Electron channeling resonance and de Broglie's internal clock

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ABSTRACT. Experimental work in electron channeling in silicon crystals has yielded a resonance suggestive of de Broglie's internal clock for the electron, but with a Monte Carlo simulated frequency twice that of de Broglie's. An explanation is provided based on a previously developed four-dimensional model of the electron. The additional spatial degree of freedom resolves a large number of physical and mathematical conceptual issues in traditional relativistic particle and wave mechanics including the mechanism for de Broglie's internal clock and the unexpected simulated channeling frequency.

KEY WORDS. Electron channeling, de Broglie clock, four-dimensional space, guided wave mechanics, geometric algebra, Dirac theory.

1 Introduction

Experimental work in electron channeling in silicon crystals [1,2] has attempted to observe, albeit indirectly, de Broglie's internal "clock" frequency in electrons. Initial results suggested that such a frequency may indeed exist. However, Monte Carlo simulations, which accurately fit the well-known "rosette motion" resonance profile at near axial directions, suggested that the fundamental interaction length is one-half the inter-atomic spacing thereby implying that the interaction frequency is twice de Broglie's clock frequency.

De Broglie's clock frequency [3] is

$$\nu_{cl} = \nu_c \alpha \tag{1}$$

where $\nu_c = mc^2 / h$, m is the mass, and $\alpha = \left(1 - \beta^2\right)^{1/2}$ is the inverse of the Lorentz factor. It is easy to see that ν_{cl} decreases with increasing relative velocity, a direct manifestation of time dilation on the clock period $\tau_{cl} = \tau_c / \alpha$. In contrast, the de Broglie wave frequency is given by

$$\nu_p = \nu_c / \alpha, \tag{2}$$

which increases with relative velocity. The difference in the form of these frequencies was one of the motivating factors that led de Broglie to develop his wave mechanics.

A key assumption in his electron model was that of a point particle whose rest frequency ν_c in (1) and (2) arises from the same periodic phenomenon, which he identified with his internal clock. One of his arguments in this regard was to show that the electron could be modeled as a periodic oscillation that obeys the Lorentz transformation

$$a_0 e^{i\nu_c t'} \rightarrow a_0 e^{i\nu\left(t - x/\upsilon_p\right)} \tag{3}$$

where $\nu_c = mc^2 / h$, $\nu = \nu_c / \alpha$, and $\upsilon_p = c / \beta$. Here de Broglie identified the left side of the transformation as that of a periodic or pulsating phenomenon that extends throughout all space while the right side is strictly a "phase" wave propagating at the speed $\upsilon_p = c / \beta$. It was therefore natural for him to identify the pulsating amplitude with a clock-like phenomenon that behaved relativistically in accordance with (1). However, given the fact that the phase wave has been observed while the clock phenomenon has not, the internal clock concept has been all but abandoned by the physics community. Although the channeling experiment is an attempt to revise the concept, the anomalous factor of two in the simulation seems to point to some other process as the responsible agent, such as zitterbewegung [1].

In this paper it is shown that the internal clock concept can be preserved if one generalizes the de Broglie model to that of a four-dimensional guided interference wave. In this case the clock and wave frequencies arise due to different physical processes that share a common rest frequency, the latter reducing to a standing wave in its rest frame. As will be seen, the model accounts for not only the observed resonant momentum at the de Broglie clock frequency but also the reason for the anomalous factor of two in the simulation.

In previous work by the author [4] it was shown that a photon propagating in a planar waveguide exhibits all of the kinematic and mass related properties of an elementary particle, albeit in two dimensions rather than three, the third dimension corresponding to the transverse dimension of the guide. In a subsequent article [5] it was shown that the traditional three-dimensional theories of relativistic particle and wave mechanics, both classical and quantum, can be viewed as *incomplete* descriptions of four-dimensional guided wave phenomena, the transverse dimension corresponding to a *fourth* Cartesian dimension. The resultant kinematics is referred to as *guided-wave mechanics*, a name intended to convey the type of propagation rather than any particular type of boundary conditions.

Although the correspondence between the equations of guided wave theory and wave mechanics has been known for some time [6] it has been exceedingly difficult to relate the two theories in any meaningful way. The difficulty can be traced to the fact that guided wave theory requires the introduction of a preferred direction in space, the transverse guide axis, which is incompatible with known particle kinematics, which is isotropic. As will be seen guided-wave mechanics removes this restriction by introducing an additional degree of freedom into the model in the form of a fourth spatial dimension. This in turn leads to intuitive and self-consistent explanations for numerous conceptual difficulties inherent in traditional theories while reducing to the latter in the three dimensions of known space.

Most noteworthy in this regard is its ability to provide models for the concept of mass, both inertial and gravitational, the equivalence principle of general relativity, and wave-particle duality in elementary matter. But just as interesting is its ability to lend insight into a variety of well-known postulates and ad hoc mathematical constructs. These include, among others, the classical relativistic Lagrangian, Lorentz invariants and four-vectors, the differential operator prescriptions of quantum mechanics, the Klein-Gordon equation, the Dirac equation, the Foldy-Wouthuysen transformation, zitterbewegung, etc. A brief overview sufficient to discuss the channeling experiment is given below. The reader is referred to [5] for more detail.

2 Overview

The guided-wave mechanical model of the electron is a generalization of traditional wave mechanics, and relativistic quantum mechanics in general, to four spatial dimensions. Although the emphasis here is on de Broglie's wave mechanics and his internal clock concept, any reinterpretations of his theory must be shown to be consistent with the Dirac theory as well. To that end a unique mathematical approach is taken whereby the Dirac alpha-beta matrices of the Hamiltonian formulation are transcribed in terms of a four-dimensional Clifford geometric elements. This must be contrasted with other works that rely exclusively on the covariant or *space-time* algebras [7,8] where the requirement for four spatial dimensions is not manifested. Clearly both descriptions are necessary for completeness and self-consistency as was demonstrated by Dirac in his matrix formulation.

The generalization to four spatial dimensions of the quantum mechanical wave equation for a photon propagating in a three-dimensional planar waveguide yields

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \frac{p_4^2}{\hbar^2}\psi, \tag{4}$$

where $p_4 = mc$ represents the *constant* component of the momentum along a fourth spatial dimension x_4 and $\nabla = \nabla_1^2 + \nabla_2^2 + \nabla_3^2$ is the three-dimensional Cartesian Laplacian. The waveform ψ in (4) represents a four-dimensional guided interference wave propagating in the x_1 direction. Its Lorentz transformation can be written

$$a_0 f\left(k_4 x_4\right) e^{i\nu_c t'} \to a_0 f\left(k_4 x_4\right) e^{i\nu\left(t - x_1/\upsilon_p\right)} \tag{5}$$

which is a straightforward generalization of (3). Here $f\left(k_4 x_4\right)$ is a harmonic function that is invariant with respect to Lorentz transformations since it is normal to all directions in three-dimensional space. It is easy to see that since $f\left(k_4 x_4\right)$ does not depend on x or t it can be divided out of (4) such that with $p_4 = mc$ the equation reduces to the Klein-Gordon equation. However, in its more general form it represents a massless particle confined to a region $\lambda_c / 2$ along a fourth spatial dimension, where $\lambda_c = h/mc$ is the Compton-de Broglie wavelength. It follows that the operator equation for the scalar momentum is given by

$$p_d^2 = p_\parallel^2 + p_\perp^2, \tag{6}$$

where $p_{\|} = \left(p_1^2 + p_2^2 + p_3^2\right)^{1/2}$ and $p_\perp = p_4$. With $p_d = E/c$ and $p_\perp = mc$ it is easy to see that (6) is equivalent to the relativistic energy momentum equation. Linearization of either (4) or (6) using Clifford geometric elements yields the linear momentum of the massless particle or *direct wave*, which in the Heisenberg representation is written

$$\boldsymbol{p}_d = p_{\|}\boldsymbol{e}_p + p_\perp \boldsymbol{e}_4, \tag{7}$$

where $\boldsymbol{p}_d^2 = p_d^2$ and a minus sign yields the solution for the counter-propagating wave along x_4. Here $\boldsymbol{e}_p$ is a unit vector in the three dimensions of known space while $\boldsymbol{e}_4$ lies along the fourth spatial dimension. The former is given by $\boldsymbol{e}_p = p_i\boldsymbol{e}_i / p_{\|}$ where the operator ratios $p_i / p_{\|}$ represent direction cosines in the subspace $\left(\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\right)$. The propagation angle relative to $\boldsymbol{e}_4$ is given by

$$\phi = \tan^{-1}\left(p_{\|} / p_\perp\right). \tag{8}$$

The basis set of elements $\boldsymbol{e}_\mu$, $\mu = 1, 2, 3, 4$, constitute unit vectors in the four-dimensional Clifford space $\Re_4$ [9]. They have the algebraic properties

$$\boldsymbol{e}_\mu^2 = 1, \quad \left\{\boldsymbol{e}_\mu, \boldsymbol{e}_\nu\right\} = 2\delta_{\mu\nu}, \quad \mu, \nu = 1, 2, 3, 4, \tag{9}$$

and are therefore isomorphic to the Dirac alpha-beta matrices, a relationship that can be written as $\boldsymbol{e}_i \leftrightarrow \alpha_i$, $i = 1, 2, 3$, and $\boldsymbol{e}_4 \leftrightarrow \beta_0$. Hence, (7) is the geometric equivalent of the Dirac Hamiltonian under the replacement $p_d = H/c$ and $p_\perp = mc$. It is easy to see that its identification as linear momentum leads to a more physically meaningful relationship with the Klein-Gordon equation than is possible in the matrix formulation. The concept of spin manifests itself through minimal coupling to the electromagnetic field in the same manner as in the matrix formulation

The free-particle Hamiltonian follows from (6) or (7). In the latter case we have

$$H_d = c\left(\boldsymbol{p}_d^2\right)^{1/2} = \pm c\left(p_{\|}^2 + p_\perp^2\right)^{1/2}. \tag{10}$$

With $p_\perp = mc$ this agrees with the Klein-Gordon Hamiltonian where the $\pm$ signs represent positive and negative energy states, respectively. It should be noted that the departure from the linearized form of the Dirac Hamiltonian is a necessary consequence of the geometric formulation since if the Hamiltonian is to represent energy it must be a scalar not a vector, a distinction that is not manifested in the Dirac or space-time algebras.

The position of a phase point on the direct wave is given by

$$\boldsymbol{x}_d = x_{\|}\boldsymbol{e}_x + x_\perp \boldsymbol{e}_4\,, \tag{11}$$

where $x_{\|}$ and $x_\perp$ are the longitudinal and transverse components of the position operator, the maximum excursion of the latter being $x_\perp = \lambda_c / 2$. The unit vector $\boldsymbol{e}_x$ is defined by $\boldsymbol{e}_x = x_i \boldsymbol{e}_i / x_{\|}$ and is therefore parallel to $\boldsymbol{e}_p$. The particle or group velocity follows from (10) and (11) in Heisenberg's equations of motion, obtaining

$$\boldsymbol{\upsilon}_d = \upsilon_{\|}\boldsymbol{e}_\upsilon + \upsilon_\perp \boldsymbol{e}_4\,. \tag{12}$$

Here

$$\upsilon_{\|} = \pm\frac{p_{\|}c^2}{|E|}, \quad \upsilon_\perp = \pm\frac{p_\perp c^2}{|E|}, \tag{13}$$

where $\upsilon_{\|}$ and $\upsilon_\perp$ are the group velocities in the longitudinal and transverse directions. Squaring (12) leads to $\upsilon_d = c$ as expected for a massless particle. Note that there is no Dirac type zitterbewegung in (12), a consequence once again of the geometric formulation where the Hamiltonian is necessarily a scalar. (Zitterbewegung can be generated in the geometric formulation simply by using the linear momentum (7), times c, in Heisenberg's equations of motion for dx/dt, but clearly this would have little if any physical meaning despite having the correct units).

Since $\boldsymbol{x}_d, \boldsymbol{\upsilon}_d$, and $\boldsymbol{p}_d$ are parallel for a massless particle their magnitudes can be written as

$$\beta^2 + \alpha^2 = 1, \tag{14}$$

where $\tan\phi = \beta/\alpha$ in agreement with (8). Care must be exercised in using (14) however since the inertial transformation properties of $\boldsymbol{x}_d, \boldsymbol{p}_d$ are different than those for $\boldsymbol{\upsilon}_d$ [5].

In addition to the group velocity there is also the phase velocity that, for the direct wave, is given by $\upsilon_\phi = c$. However, the de Broglie dispersion equation $\upsilon_p \upsilon_g = c^2$ generalizes to

$$\upsilon_\phi \upsilon_d = \upsilon_{\phi_\parallel} \upsilon_\parallel = \upsilon_{\phi_\perp} \upsilon_\perp = c^2, \tag{15}$$

where $\upsilon_{\phi_\parallel} = c/\beta$ and $\upsilon_\parallel = c\beta$ are the phase and group velocities of the direct wave in the longitudinal direction and $\upsilon_{\phi_\perp} = c/\alpha$ and $\upsilon_\perp = c\alpha$ are the corresponding velocities in the transverse direction. The former are, of course, consistent with traditional wave mechanics but the latter suggest that the de Broglie expression is incomplete.

The relativistic behavior of the direct wave frequency ν_ϕ can be traced to that of a transverse Doppler effect as seen by an observer in three-dimensional space. The purely phase-like behavior of the wave with its greater than light speed velocity is the result of the oblique intersection of phase fronts with three-dimensional space. The de Broglie matter wave is therefore not a real wave in the sense of transporting energy as de Broglie originally pointed out. It is only the underlying massless particle that may be considered as real, its projected velocity into three-dimensional space being less than c. On the other hand the clock frequency ν_{cl} has its origins in the saw-tooth or linear propagation mode of the direct wave, a process reminiscent of Einstein's optical clock. The wave and clock frequencies therefore arise out of different physical processes, the former corresponding to the direct wave frequency $\nu_\phi = \nu_c/\alpha$ and the latter the inverse of its round-trip propagation time $\nu_{cl} = \nu_c \alpha$. Thus, their rest frequencies are identical being given by $\nu_c = c/\lambda_c$ and $\nu_c = c/2x_\perp$, respectively, where $2x_\perp = \lambda_c$.

The corresponding wavelengths are

$$\lambda_d = \lambda_c \alpha, \quad \lambda_\parallel = \lambda_c \frac{\alpha}{\beta}, \quad \lambda_\perp = \lambda_c, \tag{16}$$

where $\lambda_d^{-2} = \lambda_\parallel^{-2} + \lambda_\perp^{-2}$ and

$$\Lambda_d = \lambda_c \frac{1}{\alpha}, \quad \Lambda_\parallel = \lambda_c \frac{\beta}{\alpha}, \quad \Lambda_\perp = \lambda_c, \tag{17}$$

where $\Lambda_d^2 = \Lambda_{\|}^2 + \Lambda_{\perp}^2$. It is easy to see that with $\nu_c \lambda_c = c$ multiplication of (16) and (17) by their respective frequencies yields the phase and group velocities given above.

The mass-energy equation follows from the geometric product of the momentum and velocity, that is

$$E = \boldsymbol{p}_d \boldsymbol{\upsilon}_d = \pm \frac{mc^2}{\alpha}, \tag{18}$$

where

$$\boldsymbol{p}_d = \pm \frac{m}{\alpha} \boldsymbol{\upsilon}_d \tag{19}$$

and $\boldsymbol{p}_d \boldsymbol{\upsilon}_d = \boldsymbol{p}_d \cdot \boldsymbol{\upsilon}_d$. Here the concept of mass is simply that of a parameter that relates the velocity and momentum of the *massless* particle with no physical significance beyond that. Note that the scalar product of the traditional *three-dimensional* sub-vectors for momentum and velocity do not yield the mass-energy equation as do the full four-dimensional vectors suggesting that these sub-vectors are also incomplete. In this manner one finds that many of the conceptual difficulties in traditional relativistic particle and wave (quantum) mechanics can be traced in part to a failure to include a fourth spatial dimension in the definition of dynamical variables [5].

Of particular importance to channeling resonance is the Foldy-Wouthuysen transformation [10] of the Dirac equation, a transformation that has never had a physically meaningful interpretation [11]. In the geometric formulation the isomorphisms $\boldsymbol{e}_{4p} \leftrightarrow \beta_0 \alpha_i p_i / p$ and $\boldsymbol{e}_4 \leftrightarrow \beta_0$ lead to the conclusion that it is a rotation in the $(\boldsymbol{e}_4, \boldsymbol{e}_p)$ plane of the direct wave momentum, (7), through the propagation angle ϕ given by (8). The resultant vector is thereby aligned with the positive $\boldsymbol{e}_4$-axis while its magnitude assumes a square-root form characteristic of any vector rotation to a coordinate axis. On the other hand the Cini-Touschek transformation [12] corresponds to a rotation to the $\boldsymbol{e}_p$ axis. The significance of the transformation angle is not limited to the above two examples, however, for it is also related to the Minkowski hyperbolic angle of special relativity through the Gudermannian functions. We therefore find that the Euclidean geometry of the direct wave in guided-wave mechanics implicitly contains the quasi-Euclidean geometry of traditional relativistic particle mechanics, the two manifolds being separate and distinct mathematical descriptions of a common physical entity.

3 Electron channeling

With the above preliminaries we return to the channeling experiment and note that the direct wave executes two traversals of the propagation medium (i.e., three-dimensional space) during each clock cycle resulting in two independent opportunities for interaction with the crystal lattice. The interaction frequency therefore becomes

$$\nu_{\text{int}} = 2\nu_{cl} = 2\nu_c \alpha. \tag{20}$$

Assuming an interaction length of $L/2$ where L is the inter-atomic spacing, we have at resonance

$$L = \frac{2\upsilon_{\|}}{\nu_{\text{int}}} = \lambda_c \frac{\beta}{\alpha} = \Lambda_{\|}. \tag{21}$$

Thus, with $\Lambda_{\|} = c\beta / \nu_{cl}$ we find that the clock wavelength matches the inter-atomic spacing. The resonant momentum is therefore

$$p_{\|} = p_{\perp} \frac{\beta}{\alpha} = p_{\perp} \frac{L}{\lambda_c}. \tag{22}$$

Neglecting the small shift in observed resonance due to calibration errors [1] we have $p_{\|} = 80.874$ Mev/c for an inter-atomic spacing of $L = 3.84$ Å such that with $p_{\perp} = mc = .511$ Mev/c we obtain $L/\lambda_c = \beta/\alpha = 158.266$ where $\beta = \upsilon_{\|}/c \approx 0.99998$ and $\alpha = \upsilon_{\perp}/c \approx 0.0063$.

The significance of the distance $L/2$ and its relationship to the Compton wavelength can be understood by noting that the propagation angle of the direct wave is $\phi = \tan^{-1}(\beta/\alpha) = 89.638$ deg and that $\boldsymbol{x}_d$ and $\boldsymbol{\upsilon}_d$ are parallel. For a maximum transverse excursion of $x_{\perp} = \lambda_c/2 = .0121$ Å, corresponding to a half clock cycle, the parallel component of the direct wave travels

$$x_{\|} = x_{\perp} \tan\phi = 1.92 \text{ Å}, \tag{23}$$

which is indeed one-half the inter-atomic spacing. The linear segments $x_{\|}$, which bear a striking resemblance to the propagation segments of the simulation, represent independent scattering regions since the corresponding direct wave segments x_d define independent propagation paths. In contrast to the de Broglie clock, which requires a doubling of its frequency to account for the $L/2$ interaction length [13], the present model maintains the clock frequency while doubling the interaction frequency. In this case interaction with an atomic site would occur for every other segment suggesting that a stronger resonance should exist for $x_{\|} = L$ or $p_{\|} = 161.74$ Mev/c, in which case an interaction would occur for each segment. Simulation results for $dz = L$ seem to support such a prediction [1]. Still weaker resonances should exist for $x_{\|} = L/3, L/4, \ldots$.

4 Summary and conclusions

It has been shown that de Broglie's original conjecture of an internal clock for the electron may indeed have been correct, but for somewhat different reasons. The addition of a fourth spatial dimension to relativistic wave and quantum mechanics together with the guided wave hypothesis leads to a plethora of new insights including independent physical processes for the wave and clock frequencies. The anomalous factor of two in the simulated resonance frequency has been shown to be a consequence of the particular nature of the clock waveform. Although these findings suggest that a fourth spatial dimension may indeed exist, it would be limited to one-half the Compton wavelength of the particle. This can be viewed as a lower bound on gravitational field measurements that attempt to detect the presence of additional spatial dimensions through changes in the r^{-2} fall off of the gravitational field [14].

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