximation of the Einstein equation of a gravitationnal field were : (*Fierz & Pauli 2*). This linear approximation was given by Einstein himself (*Einstein 2, 3*) and it may be found, for instance, in (*Laue,* or *Möller*). The paper of (*Einstein 2*) was the first in which Einstein formulated the idea of gravitational waves. He even alluded to a possible modification of gravitation theory by quantum effects, in analogy with the modification of Maxwell's electromagnetism.

It must be stressed that the quantum theory of gravitation, developed by L. de Broglie, M.-A. Tonnelat and G. Pétiau (see : *Broglie 4*, *Tonnelat 1,2*) on the basis of the fusion method, is not based on a particle of spin 2 but on *a particle of maximum spin 2*. This point is important for two reasons :

1) The fusion theory raises the question : is the graviton a *composite particle*, just as the photon and all the other particles of spin >1/2 ?

2) In this theory, gravitons do not appear alone. They are linked with photons. This theory is actually **a unitary theory of gravitation and electromagnetism** (at least at the linear approximation). The fields are not gathered by an extended geometry, as in other attempts but by the fusion of spins.

12.1 Why gravitation and electromagnetism are linked ?

Formally one could say that "Fields are linked by Clebsch-Gordan's theorem" because :

$$D_{\frac{1}{2}} \times D_{\frac{1}{2}} \times D_{\frac{1}{2}} \times D_{\frac{1}{2}} = D_2 + 3D_1 + 2D_0 \tag{68}$$

Therefore, in the fusion of four spin 1/2 particles, we must find : one particle of spin 2, three particles of spin 1 and two particles of spin 0. In particular, we have gravitons and photons. The zero spin photon already appeared in the first part of the present paper.

Now, it is interesting to give an intuitive argument of de Broglie's theory. He defines a particle of maximum spin 2 by the fusion of two particles of spin 1, described by quadripotentials : $A_\mu^{(1)} = \{A^{(1)}, V\}$, $A_\mu^{(2)} = \{A^{(2)}, V\}$, and invariants $I_2^{(1)}, I_2^{(2)}$ ($I_1^{(1)}, I_1^{(2)} = 0$ because $\mu_0 \neq 0$) [2]. The fusion gives:

$$A_\mu^{(1)} \times A_\mu^{(2)};\ A_\mu^{(1)} \times I_2^{(2)};\ I_2^{(1)} \times A_\mu^{(2)};\ I_2^{(1)} \times I_2^{(2)} \tag{69}$$

The first product is a tensor of rank 2 that defines a symmetric and an antisymmetric tensor:

$$A_{(\mu\nu)} = \frac{A_{\mu\nu} + A_{\nu\mu}}{2} \quad ; \quad A_{[\mu\nu]} = \frac{A_{\mu\nu} - A_{\nu\mu}}{2} \tag{70}$$

The products $A_\mu^{(1)} \times I_2^{(2)}$ and $I_2^{(1)} \times A_\mu^{(2)}$ are vector-like quantities $P_\mu^{(1)}, P_\mu^{(2)}$ and it may be hoped that they will be photon potentials. The antisymmetric tensor $A_{[\mu\nu]}$ suggests an electromagnetic field.

The symmetric tensor $A_{(\mu\nu)}$ cannot be interpreted at this level of exposition, but actually we can guess that it will be related to gravitation.

[2]Here, we consider only the electric case.

De Broglie shows, owing to a study of plane waves, that $P_\mu^{(1)}$, $P_\mu^{(2)}$ and the antisymmetric tensor $A_{[\mu\nu]}$ are related to the spin 1; $A_{(\mu\nu)}$ is linked to spin 2, only if it is reduced to a zero trace tensor because $\mathrm{Sp}\, A_{(\mu\nu)} = A_{(\mu\mu)}$ is an invariant and it will be actually related to spin 0, just as the invariant $I_2^{(1)} \times I_2^{(2)}$.

Now it will be shown later in the case of the photon, that *the splitting between different spin states is not relativistically covariant* because it is based on the total spin operator which is not a relativistic invariant. **Therefore, in the fusion theory, gravitation cannot appear without electromagnetism.** Furthermore, it will be shown that, if $\mu_0 \neq 0$, the splitting between spin 2 and spin 0 is impossible, and the interpretation of this fact is highly interesting.

12.2 The tensorial equations of a particle of maximum spin 2.

We give only the tensorial form, generalizing the theory of light. The total wave equations (type (15) for $n = 4$) would have 4^4=256 components with 168 independent quantities (*Broglie 4*). They give five equations A, $B_{1,2,3}$, C :

$$(A)\quad \begin{aligned} \partial_\mu \phi_{(\nu\rho)} - \partial_\nu \phi_{(\mu\rho)} &= k_0 \phi_{[\mu\nu]\rho} \\ \partial_\rho \phi_{[\rho\mu]\nu} &= k_0 \phi_{(\mu\nu)} \\ \partial_\mu \phi_{[\rho\sigma]\nu} - \partial_\nu \phi_{[\rho\sigma]\mu} &= k_0 \phi_{[\mu\nu][\rho\sigma]} \\ \partial_\varepsilon \phi_{([\varepsilon\rho][\mu\nu])} &= k_0 \phi_{[\mu\nu]\rho} \end{aligned} \tag{71}$$

$\phi_{(\mu\nu)}$ is a symmetric tensor of rank 2, $\phi_{[\mu\nu]\rho}$ a tensor of rank 3 antisymmetric with respect to the two first indices, $\phi_{[\mu\nu][\rho\sigma]}$ a tensor of rank 4 antisymmetric with respect to $\mu\nu$ and $\rho\sigma$ but symmetric with respect to these pairs. As a consequence of (71) we have :

$$\partial_\nu \phi_{(\mu\nu)} = \partial_\rho \partial_\nu \phi_{[\rho\mu]\nu} = 0\ ;\ \phi_{[\rho\rho]} = \frac{1}{2} \phi_{[\mu\rho][\mu\rho]}\ ;\ \partial_\nu \phi_{(\rho\rho)} = k_0 \phi_{[\nu\rho]\rho} \tag{72}$$

Now, the group (B) is divided in three sub-groups where new tensors of rank 2, 3, 4 appear . The equations (B_2) and (B_3) are identical :

$$(B_1)\quad \begin{aligned} \partial_\mu \phi^{(1)}_{(\nu\rho)} - \partial_\nu \phi^{(1)}_{(\mu\rho)} &= k_0 \phi^{(1)}_{[\mu\nu]\rho} \\ \tfrac{1}{2}\left(\partial_\rho \phi^{(1)}_{[\rho\mu]\nu} - \partial_\rho \phi^{(1)}_{[\rho\nu]\mu}\right) &= k_0 \phi^{(1)}_{[\mu\nu]} \\ \partial_\mu \phi^{(1)}_{[\rho\sigma]\nu} - \partial_\nu \phi^{(1)}_{[\rho\sigma]\mu} &= k_0 \phi^{(1)}_{[\mu\nu][\rho\sigma]} \\ \partial_\varepsilon \phi^{(1)}_{([\varepsilon\rho][\mu\nu])} &= k_0 \phi^{(1)}_{[\mu\nu]\rho} \end{aligned} \tag{73}$$

$$(B_2, B_3) \qquad \begin{aligned} \partial_\mu \chi_\nu^{(1)} - \partial_\nu \chi_\mu^{(1)} &= k_0 \chi_{[\mu\nu]}^{(1)} \\ \partial_\rho \chi_{[\rho\nu]}^{(1)} &= k_0 \chi_\nu^{(1)} \\ \partial_\mu \chi_\nu^{(1)} &= k_0 \chi_{\rho\nu}^{(1)} \\ \partial_\rho \chi_{[\mu\nu]}^{(1)} &= k_0 \chi_{[\mu\nu]\rho}^{(1)} \end{aligned}$$

From (B_1) we deduce the identities :

$$\phi_{[\nu\mu]\nu}^{(1)} = \phi_{([\mu\nu][\rho\nu])}^{(1)} = 0 \tag{74}$$

In (B_2), (B_3), $\chi_{\rho\nu}^{(1)}$ is neither symmetric nor antisymmetric and (74) entails :

$$\begin{aligned} &\chi_{\rho\rho}^{(1)} = 0\,; \quad \chi_{\mu\nu}^{(1)} - \chi_{\nu\mu}^{(1)} = \chi_{[\mu\nu]}^{(1)} \\ &\chi_{[\nu\rho]\rho}^{(1)} = -\chi_\nu^{(1)}\,; \quad \chi_{[\mu\nu]\rho}^{(1)} + \chi_{[\nu\rho]\mu}^{(1)} + \chi_{[\rho\mu]\nu}^{(1)} = 0 \end{aligned} \tag{75}$$

Finally the last group of equations is :

$$(C) \qquad \begin{aligned} \partial_\mu \phi_\nu^{(0)} = \partial_\nu \phi_\mu^{(0)} &= k_0 \phi_{(\mu\nu)}^{(0)} \\ \partial_\mu \phi_\mu^{(0)} &= k_0 \partial_\mu \phi^{(0)} \\ \partial_\mu \phi^{(0)} &= k_0 \phi_\mu^{(0)} \end{aligned} \tag{76}$$

The equations (B_1), (B_2), (B_3) are three realizations of total spin 1. **It is evident for (B_2), (B_3)** because putting:

$$F_\mu = k_0 \chi_\mu^{(1)}\,;\ F_{[\mu\nu]} = k_0 \chi_{\mu\nu}^{(1)} \tag{77}$$

and defining potentials and fields as we did in (29, Part 1), we find Maxwell equations with mass (nevertheless, we shall see that it needs some comments).

The correspondence is less evident for (B_1). Instead of (77), we must write:

$$F_\mu = \frac{k_0}{6} \varepsilon_{\mu\lambda\nu\rho} \phi_{[\lambda\nu]\rho}^{(1)}\,;\ F_{[\mu\nu]} = k_0 \phi_{\mu\nu}^{(1)} \tag{78}$$

($\varepsilon_{\mu\lambda\nu\rho}$ = Levi-Civita symbol). Applying (29, Part 1), we find Maxwell equations (same remark as above).

Now, (C) is a realization of spin 0, as it may be seen by comparison of (76) with (31, Part 1). But here we find a difficulty (that justifies the preceding remarks) :

De Broglie didn't know the magnetic monopole. He considered only the electric photon : (30, Part 1) and he identified (76) with the non maxwellian equations (31, Part 1). **But this implies the identity $\phi^{(0)} = I_2$, where $\phi^{(0)}$ is a *scalar* while I_2 is a pseudoscalar.**

At the time of the reference : 1943 *(Broglie 4)*, people were less careful than now with parity and de Broglie wrote that (76) and (31 Part 1) **"are entirely equivalent" ("at least when vectors and pseudo-vectors are assimilated" he said).** Today, we pay great attention to parity and we cannot neglect such a discrepancy : **an equality like $\phi^{(0)} = I_2$ is unacceptable**. Two solutions may be suggested:

1) We can admit that $\phi^{(0)} = I_2$, if $\phi^{(0)} = I_2 = 0$. Thus the spin 0 component (C) vanishes. But there is a second spin 0 component, hidden in the equations (A) in the form of an invariant $\phi^{(0)}$, a vector $\phi^{(0)}_\mu$ and a symmetric tensor $\phi^{(0)}_{(\mu\nu)}$, which might be defined as :

$$\phi^{(0)} = \phi^{(0)}_{(\rho\rho)} \ ; \ \phi^{(0)}_\mu = \phi^{(0)}_{[\mu\rho]\rho} \ ; \ \phi^{(0)}_{(\mu\nu)} = \phi_{([\mu\rho][\nu\rho])} - \phi_{(\mu\nu)} \qquad (79)$$

One can show, using (71), that these tensors obey the group C of equations (76). But, once more, if $\phi^{(0)}$ is a true scalar, we can write $\phi^{(0)} = I_2$ only if $\phi^{(0)} = I_2 = 0$.

This implies that (71) is submitted to the condition $\mathrm{Sp}\, \phi^{(0)}_{(\rho\rho)} = 0$. It was *a priori* supposed by Firz and Pauli who based their theory on a pure spin 2 (and not maximum spin 2) particle. De Broglie criticized this postulate as artificial and the above suggestion, based on parity, could be considered as a justification.

But, the objection is : as it will be shown later, the splitting of spin components is not covariant, which is thus true for the condition : $\phi^{(0)} = I_2 = 0$, even if, on another side, the equality $\mathrm{Sp}\, \phi^{(0)}_{(\rho\rho)} = 0$ is covariant : the problem thus remains unsolved. But there is a second possibility:

2) We can ask the question: is $\phi^{(0)} = I_2$ the good equality? Perhaps it is rather $\phi^{(0)} = I_1$, which is covariant because I_1 is a true invariant? In such a case, (76) must not be identified with (31), but with (36). Is it possible? : Yes.

Let us go back to (69). The products $A^{(1)}_\mu \times I^{(2)}_2$ and $I^{(1)}_2 \times A^{(2)}_\mu$, denoted $P^{(1)}_\mu$, $P^{(2)}_\mu$, were considered by de Broglie as vectors, but he said, more prudently, "vector-like": **they are pseudo-vectors, be-**

cause they are the products of a polar vector by a pseudo-scalar. Thus $P_\mu^{(1)}$ and $P_\mu^{(2)}$ are not polar potentials but pseudo-potentials as in (35). On the contrary, the product $I_2^{(1)} \times I_2^{(2)}$ of two pseudo-scalars is a *true* scalar, of the same type as I_1, which appears in (29) and they can be identified.

The answer to the difficulty is that the third photon associated to the graviton is not electric but magnetic.

Now, suppose that, instead of electric photons, we introduce magnetic photons in the symbolic formulae (69) : pseudo-potentials $B_\mu^{(1)}$, $B_\mu^{(2)}$ and pseudoscalars $I_2^{(1)}, I_2^{(2)}$. The fusion gives:

$$B_\mu^{(1)} \times B_\mu^{(2)} \quad ; B_\mu^{(1)} \times I_1^{(2)} \quad ; I_1^{(1)} \times B_\mu^{(2)} \quad ; I_1^{(1)} \times I_1^{(2)} \tag{80}$$

and we see that:

- The spin 2 product $B_\mu^{(1)} \times B_\mu^{(2)}$ has the same symmetry as $A_\mu^{(1)} \times A_\mu^{(2)}$, because the axial character of $B_\mu^{(1)}, B_\mu^{(2)}$ is annihilated by the product;
- For the same reason, the spin 0 product $I_1^{(1)} \times I_1^{(2)}$ is a scalar, so as $I_2^{(1)} \times I_2^{(2)}$;
- The spin 1 products $B_\mu^{(1)} \times I_1^{(2)}$; $I_1^{(1)} \times B_\mu^{(2)}$ are pseudo-vectors, as $A_\mu^{(1)} \times I_2^{(2)}$; $I_2^{(1)} \times A_\mu^{(2)}$: they are products of a pseudo-vector by a scalar, while the latter were products of a polar vector by a pseudo-scalar.

Thus we find a magnetic photon of the (35) type, wether we start from electric or from magnetic photons: **among the three photons associated to the graviton, two are electric and one is magnetic.**

12.3 Gravitation.

Now, we shall follow de Broglie and Tonnelat and consider the general equations (A) when $\mathrm{Sp}\,\phi_{(\rho\rho)}^{(0)} \neq 0$. But we shall not be able to separate the spin 2 component from its spin 0 part !

We start from (71), (72) and the Klein-Gordon equation, verified by all the field quantities :

$$\Box\phi = -k_0^2\phi \quad (\Box = -\partial_\rho\partial_\rho) \tag{81}$$

The metric tensor $g_{(\mu\nu)}$ will be taken at the linear approximation:

$$g_{(\mu\nu)} = \delta_{\mu\nu} + h_{(\mu\nu)} \quad \left(h_{(\mu\nu)} \ll 1\right) \tag{82}$$

At this limit, the propagation of gravitation waves is given by:

$$\Box g_{(\mu\nu)} = -2R_{(\mu\nu)} \quad \left(R_{(\mu\nu)} = g^{\rho\sigma} R_{([\mu\rho][\nu\sigma])}\right) \tag{83}$$

Where $R_{([\mu\rho][\nu\sigma])}$ is the tensor of Riemann-Christoffel ; in the euclidian regions of space-time we have the d'Alembert equation $\Box g_{(\mu\nu)} = 0$ without second member. This is true if we use "isothermic" coordinates x_μ, for which $D_2 x_\mu = 0$; D_2 is the second order Beltrami differential parameter: $D_2 f = -g^{-1/2}\partial_i(g^{1/2} g^{ij} \partial_j f)$.

Now it could seem that metrics may be defined by:

$$g_{(\mu\nu)} = \phi_{(\mu\nu)} \tag{84}$$

But Tonnelat remarked that, according to (72), this implies: $\partial_\mu g_{(\mu\nu)} = 0$, which is wrong because "isothermic" coordinates obey the relation[3] :

$$\partial_\mu g_{(\mu\nu)} = \frac{1}{2}\partial_\nu g_{(\rho\rho)} \quad \left(g_{(\rho\rho)} = g_{(\mu\nu)}\delta^{(\mu\nu)}\right) \tag{85}$$

and the second member is not equal to zero, thus contradicts (84). This is why Tonnelat suggested the following metrics (which is possible because $k_0 \neq 0$):

$$g_{(\mu\nu)} = \phi_{([\mu\rho][\nu\rho])} = \phi_{(\mu\nu)} + \frac{1}{k_0^2}\partial_\mu\partial_\nu\phi_{(\rho\rho)} \tag{86}$$

from which follows immediately:

$$\partial_\mu g_{(\mu\nu)} = \partial_\mu \phi_{([\mu\rho][\nu\rho])} = \partial_\nu \phi_{(\rho\rho)} \tag{87}$$

So we get from (72), (86), (87) :

$$g_{(\rho\rho)} = 2\phi_{(\rho\rho)} \rightarrow \partial_\mu g_{(\mu\nu)} = \frac{1}{2}\partial_\nu g_{(\rho\rho)} \tag{88}$$

in accordance with (85).

Now, from (86) we deduce that $g_{(\mu\nu)}$ obey the Klein-Grordon equation, as all the field quantities:

$$\Box g_{(\mu\nu)} = -k_0^2 g_{(\mu\nu)} \tag{89}$$

[3] It must be noted that we have not: $g^\rho_\rho = g_{\rho\sigma}g^{\rho\sigma}$, because this quantity, *in the present case,* is equal to 4.

We have to identify (89) with (83), and we get:

$$R_{(\mu\nu)} = \frac{1}{2}k_0^2 g_{(\mu\nu)} \tag{90}$$

Now, the tensor of Riemann-Christoffel may be deduced at the linear approximation, from (86), (71) and (72) :

$$\phi_{([\mu\rho][\nu\sigma])} \approx \frac{2}{k_0^2}\ R_{([\mu\rho][\nu\sigma])} \tag{91}$$

This formula is possible only if $\mu_0 \neq 0$, which imposes a curvature of the universe. Indeed, $\frac{k_0^2}{2}$ is nothing but the cosmological constant, defined by :

$$R_{(\mu\nu)} = \lambda g_{(\mu\nu)} \tag{92}$$

λ is related to a "natural curvature" of space-time. In the euclidean space: $\lambda = 0$; in a de Sitter space of radius R we have : $\lambda = \frac{3}{R^2}$. Therefore:

$$\lambda = \frac{k_0^2}{2} = \frac{\mu_0^2 c^2}{2\hbar^2} \tag{93}$$

and the mass of the graviton is related to a natural curvature of radius R:

$$\mu_0 = \frac{\hbar\sqrt{6}}{Rc} \tag{94}$$

If $R = 10^{26}$cm, the graviton (and photon) mass is:

$$\mu_0 = 10^{-66} g \tag{95}$$

Now let us go back to the definitions (76) that gave the spin 0 part of (71).
In order to separate a "pure" spin 2 component, we could write:

$$\begin{aligned} \phi_{(\mu\nu)} &= \phi^{(2)}_{(\mu\nu)} + \phi'_{(\mu\nu)} \\ \phi^{(2)}_{(\mu\nu)} &= \phi_{(\mu\nu)} - \delta_{\mu\nu}\phi_0, \quad \phi'_{(\mu\nu)} = \delta_{\mu\nu}\phi_0 \end{aligned} \tag{96}$$

Thus we have:

$$\phi^{(2)}_{(\mu\mu)} = 0, \quad \phi'_{(\mu\mu)} = \phi_0 \tag{97}$$

It looks quite well, as a separation between spin 2 and spin 0, and it is easy to find the same decomposition for all the other tensors (*Broglie* 7). But, unfortunately, in the general case, the spin 2 components of the type $\phi^{(2)}$ do not obey the equations (71): there are additional terms of the type $\phi^{(0)}$, corresponding to a zero spin.

The spin 0 may be eliminated from the equations of spin 2 only in two cases:
- either by the a priori supposition that $\phi^{(0)} = 0$ (Firz equations),
- or *in the limit case* $\mu_0 = 0$, when the radius of the universe is infinite: the euclidean case.

In conclusion, the quantum theory of gravitation based on de Broglie's fusion theory raises the important question of a composite nature of photon and graviton and above all, this theory furnishes the beginning of a quantum unitary field theory of electromagnetism and gravitation. Only the beginning because it is linear.

Two remarks may be made about all that:

- It could be asked if the obstinate efforts of Einstein and other great physicists and mathematicians towards a unitary field theory had any sense, given that we know hundreds of particles and it seems that there is no reason to pay a particular attention to two of them : photon and graviton. De Broglie's theory gives a reason: these particles are those which appear, linked by spin properties, in the fusion procedure.

  The strength of this argument is that it is absolutely exterior (at least it seems so!) to the geometrical path followed by Einstein.

- The second remark concerns symmetry : the fact that a photon associated with the graviton could be magnetic instead of electric, as was suggested above, signifies the intrusion of duality, chirality, magnetic monopoles instead of electric charges and so on. **It is certainly of interest that a photon is perhaps not the one that was expected.**

13 Some words about other theories.

First of all, we must emphasize the priority of Louis de Broglie in the quantum theory of photon considered as a *composite particle.* His first paper appeared in 1934 (*Broglie 4*) and the idea of a *fusion* of Dirac particles is the starting point of the theory of particles of higher spin. Unlike the others, de Broglie's initial aim was not a generalization of Dirac's equation but a theory of light. This is why he didn't introduce any electromagnetic interaction.

For reasons given above, he was the only one to suppose a massive photon, contrary to other authors who considered a massless photon as an evidence. As for him, he never tried to extend his theory to massless particles and even scarcely alluded to this possibillity. The often cited Proca equations with mass, actually do not concern photons but electrons : an attempt to eliminate negative energies.

14 Relativistic non invariance of the decomposition spin 1–spin 0 in the theory of light.

The spin operators $s_j = \varepsilon_{jik} S_{ik}$ obey the commutation rules of an angular momentum and they have the eigenvalues $\{-1,\ 0,\ 1\}$. The total spin $\mathbf{s}^2$ has the eigenvalues $l\,(l+1) = (2,0)$, corresponding to $l = 1, 0$.

In the case of a plane wave in (30), (31), and (35), (36), one can show that the group of equations (M) is associated with $l = 1$, with projections $s = -1, 0, +1$ on the direction of propagation of the wave: $s = -1 \Leftrightarrow$ *right circular wave*, s=+1 $\Leftrightarrow$ *left circular wave*. For $s = 0$, we have in both cases (due to the mass), a small longitudinal *electric* wave for the electric photon, and a small longitudinal *magnetic* wave for the magnetic photon. The group (NM) is associated with $l = 0$. In this sense, we can speak of (M) as of a "spin 1 particle" and of (NM) as of a "spin 0 particle".

However, de Broglie made an important remark (*Broglie 4, Ch. VIII*): *although the equations (M) and (NM) are relativistically invariant, their separation in "spin 1" and "spin 0" is not covariant (Broglie 4).* The reason is that the distinction between values 2 and 0 of the total spin is based on the operator $\mathbf{s}^2 = \mathrm{s}_1^2 + \mathrm{s}_2^2 + \mathrm{s}_3^2$ which is not a relativistic invariant.

Now, if we examine field quantities and eigenvalues of $\mathbf{s}^2$ we find the following correspondences :

1) For the electric photon:

$$\begin{array}{cccc} \mathbf{A} & V & \mathbf{E} & \mathbf{H} \\ 2 & 0 & 2 & 2 \end{array} \quad \begin{array}{cccccc} I_1 & \mathbf{B} & W & I_2 & \mathbf{E}' & \mathbf{H}' \\ 0 & 2 & 0 & 0 & 2 & 2 \end{array} \tag{98}$$

2) For the magnetic photon:

$$\begin{array}{cccc} \mathbf{B}' & W' & \mathbf{H}' & \mathbf{E}' \\ 2 & 0 & 2 & 2 \end{array} \quad \begin{array}{cccccc} I_2 & \mathbf{A}' & V' & I_1 & \mathbf{H} & \mathbf{E} \\ 0 & 2 & 0 & 0 & 2 & 2 \end{array} \tag{99}$$

In both cases, the first group corresponds to (M) equations and the second group to (NM). We can note, when passing from (98) to (99), the exchanges :

- between potentials **A**, V and pseudopotentials **B'**, W' and conversely;
- between fields **E**, **H** and anti-fields **E'**, **H'** ; we know that **E'**, **H'**=0 in (31)) and **E**, **H**=0 in (36).
- between the invariant $I_1$and the pseudo-invariant I_2, inside the group (NM) (with $I_1 = 0$ in (31) and $I_2 = 0$ in (36).

But the most important fact is, that there are in both groups (M) and (NM), field quantities with s^2 = 2 and s^2 = 0. This means that *in both groups (M) and (NM), there are spin 1 and spin 0 components*: there is no true separation between the two values of spin. Following de Broglie one can show (and now we know that it is true for both photons) that this separation occurs only in the proper system, because :

a) For the electric photon, the potential $(\mathbf{A}\,, V)$ is *spacelike*, and the pseudopotential $(\mathbf{B}\,, W)$is *timelike*, so that V and **B** disappear from (98) and only $\mathbf{s}^2 = 2$ remains in (M) and $\mathbf{s}^2 = 0$ in (NM) because in (NM), we know that $\mathbf{E}' = \ \mathbf{H}' = 0$.

b) For the magnetic photon, the same happens, because this case follows from the preceding by multiplying an electric solution by γ_5, exchanging polar and axial quantities :

$$(\mathbf{E}, \mathbf{H}) \leftrightarrow (\mathbf{H}', \mathbf{E}')\,;\ (V, \mathbf{A}) \leftrightarrow (W, \mathbf{B})\,;\ (I_1, I_2) \leftrightarrow (I_2, I_1) \qquad (100)$$

So that the potential $(\mathbf{A}, V)$becomes *timelike* and the pseudopotential (**B**, W) *spacelike*. And we have once more, in the proper frame, $\mathbf{s}^2 = 2$ in (M) and $\mathbf{s}^2 = 0$ in (NM), taking into account that we have $\mathbf{E} = \mathbf{H} = \mathbf{0}$ instead of $\mathbf{E}' = \mathbf{H}' = \mathbf{0}$. In conclusion, the (M) and (NM) groups of equations cannot be rigorously separated, except in the proper frame, and they must be considered as forming one block, for two reasons:

1) The difficulty to separate spin 1 and spin 0 finally signifies that the composite photon cannot be considered as a spin 1 particle, but as a particle with a *maximum spin 1*, just as a two-electron atom or a two-atom molecule. It is noteworthy that the proper state, in which

the components 1 and 0 are separated, is obviously the same for both components.

2) On the contrary, the presence of two photons (electric and magnetic) is inscribed in the very structure of the theory, their separation is covariant and seems more radical than the separation of spin states. The simultaneous presence, in (M) and (NM) equations, of potentials and pseudopotentials, of fields and anti-fields (even if half of them equals zero) and the “migration” of these quantities from one group of equations to the other, according to the type of photon, all these points constitute another link.

Last remark : the preceding analysis was given by de Broglie only in the photon theory, and it must be completed for the case of a graviton, but there is no doubt about the generality of a non relativistic covariance of the separation of spins.

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[4]The book of Akira Tonomura, written in non-technical terms, is in principle, a popular book but it is so clear and so profound that it must be included in a bibliography on the Aharonov-Bohm effect.