On the variance of light

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RÉSUMÉ. Dans le cadre de l'algèbre de Clifford de l'espace physique Cl_3 on reprend la théorie de la lumière de Louis de Broglie. On retrouve les quatre photons de G. Lochak. La théorie est invariante sous le groupe Cl_3^*, plus vaste que le groupe de Lorentz.

In the frame of the Cl_3 Clifford algebra of the physical space we resume the Louis de Broglie's theory of light. We get all four photons of G. Lochak. The theory is invariant under the Cl_3^* group, a greater group than the Lorentz group.

We link here Louis de Broglie's theory of light [1], its generalization by G. Lochak as a theory of four kinds of photons [2], to the invariance under the Cl_3^* group [3] which is the true invariance group of all electromagnetism, Dirac wave equation included.

For his construction of the wave of a photon Louis de Broglie started from two Dirac spinors. In the frame of the initial formalism used by de Broglie his two spinors read

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \;;\quad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}. \tag{1}$$

They are solutions of the Dirac wave equation for a particle without charge

$$\partial_0\psi = (\alpha_1\partial_1 + \alpha_2\partial_2 + \alpha_3\partial_3 + i\frac{m}{2}\alpha_4)\psi \tag{2}$$

and of the wave equation for its antiparticle

$$\partial_0\varphi = (\alpha_1\partial_1 - \alpha_2\partial_2 + \alpha_3\partial_3 - i\frac{m}{2}\alpha_4)\varphi \tag{3}$$

where

$$x^0 = ct \; ; \quad \partial_\mu = \frac{\partial}{\partial x^\mu} \; ; \quad m = \frac{m_0 c}{\hbar} \tag{4}$$

$$\alpha_j \alpha_k + \alpha_k \alpha_j = 2\delta_{jk}. \tag{5}$$

It is well known that these matrix relations are not enough to define uniquely α_μ. We can choose different sets of α_μ matrices. We choose here a set working with Weyl spinors which are used to get the relativistic invariance :

$$\alpha_j = \begin{pmatrix} -\sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}, \; j = 1,2,3 \; ; \; \alpha_4 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \; ; \; I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

where σ_j are Pauli matrices and we let

$$\xi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \; ; \quad \eta = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

$$\zeta^* = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} \zeta_1^* \\ \zeta_2^* \end{pmatrix} \; ; \quad \lambda^* = \begin{pmatrix} \varphi_3 \\ \varphi_4 \end{pmatrix} = \begin{pmatrix} \lambda_1^* \\ \lambda_2^* \end{pmatrix} \tag{7}$$

where a^* is the complex conjugate of a. With

$$\vec{\partial} = \sigma_1 \partial_1 + \sigma_2 \partial_2 + \sigma_3 \partial_3$$

$$\vec{\partial}^* = \sigma_1 \partial_1 - \sigma_2 \partial_2 + \sigma_3 \partial_3 \tag{8}$$

the wave equation (2) is equivalent to the system

$$(\partial_0 + \vec{\partial})\xi + i\frac{m}{2}\eta = 0 \tag{9}$$

$$(\partial_0 - \vec{\partial})\eta + i\frac{m}{2}\xi = 0. \tag{10}$$

ξ and η are Weyl spinors of the wave ψ and the wave equation of the anti-particle (3) is equivalent to the system

$$(\partial_0 + \vec{\partial}^*)\zeta^* - i\frac{m}{2}\lambda^* = 0$$

$$(\partial_0 - \vec{\partial}^*)\lambda^* - i\frac{m}{2}\zeta^* = 0. \tag{11}$$

By complex conjugation we get

$$(\partial_0 + \vec{\partial})\zeta + i\frac{m}{2}\lambda = 0 \tag{12}$$

$$(\partial_0 - \vec{\partial})\lambda + i\frac{m}{2}\zeta = 0. \tag{13}$$

This system is identical to (9)-(10) if we replace ζ by ξ and λ by η. Now we take the complex conjugate of (10) and we multiply by $-i\sigma_2$ on the left side :

$$(-i\sigma_2)[(\partial_0 - \vec{\partial}^*)\eta^* - i\frac{m}{2}\xi^*] = 0 \tag{14}$$

and we get

$$(\partial_0 + \vec{\partial})(-i\sigma_2)\eta^* - i\frac{m}{2}(-i\sigma_2)\xi^* = 0. \tag{15}$$

We get with (9) and (15)

$$\begin{aligned}(\partial_0 + \vec{\partial})\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix} + i\frac{m}{2}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} &= 0\\ (\partial_0 + \vec{\partial})\begin{pmatrix}-\eta_2^*\\ \eta_1^*\end{pmatrix} + i\frac{m}{2}\begin{pmatrix}\xi_2^*\\ -\xi_1^*\end{pmatrix} &= 0.\end{aligned} \tag{16}$$

This system is equivalent to the equation

$$(\partial_0 + \vec{\partial})\begin{pmatrix}\xi_1 & -\eta_2^*\\ \xi_2 & \eta_1^*\end{pmatrix} + \frac{m}{2}\begin{pmatrix}\eta_1 & -\xi_2^*\\ \eta_2 & \xi_1^*\end{pmatrix}\begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} = 0. \tag{17}$$

We let [3]

$$\phi_1 = \sqrt{2}\begin{pmatrix}\xi_1 & -\eta_2^*\\ \xi_2 & \eta_1^*\end{pmatrix} \ ; \ \ \phi_2 = \sqrt{2}\begin{pmatrix}\zeta_1 & -\lambda_2^*\\ \zeta_2 & \lambda_1^*\end{pmatrix} \tag{18}$$

which have their value in the algebra generated by Pauli matrices σ_j. This algebra named Cl_3 is also the Clifford algebra of the physical space. Any element in Cl_3 is the sum of a scalar s, a vector $\vec{v}$, a pseudo-vector $i\vec{w}$ and a pseudo-scalar ip.

$$\phi = s + \vec{v} + i\vec{w} + ip. \tag{19}$$

The main automorphism P in Cl_3

$$P : \phi \mapsto \widehat{\phi} = s - \vec{v} + i\vec{w} - ip \tag{20}$$

satisfies

$$\widehat{\phi}_1 = \sqrt{2}\begin{pmatrix}\eta_1 & -\xi_2^*\\ \eta_2 & \xi_1^*\end{pmatrix} \ ; \ \ (\widehat{A+B}) = \widehat{A} + \widehat{B} \ ; \ \ \widehat{AB} = \widehat{A}\widehat{B}. \tag{21}$$

We need

$$\begin{aligned}&\nabla = \sigma^\mu\partial_\mu = \partial_0 - \vec{\partial} \ ; \ \ \widehat{\nabla} = \partial_0 + \vec{\partial} \ ; \ \ \sigma_0 = \sigma^0 = I\\ &\sigma_{12} = \sigma_1\sigma_2 = i\sigma_3.\end{aligned} \tag{22}$$

With (18) and (21) equation (17) therefore reads

$$\widehat{\nabla}\phi_1 + \frac{m}{2}\widehat{\phi}_1\sigma_{12} = 0 \tag{23}$$

or using again the main automorphism P

$$\nabla\widehat{\phi}_1 + \frac{m}{2}\phi_1\sigma_{12} = 0. \tag{24}$$

Similarly the system (12)-(13) is equivalent to

$$\nabla\widehat{\phi}_2 + \frac{m}{2}\phi_2\sigma_{12} = 0. \tag{25}$$

De Broglie's half-photons ψ and φ are linked, they have the same energy and the same momentum, so he supposed [1] that they satisfy

$$\varphi_k\partial_\mu\psi_i = (\partial_\mu\varphi_k)\psi_i = \frac{1}{2}\partial_\mu(\varphi_k\psi_i)\ ,\ k,j = 1,2,3,4\ ;\ \mu = 0,1,2,3. \tag{26}$$

This is equivalent, with (1) and (7), to

$$\begin{aligned}
\xi_k(\partial_\mu\zeta_i^*) &= (\partial_\mu\xi_k)\zeta_i^* = \frac{1}{2}\partial_\mu(\xi_k\zeta_i^*)\\
\xi_k(\partial_\mu\lambda_i^*) &= (\partial_\mu\xi_k)\lambda_i^* = \frac{1}{2}\partial_\mu(\xi_k\lambda_i^*)\\
\eta_k(\partial_\mu\zeta_i^*) &= (\partial_\mu\eta_k)\zeta_i^* = \frac{1}{2}\partial_\mu(\eta_k\zeta_i^*)\\
\eta_k(\partial_\mu\lambda_i^*) &= (\partial_\mu\eta_k)\lambda_i^* = \frac{1}{2}\partial_\mu(\eta_k\lambda_i^*).
\end{aligned} \tag{27}$$

Wave equations (24) and (25) are form invariant [3] under the Lorentz dilation D defined by

$$\begin{aligned}
&x' = D(x) = MxM^\dagger\ ;\quad \phi_1' = M\phi_1\ ;\quad \phi_2' = M\phi_2\\
&\nabla = \overline{M}\nabla'\widehat{M}\ ;\quad \nabla' = \sigma^\mu\partial_\mu'\ ;\quad m = \det(M)m'\ ;\quad \overline{M} = \widehat{M}^\dagger
\end{aligned} \tag{28}$$

where M is any element in Cl_3^* that is to say any invertible element in Cl_3 and $M^\dagger$ is the adjoint of M. We get indeed as $\det(M) = M\overline{M} = \overline{M}M$ for any M in Cl_3

$$\begin{aligned}
\nabla\widehat{\phi}_j + \frac{m}{2}\phi_j i\sigma_3 &= \overline{M}\nabla'\widehat{M}\widehat{\phi}_j + \frac{m'}{2}\det(M)\phi_j i\sigma_3\\
&= \overline{M}\nabla'\widehat{\phi}_j' + \overline{M}M\frac{m'}{2}\phi_j i\sigma_3\\
&= \overline{M}(\nabla'\widehat{\phi}_j' + \frac{m'}{2}\phi_j' i\sigma_3).
\end{aligned} \tag{29}$$

1- The electromagnetism of the photon

We start here from the fact seen in [3] that the electromagnetic potential A is a contra-variant space-time vector, that is to say a vector transforming as x :

$$A' = MAM^\dagger. \tag{30}$$

We know in addition that Pauli's principle rules that products must be antisymmetric. We also know that the σ_3 term is privileged with the Dirac equation. We then consider

$$A = \phi_1 i\sigma_3 \phi_2^\dagger - \phi_2 i\sigma_3 \phi_1^\dagger \tag{31}$$

$$F_e = \nabla \widehat{A} \tag{32}$$

The variance of A and the variance of the electromagnetic field F_e are expected because

$$\begin{aligned} A' &= \phi_1' i\sigma_3 {\phi_2'}^\dagger - \phi_2' i\sigma_3 {\phi_1'}^\dagger \\ &= (M\phi_1) i\sigma_3 (M\phi_2)^\dagger - (M\phi_2) i\sigma_3 (M\phi_1)^\dagger \\ &= M(\phi_1 i\sigma_3 \phi_2^\dagger - \phi_2 i\sigma_3 \phi_1^\dagger)M^\dagger = MAM^\dagger \end{aligned} \tag{33}$$

$$\begin{aligned} F_e &= \nabla\widehat{A} = \overline{M}\nabla'\widehat{M}\widehat{A} = \overline{M}\nabla'\widehat{MAM^\dagger}(\widehat{M}^\dagger)^{-1} \\ &= \overline{M}(\nabla'\widehat{A}')\overline{M}^{-1} = M^{-1}M\overline{M}F_e'\overline{M}^{-1}M^{-1}M \\ &= M^{-1}\det(M)F_e'\det(M^{-1})M = M^{-1}F_e'M \\ F_e' &= MF_eM^{-1}. \end{aligned} \tag{34}$$

A is actually a space-time vector because

$$\begin{aligned} A^\dagger &= (\phi_1 i\sigma_3 \phi_2^\dagger - \phi_2 i\sigma_3 \phi_1^\dagger)^\dagger \\ &= \phi_2(-i\sigma_3)\phi_1^\dagger - \phi_1(-i\sigma_3)\phi_2^\dagger = A. \end{aligned} \tag{35}$$

The calculation of A with (18) and usual Pauli matrices gives

$$\frac{\widehat{A}}{2i} = \begin{pmatrix} \eta_1\lambda_1^* - \xi_2^*\zeta_2 - \lambda_1\eta_1^* + \zeta_2^*\xi_2 & \eta_1\lambda_2^* + \xi_2^*\zeta_1 - \lambda_1\eta_2^* - \zeta_2^*\xi_1 \\ \eta_2\lambda_1^* + \xi_1^*\zeta_2 - \lambda_2\eta_1^* - \zeta_1^*\xi_2 & \eta_2\lambda_2^* - \xi_1^*\zeta_1 - \lambda_2\eta_2^* + \zeta_1^*\xi_1 \end{pmatrix}. \tag{36}$$

We then remark that each product is one of products in (27) and this gives

$$\begin{aligned} \partial_\mu\widehat{A} &= \partial_\mu(\widehat{\phi}_1 i\sigma_3\overline{\phi}_2 - \widehat{\phi}_2 i\sigma_3\overline{\phi}_1) = 2(\partial_\mu\widehat{\phi}_1)i\sigma_3\overline{\phi}_2 - 2(\partial_\mu\widehat{\phi}_2)i\sigma_3\overline{\phi}_1 \\ \nabla\widehat{A} &= 2[(\nabla\widehat{\phi}_1)i\sigma_3\overline{\phi}_2 - (\nabla\widehat{\phi}_2)i\sigma_3\overline{\phi}_1]. \end{aligned} \tag{37}$$

Dirac equations (24) and (25) give then

$$\begin{aligned} F_e &= m\phi_1(-i\sigma_3)i\sigma_3\overline{\phi}_2 - m\phi_2(-i\sigma_3)i\sigma_3\overline{\phi}_1 \\ &= m(\phi_1\overline{\phi}_2 - \phi_2\overline{\phi}_1). \end{aligned} \tag{38}$$

As any element in the Cl_3 algebra F_e is a sum

$$F_e = s + \vec{E} + i\vec{H} + ip \tag{39}$$

where s is a scalar, $\vec{E}$ is a vector, $i\vec{H}$ is a pseudo-vector and ip is a pseudo-scalar. But we get

$$\begin{aligned} \overline{F}_e = s - \vec{E} - i\vec{H} + ip &= m\overline{(\phi_1\overline{\phi}_2 - \phi_2\overline{\phi}_1)} = m(\phi_2\overline{\phi}_1 - \phi_1\overline{\phi}_2) \\ &= -m(\phi_1\overline{\phi}_2 - \phi_2\overline{\phi}_1) = -F_e = -s - \vec{E} - i\vec{H} - ip. \end{aligned} \tag{40}$$

F_e is therefore a pure bivector :

$$s = 0\ ; \quad p = 0\ ; \quad F_e = \vec{E} + i\vec{H}. \tag{41}$$

This agrees with all that we know about electromagnetism and optics. Now (32) reads

$$\vec{E} + i\vec{H} = (\partial_0 - \vec{\partial})(A^0 - \vec{A}) = \partial_0 A^0 - \vec{\partial}A^0 - \partial_0\vec{A} + \vec{\partial}\vec{A} \tag{42}$$

and as

$$\vec{\partial}\vec{A} = \vec{\partial}\cdot\vec{A} + i\vec{\partial}\times\vec{A}\ ; \quad \partial_0 A^0 + \vec{\partial}\cdot\vec{A} = \partial_\mu A^\mu \tag{43}$$

(32) is equivalent to the system

$$0 = \partial_\mu A^\mu \tag{44}$$

$$\vec{E} = -\vec{\partial}A^0 - \partial_0\vec{A} \tag{45}$$

$$\vec{H} = \vec{\partial}\times\vec{A}. \tag{46}$$

(45) and (46) are well known relations between electric and magnetic fields and potential terms. (44) is the relation known as the Lorentz gauge which is in the frame of the theory of light a necessary condition. With (38) we get

$$\widehat{\nabla}F_e = m\widehat{\nabla}(\phi_1\overline{\phi}_2 - \phi_2\overline{\phi}_1). \tag{47}$$

A detailed calculation of these matrices shows as in (36) only products present in (27) and this gives

$$\widehat{\nabla}F_e = 2m[(\widehat{\nabla}\phi_1)\overline{\phi}_2 - (\widehat{\nabla}\phi_2)\overline{\phi}_1] \tag{48}$$

And we get with (24) and (25)

$$\widehat{\nabla}\phi_1 = \frac{m}{2}\widehat{\phi}_1\sigma_{21} \; ; \quad \widehat{\nabla}\phi_2 = \frac{m}{2}\widehat{\phi}_2\sigma_{21} \tag{49}$$

$$\Box\widehat{A} = \widehat{\nabla}\nabla\widehat{A} = \widehat{\nabla}F_e = m^2[\widehat{\phi}_1(-i\sigma_3)\overline{\phi}_2 - \widehat{\phi}_2(-i\sigma_3)\overline{\phi}_1]$$

$$\widehat{\nabla}F_e = -m^2\widehat{A} \tag{50}$$

This is the expected law for the electromagnetism of the photon first obtained by Louis de Broglie since it gives

$$\begin{aligned}(\partial_0 + \vec{\partial})(\vec{E} + i\vec{H}) &= -m^2(A^0 - \vec{A})\\ &= \vec{\partial}\cdot\vec{E} + (\partial_0\vec{E} - \vec{\nabla}\times\vec{H}) + i(\partial_0\vec{H} + \vec{\nabla}\times\vec{E}) + i\vec{\partial}\cdot\vec{H}.\end{aligned} \tag{51}$$

Separating the scalar, vector, pseudo-vector and pseudo-scalar parts, (50) is equivalent to the system

$$\vec{\partial}\cdot\vec{E} = -m^2A^0 \tag{52}$$

$$\partial_0\vec{E} - \vec{\nabla}\times\vec{H} = m^2\vec{A} \tag{53}$$

$$\partial_0\vec{H} + \vec{\nabla}\times\vec{E} = 0 \tag{54}$$

$$\vec{\partial}\cdot\vec{H} = 0 \tag{55}$$

(44) to (46) and (52) to (55) are exactly laws of the electromagnetism of Maxwell in the void, completed by new terms found by Louis de Broglie containing the very small proper mass $m_0 = \frac{m\hbar}{c}$ of the photon. These seven laws are exactly the same, but quantities are here only real or with real components, F_e is therefore the electromagnetic field of classical electromagnetism and optics. The photon does not need complex fields.

Differential laws (32) and (50) are form invariant under dilations defined by (28). This invariance under Cl_3^* induces that they are invariant under the restricted Lorentz group [3].

2- Three other photons of G. Lochak

Following the example of (31) seven other space-time vectors should be possible since Cl_3 algebra is 8-dimensional. They are built with $\phi_1X\phi_2^\dagger - \phi_2X\phi_1^\dagger$. Only three of these seven choices : $X = -\sigma_3$, $X = i$, $X = 1$ are compatible with (27) and we will see now that this gives Lochak's three other photons.

2-1 Case $X = \widehat{\sigma}_3$

We start now from

$$iB = \phi_1\widehat{\sigma}_3\phi_2^\dagger - \phi_2\widehat{\sigma}_3\phi_1^\dagger. \tag{56}$$

It is a pseudo-vector, not a vector, because

$$(iB)^\dagger = \phi_2\widehat{\sigma}_3\phi_1^\dagger - \phi_1\widehat{\sigma}_3\phi_2^\dagger = -iB. \tag{57}$$

The variance of B is the same as for A :

$$B' = MBM^\dagger \tag{58}$$

B must not be confused with the magnetic induction. It reads

$$B = B^0 + \vec{B} \tag{59}$$

where B^0 is the magnetic potential and $\vec{B}$ is the magnetic potential vector which are present in the theory of the magnetic monopole [2]. We let now

$$F_m = \nabla\widehat{iB} \tag{60}$$

This gives for the variance of F_m

$$F'_m = MF_mM^{-1}. \tag{61}$$

The detailed calculation of (56) shows that each product is one of products present in (27). We therefore get

$$\begin{aligned} F_m &= \nabla(\widehat{\phi}_1\sigma_3\overline{\phi}_2 - \widehat{\phi}_2\sigma_3\overline{\phi}_1) \\ &= 2[(\nabla\widehat{\phi}_1)\sigma_3\overline{\phi}_2 - (\nabla\widehat{\phi}_2)\sigma_3\overline{\phi}_1)] \\ &= m[\phi_1(-i\sigma_3)\sigma_3\overline{\phi}_2 - \phi_2(-i\sigma_3)\sigma_3\overline{\phi}_1] \\ &= -im(\phi_1\overline{\phi}_2 - \phi_2\overline{\phi}_1) \end{aligned} \tag{62}$$

We then get

$$\overline{F}_m = -im\overline{\phi_1\overline{\phi}_2 - \phi_2\overline{\phi}_1} = -im(\phi_2\overline{\phi}_1 - \phi_1\overline{\phi}_2) = -F_m \tag{63}$$

F_m is also a pure bivector and we can let

$$F_m = \vec{E}_m + i\vec{H}_m \tag{64}$$

which gives in (60)

$$\begin{aligned}\vec{E}_m + i\vec{H}_m &= (\partial_0 - \vec{\partial})(-iB^0 + i\vec{B}) \\ &= \vec{\partial} \times \vec{B} + i(\partial_0 \vec{B} + \vec{\partial} B^0) - i(\partial_0 B^0 + \vec{\partial} \cdot \vec{B}). \end{aligned} \tag{65}$$

And this is equivalent to the system

$$\vec{E}_m = \vec{\partial} \times \vec{B} \tag{66}$$

$$\vec{H}_m = \partial_0 \vec{B} + \vec{\partial} B^0 \tag{67}$$

$$0 = \partial_\mu B^\mu \tag{68}$$

The Lorentz gauge (68) is also a necessary condition for magnetic potentials. (66) and (67) are relations linking electric and magnetic fields to magnetic potentials in the theory of the magnetic monopole [2]. We then get

$$\begin{aligned}\Box \widehat{iB} = \widehat{\nabla} F_m &= -im\widehat{\nabla}(\phi_1 \overline{\phi}_2 - \phi_2 \overline{\phi}_1) \\ &= -2im[(\widehat{\nabla}\phi_1)\overline{\phi}_2 - (\widehat{\nabla}\phi_2)\overline{\phi}_1] \\ &= -im^2[\widehat{\phi}_1(-i\sigma_3)\overline{\phi}_2 - \widehat{\phi}_2(-i\sigma_3)\overline{\phi}_1] \\ &= -m^2 \widehat{iB}. \end{aligned} \tag{69}$$

We then get

$$\begin{aligned} & m^2(iB^0 - i\vec{B}) = (\partial_0 + \vec{\partial})(\vec{E}_m + i\vec{H}_m) \\ & = \vec{\partial} \cdot \vec{E}_m + (\partial_0 \vec{E}_m - \vec{\partial} \times \vec{H}_m) + i(\partial_0 \vec{H}_m + \vec{\partial} \times \vec{E}_m) + i\vec{\partial} \cdot \vec{H}_m \end{aligned} \tag{70}$$

equivalent to the system

$$\vec{\partial} \cdot \vec{E}_m = 0 \tag{71}$$

$$\partial_0 \vec{E}_m - \vec{\nabla} \times \vec{H}_m = 0 \tag{72}$$

$$\partial_0 \vec{H}_m + \vec{\nabla} \times \vec{E}_m = -m^2 \vec{B} \tag{73}$$

$$\vec{\partial} \cdot \vec{H}_m = m^2 B^0 \tag{74}$$

These equations (66) to (68) and (71) to (74) are exactly seven laws found by G. Lochak for the second kind of photon, the magnetic photon. We can notice that as with the electric photon each quantity is real or with real components.

Now it is possible to consider a total field $F = F_e + F_m$ satisfying

$$F = \nabla(\widehat{A + iB}) \tag{75}$$
$$\widehat{\nabla} F = -m^2(\widehat{A + iB}) \tag{76}$$

which are laws of the electromagnetism with electric charges and magnetic monopoles and densities of electric current j and magnetic current k satisfying

$$j = -\frac{cm^2}{4\pi} A \ ; \quad k = -\frac{cm^2}{4\pi} B \tag{77}$$

very small since m_0 is very small. Even if A and B are contra-variant vectors, the variance of m allows j and k to be covariant vectors [3], varying as ∇, not as x.

2-2 Case $X = i$

We start now from

$$A_{(i)} = \phi_1 i \phi_2^\dagger - \phi_2 i \phi_1^\dagger. \tag{78}$$

It is a contra-variant vector in space-time since we get

$$A_{(i)}^\dagger = \phi_2(-i)\phi_1^\dagger - \phi_1(-i)\phi_2^\dagger = A_{(i)} \tag{79}$$
$$A'_{(i)} = M A_{(i)} M^\dagger. \tag{80}$$

Now we let as previously

$$F_{(i)} = \nabla \widehat{A}_{(i)} \tag{81}$$

so we get as variance under Cl_3^*

$$F'_{(i)} = M F_{(i)} M^{-1}. \tag{82}$$

The detailed calculation of the product of matrices in (81) shows only products contained in (27) and we then get

$$\begin{aligned} F_{(i)} &= \nabla(\widehat{\phi}_1 \widetilde{i\phi}_2 - \widehat{\phi}_2 \widetilde{i\phi}_1) \\ &= 2[(\nabla\widehat{\phi}_1)(-i)\overline{\phi}_2 - (\nabla\widehat{\phi}_2)(-i)\overline{\phi}_1] \\ &= m[\phi_1(-i\sigma_3)(-i)\overline{\phi}_2 - \phi_2(-i\sigma_3)(-i)\overline{\phi}_1] \\ &= -m(\phi_1\sigma_3\overline{\phi}_2 - \phi_2\sigma_3\overline{\phi}_1). \end{aligned} \tag{83}$$

This gives

$$\overline{F}_{(i)} = -m\overline{(\phi_1\sigma_3\overline{\phi}_2 - \phi_2\sigma_3\overline{\phi}_1)} = -m[\phi_2(-\sigma_3)\overline{\phi}_1 - \phi_1(-\sigma_3)\overline{\phi}_2]$$
$$= -m(\phi_1\sigma_3\overline{\phi}_2 - \phi_2\sigma_3\overline{\phi}_1) = F_{(i)} \tag{84}$$

with an opposite result in comparison with (40) and (63). We then get

$$F_{(i)} = s + \vec{u} + i\vec{v} + ip = \overline{F}_{(i)} = s - \vec{u} - i\vec{v} + ip$$
$$\vec{u} = 0\ ;\quad \vec{v} = 0\ ;\quad F_{(i)} = s + ip \tag{85}$$

But then we get from (82)

$$s' + ip' = M(s + ip)M^{-1} = (s + ip)MM^{-1} = s + ip \tag{86}$$

This case corresponds to a field completely invariant under Cl_3^*. Now (81) reads

$$s + ip = (\partial_0 - \vec{\partial})(A^0_{(i)} - \vec{A}_{(i)})$$
$$= \partial_0 A^0_{(i)} + \vec{\partial}\cdot\vec{A}_{(i)} - (\partial_0\vec{A}_{(i)} + \vec{\partial}A^0_{(i)}) + i\vec{\partial}\times\vec{A}_{(i)} + 0i. \tag{87}$$

This is equivalent to the system

$$s = \partial_\mu A^\mu_{(i)} \tag{88}$$
$$0 = \partial_0\vec{A}_{(i)} + \vec{\partial}A^0_{(i)} \tag{89}$$
$$0 = \vec{\partial}\times\vec{A}_{(i)} \tag{90}$$
$$p = 0 \tag{91}$$

The field is therefore a scalar and invariant field. Next we get

$$\widehat{\nabla}F_{(i)} = -m\widehat{\nabla}(\phi_1\sigma_3\overline{\phi}_2 - \phi_2\sigma_3\overline{\phi}_1)$$
$$= -2m[(\widehat{\nabla}\phi_1)\sigma_3\overline{\phi}_2 - (\widehat{\nabla}\phi_2)\sigma_3\overline{\phi}_1]$$
$$= -m^2[\widehat{\phi}_1(-i\sigma_3)\sigma_3\overline{\phi}_2 - \widehat{\phi}_2(-i\sigma_3)\sigma_3\overline{\phi}_1]$$
$$= -m^2[\widehat{\phi}_1(-i)\overline{\phi}_2 - \widehat{\phi}_2(-i)\overline{\phi}_1] \tag{92}$$
$$\Box\widehat{A}_{(i)} = \widehat{\nabla}F_{(i)} = -m^2\widehat{A}_{(i)}. \tag{93}$$

This is equivalent to

$$(\partial_0 + \vec{\partial})s = -m^2(A^0_{(i)} - \vec{A}_{(i)})$$
$$\partial_0 s = -m^2 A^0_{(i)} \tag{94}$$
$$\vec{\partial}s = m^2\vec{A}_{(i)} \tag{95}$$

(88) to (91) and (94)-(95) are non-Maxwellian equations of the Lochak's magnetic photon, with

$$s = I_1 \ ; \quad m = k_0 \ ; \quad \mathbf{A}' = m\vec{A}_{(i)} \ ; \quad V' = mA^0_{(i)} \tag{96}$$

We can remark that this invariant scalar field is obtained in a largely independent way from the electromagnetic field. Here also everything is with real value or component. In this case physical objects are an invariant scalar field and a contra-variant vector. There is no apparent reason to identify A and $A_{(i)}$, even if they are two contra-variant vectors. Many physicists are searching now a scalar Higgs field rather similar to the s field encountered here.

2-3 Case $X = 1$

We start now from

$$iB_{(1)} = \phi_1\phi_2^\dagger - \phi_2\phi_1^\dagger. \tag{97}$$

It is now a contra-variant pseudo-vector in the space-time since we get

$$(iB_{(1)})^\dagger = \phi_2\phi_1^\dagger - \phi_1\phi_2^\dagger = -iB_{(1)} \tag{98}$$

$$B'_{(1)} = MB_{(1)}M^\dagger. \tag{99}$$

Now we let as previously

$$F_{(1)} = \nabla\widehat{iB_{(1)}} \tag{100}$$

so we get again as variance under Cl_3^*

$$F'_{(1)} = MF_{(1)}M^{-1}. \tag{101}$$

The detailed calculation of (97) shows that all products are in (27). We then get again

$$\begin{aligned} F_{(1)} &= \nabla(\widehat{\phi}_1\overline{\phi}_2 - \widehat{\phi}_2\overline{\phi}_1) = 2[(\nabla\widehat{\phi}_1)\overline{\phi}_2 - (\nabla\widehat{\phi}_2)\overline{\phi}_1] \\ &= m[\phi_1(-i\sigma_3)\overline{\phi}_2 - \phi_2(-i\sigma_3)\overline{\phi}_1] \end{aligned} \tag{102}$$

And we get

$$\begin{aligned} \overline{F}_{(1)} &= m\overline{[\phi_1(-i\sigma_3)\overline{\phi}_2 - \phi_2(-i\sigma_3)\overline{\phi}_1]} \\ &= m[\phi_2(i\sigma_3)\overline{\phi}_1 - \phi_1(i\sigma_3)\overline{\phi}_2 = F_{(1)} \end{aligned} \tag{103}$$

$$F_{(1)} = s + ip \tag{104}$$

With (86) $F_{(1)}$ is therefore also an invariant field under Cl_3^*. Now (100) reads

$$\begin{aligned} s+ip &= (\partial_0 - \vec{\partial})(-i)(B^0_{(1)} - \vec{B}_{(1)}) \\ &= 0 + \vec{\partial}\times\vec{B}_{(1)} + i(\vec{\partial}B^0_{(1)} + \partial_0\vec{B}_{(1)}) - i\partial_\mu B^\mu_{(1)} \end{aligned} \tag{105}$$

that is

$$s = 0 \tag{106}$$

$$0 = \vec{\partial}\times\vec{B}_{(1)} \tag{107}$$

$$0 = \vec{\partial}B^0_{(1)} + \partial_0\vec{B}_{(1)} \tag{108}$$

$$p = -\partial_\mu B^\mu_{(1)} \tag{109}$$

So this field is pseudo-scalar and invariant :

$$F_{(1)} = ip = ip' = -i\partial_\mu B^\mu_{(1)} \tag{110}$$

Next we get

$$\begin{aligned} \widehat{\nabla}F_{(1)} &= m\widehat{\nabla}[\phi_1(-i\sigma_3)\overline{\phi}_2 - \phi_2(-i\sigma_3)\overline{\phi}_1] \\ &= 2m[(\widehat{\nabla}\phi_1)(-i\sigma_3)\overline{\phi}_2 - (\widehat{\nabla}\phi_2)(-i\sigma_3)\overline{\phi}_1] \\ &= m^2[\widehat{\phi}_1(-i\sigma_3)^2\overline{\phi}_2 - \widehat{\phi}_2(-i\sigma_3)^2\overline{\phi}_1] \\ &= -m^2(\widehat{\phi}_1\overline{\phi}_2 - \widehat{\phi}_2\overline{\phi}_1) \end{aligned} \tag{111}$$

$$\Box\widehat{iB_{(1)}} = \widehat{\nabla}F_{(1)} = -m^2\widehat{iB_{(1)}} \tag{112}$$

This reads

$$(\partial_0 + \vec{\partial})(ip) = im^2(B^0_{(1)} - \vec{B}_{(1)}) \tag{113}$$

which is equivalent to

$$\vec{\partial}p = -m^2\vec{B}_{(1)} \tag{114}$$

$$\partial_0 p = m^2 B^0_{(1)} \tag{115}$$

(107) to (109) and (114)-(115) are non-Maxwellian equations of the Louis de Broglie's photon with $p = -I_2$. This is reduced to an invariant pseudo-scalar and a contravariant pseudo-vector. There is no evident reason to identify $B_{(1)}$ to the magnetic potential B. If the Higgs sector of the

quantum field theory needs also an invariant pseudo-scalar, p could be a good candidate.

We then see that it is easy to get in the frame of Cl_3 all four photons of the theory of de Broglie - Lochak and the whole thing is form invariant under Cl_3^*. There are differences in comparison with the construction based on Dirac matrices : All physical quantities are real or have real components and they are obtained by antisymmetric products. This is very easy to get with the internal multiplication of the Cl_3 algebra and was very difficult to make with complex unicolumn matrices. These two differences are advantages because vectors and tensors of classical electromagnetism and optics have only real components. And Louis de Broglie had understood very early that antisymmetric products are enough to get the Bose-Einstein statistics for bosons made of an even number of fermions. The scalar field of G. Lochak and the pseudo-scalar field for which Louis de Broglie was cautious are perhaps to bring together with the scalar Higgs boson that physicists search experimentally today. As fields s and p are obtained here independently from the field of the electric photon and of the magnetic photon, their mass is not necessarily very small and may be huge. Curiously it was the first idea of Louis de Broglie about the non-Maxwellian part of his theory. Were Higgs bosons seen as soon as 1934?

Concluding remarks

The quantum wave of a photon is actually an electromagnetic wave.

The theory of the electron was built in the frame of a field theory using complex numbers and Hilbert spaces. Here fields have only real components, as classical fields. But everything is built in the frame of a Clifford algebra isomorphic to the Pauli algebra. We can also say that $F = \vec{E} + i\vec{H}$ has complex values or that F which is a 2×2 matrix is a linear operator. The usual formalism of quantum mechanics is still available.

All fields were defined from antisymmetric products of spinors. They can disappear as soon as these two spinors are equal. They can appear as soon as they are not equal.

The first difference in comparison with classical electromagnetism is about potentials. They are not convenient vectors coming from adequate calculation. They are essential quantities. Fields are computed from them. It is a reinforcement of the position of potentials in quantum theory. We must recall that wave equations of quantum mechanics contain potentials. Fields are second.

The second difference in comparison with quantum field theory, where gauge invariances play a fundamental role, comes from the fact that the electromagnetism of the photon is not gauge invariant. This was never considered as bad by Louis de Broglie, on the contrary. In fact the invariance of the theory is the form invariance under Cl_3^*. Everything about electromagnetism can be made form invariant under this group, which extends the relativistic invariance. The Dirac theory contains things which are gauge invariant and things which are not gauge invariant. This was largely hidden in the Dirac theory by the matrix formalism which put in evidence only 16 densities without derivative from the 36 existing [3].

The result of conditions (26) of Louis de Broglie is a linearisation of the derivation of products which gives linear equations for bosons built from fermions. This is how the linear operator ∇ acts both in the Dirac equation and in Maxwell equations.

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