Velocity and proper mass of muonic neutrinos

Claude Daviau, Jacques Bertrand

email : claude.daviau@nordnet.fr, bertrandjacques-m@orange.fr
Fondation Louis de Broglie, 23 rue Marsoulan, 75012 Paris

RÉSUMÉ. Résolvant en ondes planes l'équation de Dirac pour un neutrino muonique, nous rendons compte de la vitesse mesurée de ces neutrinos. La vitesse mesurée permet d'obtenir la masse propre des neutrinos muoniques.

ABSTRACT. Resolving with plane waves the Dirac equation for muonic neutrinos we account for the measured velocity of these neutrinos. The measured velocity allows to get the proper mass of muonic neutrinos.

A recent experiment [1] found for the velocity v of muonic neutrinos from CERN to Gran Sasso a little more than the light speed :

$$\epsilon = \frac{v-c}{c} = [2.48 \pm 0.28(\text{stat}) \pm 0.30(\text{sys.})] \times 10^{-5} \tag{1}$$

And the dispersion of velocities as a function of the energy is weak. This does not allow to rely the measurement to tachyonic physics. The aim of this paper is to see how the measured velocity is consistent with both relativistic quantum mechanics and a rotating Earth. [1]

As neutrinos are neutral spin 1/2 particles, they follow a Dirac equation without charge term. The Dirac wave is made of two Weyl spinors and may be reduced to only one of these spinors if the proper mass is null. The standard model uses a null mass for neutrinos or antineutrinos, but countings of solar neutrinos and other experiments of neutrinic oscillations imply a not null proper mass. We suppose then here a wave

[1] Authors of this paper recognize now an error and think their experimental results as compatible with the light velocity. But this changes nothing to the Dirac equation and to the beginning of our computation.

with two Weyl spinors and a mass term. ψ is the matrix column of the wave, ξ and η are the Weyl spinors

$$\psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix} \ ; \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \ ; \quad \eta = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \tag{2}$$

We use here, as in [2], the matrix algebra generated by the Pauli matrices. This algebra is called Cl_3, because it is (isomorphic to) the Clifford algebra of the physical space. We note Cl_3^* the subset of all invertible elements in this Pauli algebra, that is the set of the 2×2 complex matrices M such as $\det(M) \neq 0$. We use

$$\phi = \sqrt{2} \begin{pmatrix} \xi_1 & -\eta_2^* \\ \xi_2 & \eta_1^* \end{pmatrix} \ ; \quad \widehat{\phi} = \sqrt{2} \begin{pmatrix} \eta_1 & -\xi_2^* \\ \eta_2 & \xi_1^* \end{pmatrix}. \tag{3}$$

ϕ and $\widehat{\phi}$ have their values in the Pauli algebra and are invertible if

$$\det(\phi) \neq 0 \tag{4}$$

In this frame the Dirac equation reads [2][3]

$$\begin{gathered} 0 = \nabla\widehat{\phi} + m\phi\sigma_{12} \ ; \quad m = \frac{m_0 c}{\hbar} \ ; \quad \sigma_{12} = \sigma_1\sigma_2 = i\sigma_3 \\ \nabla = \sigma^\mu \partial_\mu \ ; \quad \partial_\mu = \frac{\partial}{\partial x^\mu} \ ; \quad \sigma_0 = \sigma^0 = 1 \ ; \quad \sigma^j = -\sigma_j \end{gathered} \tag{5}$$

where m_0 is the tiny proper mass of a muonic neutrino[2]. To get the form invariance of the Dirac equation it is necessary, in any formalism, to consider D so defined :

$$\begin{aligned} &D : x \mapsto x' = MxM^\dagger \ ; \quad M \in Cl_3^* \\ &x = x^0 + \vec{x} \ ; \quad x' = x'^0 + \vec{x}' \ ; \quad \vec{x} = x^j\sigma_j \ ; \quad \vec{x}' = x'^j\sigma_j \end{aligned} \tag{6}$$

[2]Here $\widehat{\phi}$ denotes the conjugation $P : M \mapsto \widehat{M}$ satisfying

$$M = s + \vec{v} + i\vec{w} + ip = \begin{pmatrix} a_1 & -b_2^* \\ a_2 & b_1^* \end{pmatrix} \mapsto$$

$$\widehat{M} = s - \vec{v} + i\vec{w} - ip = \begin{pmatrix} b_1 & -a_2^* \\ b_2 & a_1^* \end{pmatrix},$$

where s is a real scalar, $\vec{v} = v^j\sigma_j$ is a vector, $i\vec{w} = iw^j\sigma_j$ is a pseudo-vector, ip with $p \in \mathbb{R}$ is a pseudo-scalar, and b^* is the complex conjugate of b.

If $\det(M) = 1$ it is well known that D is a Lorentz rotation conserving space and time orientation. We need also the identity

$$\det(M) = M\overline{M} = \overline{M}M \ ; \quad \overline{M} = \widehat{M}^\dagger \tag{7}$$

which gives if $\det(M) = 1 : M^{-1} = \overline{M}$. We have also

$$\nabla = \overline{M}\nabla'\widehat{M} \ ; \quad \nabla' = \sigma^\mu \partial'_\mu \ ; \quad \partial'_\mu = \frac{\partial}{\partial x'^\mu}. \tag{8}$$

We get indeed

$$\begin{aligned} 0 &= \nabla\widehat{\phi} + m\phi\sigma_{12} = \overline{M}\nabla'\widehat{M}\widehat{\phi} + m\phi\sigma_{12} \\ &= \overline{M}[\nabla'(\widehat{M\phi}) + m(M\overline{M})^{-1}(M\phi)\sigma_{12}]. \end{aligned} \tag{9}$$

So with

$$\phi' = M\phi \tag{10}$$

(9) becomes

$$0 = \nabla\widehat{\phi} + m\phi\sigma_{12} = \overline{M}[\nabla'\widehat{\phi}' + m\phi'\sigma_{12}]. \tag{11}$$

This implies the form invariance of the wave equation (5).

The Dirac equation is not form invariant under the uniform rotation move obtained with a not fixed M which is a function of the time. We get

$$\nabla\widehat{\phi} = \overline{M}[\nabla'\widehat{\phi}' - (\nabla'\widehat{M})\widehat{M}^{-1}\widehat{\phi}']. \tag{12}$$

The CERN experiment measures the velocity of muonic neutrinos in the frame of the Earth which is not Galilean, because the Earth rotates with one turn in a sidereal day (86164.09s). We take the rotation axis as the third direction and we get

$$\begin{aligned} &M = e^{ax^0 i\sigma_3} = e^{iact\sigma_3} \ ; \quad x' = MxM^\dagger \\ &x'^0 = x^0 \ ; \quad x'^3 = x^3 \ ; \quad x'^1 + ix'^2 = e^{-2iact}(x^1 + ix^2) \end{aligned} \tag{13}$$

As $e^{-2iact} = 1 = e^{-2i\pi}$ with $t = 86164.09\text{s}$ we get

$$a = \frac{\pi}{86164.09cs}. \tag{14}$$

We cut out the $'$ to simplify notations. We search a plane wave solution of

$$0 = \nabla\widehat{\phi} - (\nabla\widehat{M})\widehat{M}^{-1}\widehat{\phi} + m\phi\sigma_{12}. \tag{15}$$

As the velocity of neutrinos is nearly orthogonal to the axis of the Earth we seek a solution verifying

$$\phi = \phi_0 e^{-\varphi\sigma_{12}} \ ; \quad \varphi = m v_\mu x^\mu \ ; \quad v = v_\mu \sigma^\mu = v_0 + v_1\sigma^1 \tag{16}$$

where ϕ_0 is a fixed invertible term. And we get

$$\begin{aligned} \nabla\widehat{M} &= \partial_0(e^{iax^0\sigma_3}) = ia\sigma_3 e^{iax^0\sigma_3} \\ (\nabla\widehat{M})\widehat{M}^{-1} &= ia\sigma_3 \end{aligned} \tag{17}$$

Wave equation (15) gives

$$\begin{aligned} 0 &= (\sigma^\mu\partial_\mu)(\widehat{\phi}_0 e^{-\varphi\sigma_{12}}) - ia\sigma_3\widehat{\phi}_0 e^{-\varphi\sigma_{12}} + m\phi_0 e^{-\varphi\sigma_{12}}\sigma_{12} \\ &= [(\sigma^\mu\partial_\mu\varphi)\widehat{\phi}_0(-\sigma_{12}) - ia\sigma_3\widehat{\phi}_0 + m\phi_0\sigma_{12}]e^{-\varphi\sigma_{12}}. \end{aligned} \tag{18}$$

We get

$$0 = (\sigma^0\partial_0 + \sigma^1\partial_1)\varphi\widehat{\phi}_0 + a\sigma_3\widehat{\phi}_0\sigma_3 - m\phi_0 \tag{19}$$

But we have :

$$(\sigma^0\partial_0 + \sigma^1\partial_1)\varphi = m(v_0 + v_1\sigma^1) = mv \tag{20}$$

The wave equation is reduced to

$$\begin{aligned} 0 &= mv\widehat{\phi}_0 + a\sigma_3\widehat{\phi}_0\sigma_3 - m\phi_0 \\ \phi_0 &= v\widehat{\phi}_0 + \frac{a}{m}\sigma_3\widehat{\phi}_0\sigma_3. \end{aligned} \tag{21}$$

Conjugating we get

$$\widehat{\phi}_0 = \widehat{v}\phi_0 + \frac{a}{m}\sigma_3\phi_0\sigma_3. \tag{22}$$

This gives together

$$\begin{aligned} \phi_0 &= v[(\widehat{v}\phi_0 + \frac{a}{m}\sigma_3\phi_0\sigma_3)] + \frac{a}{m}\sigma_3[(\widehat{v}\phi_0 + \frac{a}{m}\sigma_3\phi_0\sigma_3)]\sigma_3 \\ \phi_0 &= v\widehat{v}\phi_0 + \frac{a}{m}v\sigma_3\phi_0\sigma_3 + \frac{a}{m}\sigma_3\widehat{v}\phi_0\sigma_3 + \frac{a^2}{m^2}\phi_0 \\ (1 - v\widehat{v} - \frac{a^2}{m^2})\phi_0 &= \frac{a}{m}(v\sigma_3 + \sigma_3\widehat{v})\phi_0\sigma_3. \end{aligned} \tag{23}$$

We use now

$$\begin{aligned} v\sigma_3 + \sigma_3\widehat{v} &= (v_0 - v_1\sigma_1)\sigma_3 + \sigma_3(v_0 + v_1\sigma_1) \\ &= 2(v_0\sigma_3 + iv_1\sigma_2) \end{aligned} \tag{24}$$

which gives

$$
\begin{aligned}
(1 - v\widehat{v} - \frac{a^2}{m^2})\phi_0 &= \frac{2a}{m}(v_0\sigma_3 + iv_1\sigma_2)\phi_0\sigma_3 \\
(1 - v\widehat{v} - \frac{a^2}{m^2})^2\phi_0 &= \frac{2a}{m}(v_0\sigma_3 + iv_1\sigma_2)(1 - v\widehat{v} - \frac{a^2}{m^2})\phi_0\sigma_3 \\
&= \frac{2a}{m}(v_0\sigma_3 + iv_1\sigma_2)\frac{2a}{m}(v_0\sigma_3 + iv_1\sigma_2)\phi_0\sigma_3^2 \\
&= \frac{4a^2}{m^2}(v_0^2 - v_1^2)\phi_0 \\
&= \frac{4a^2}{m^2}v\widehat{v}\phi_0. \qquad (25)
\end{aligned}
$$

And as ϕ_0 is invertible we get

$$
\begin{aligned}
&(1 - v\widehat{v} - \frac{a^2}{m^2})^2 = \frac{4a^2}{m^2}v\widehat{v} \\
&1 + (v\widehat{v})^2 + \frac{a^4}{m^4} - 2v\widehat{v} - 2\frac{a^2}{m^2} + 2\frac{a^2}{m^2}v\widehat{v} = 4\frac{a^2}{m^2}v\widehat{v} \\
&1 + (v\widehat{v})^2 + \frac{a^4}{m^4} - 2v\widehat{v} + 2\frac{a^2}{m^2} - 2\frac{a^2}{m^2}v\widehat{v} = 4\frac{a^2}{m^2} \\
&(1 - v\widehat{v} + \frac{a^2}{m^2})^2 = \left(\frac{2a}{m}\right)^2 \\
&\quad 1 - v\widehat{v} + \frac{a^2}{m^2} = \pm\frac{2a}{m} \qquad (26)
\end{aligned}
$$

This gives

$$
1 \mp \frac{2a}{m} + \frac{a^2}{m^2} = v\widehat{v} = v \cdot v \qquad (27)
$$

Two signs are possible corresponding to the possibility of velocities adding or subtracting. The + sign is required for the CERN experiment where neutrinos travel in the direction of the rotating Earth. We then get

$$
\| v \| = 1 + \frac{a}{m}. \qquad (28)
$$

This must be compared to the usual result $\| v \| = 1$ obtained when we calculate plane waves without rotation move. This gives with (16)

$$
\| mv \| = \| \frac{m_0 c}{\hbar} v \| = \frac{m_0 c}{\hbar}(1 + \frac{a}{m}). \qquad (29)
$$

The limit speed c which is the light speed if the proper mass of the photon is null or negligible is replaced by

$$c' = c(1 + \frac{a}{m}). \tag{30}$$

And we get

$$mv = \frac{m_0 c'}{\hbar} v' \ ; \quad \| v' \| = 1 \tag{31}$$

It is the reduced speed v' which has properties of the usual v reduced speed, for instance with the usual v speed :

$$v'_0 = \frac{1}{\sqrt{1 - \frac{\mathrm{v}^2}{c'^2}}} \ ; \quad v'_j = \frac{\frac{\mathrm{v_j}}{c'}}{\sqrt{1 - \frac{\mathrm{v}^2}{c'^2}}} \tag{32}$$

As the energy of the muonic neutrinos is very high, $\mathrm{v} \approx \mathrm{c}'$. If the proper mass of electronic or tauic neutrinos is different, the limit speed of these neutrinos may be different.

This interpretation of experimental results [1] is easy to check : if the same experiment is made with the velocity of muonic neutrinos in the inverse direction of rotation of the Earth, we will get a velocity lower than the light speed.

The measured value of the velocity of muonic neutrinos is linked to the proper mass of these particles. Therefore it allows to calculate this proper mass.

$$\epsilon = \frac{a}{m} = \frac{a\hbar}{m_0 c} \tag{33}$$

which gives

$$m_0 = \frac{a\hbar}{\epsilon c} \approx 1.715 \times 10^{-51}\mathrm{kg} \tag{34}$$

Uncertainties on m_0 come firstly from uncertainties (1) on the experimental value ϵ. It should be also useful to account for the fact that CERN and Gran Sasso have not the same latitude. [3]

Proper mass of the muonic neutrino appears very small, even compared with the electronic proper mass. In the case of the electron, it is

[3] This calculated mass is too little if the experimental result is false. It is possible that only electronic neutrinos have a very small mass. The effect studied here does not apply to neutrinos coming from stars, which are not concerned by the rotation of the Earth, nor to photons because they follow a wave equation different from the Dirac equation.

useless to account for the rotating Earth : As the proper mass is much greater $\frac{a}{m}$ is negligible.

A such very little mass is compatible with the theory of the photon as a compound of two neutrinic waves. This theory was developed by Louis de Broglie [4] and enlarged by G. Lochak [5].

References

[1] T.Adam ... A. Zguiche : *Measurement of the neutrino velocity with the OPERA detector in the CNGS beam*, http ://arxiv.org/abs/1109.4897v2.

[2] C. Daviau : *L'espace-temps double*, Ed. JePublie Paris 2011. English translation : *Nonlinear Dirac Equation, Magnetic Monopoles and Double Space-Time*, in printing.

[3] C. Daviau : *Cl_3^* Invariance of the Dirac Equation and of Electromagnetism*, Advances in Applied Clifford Algebras, accepted for publication.

[4] L. de Broglie : *La mécanique du photon, Une nouvelle théorie de la Lumière : tome 1 La Lumière dans le vide*, Hermann, Paris 1940

tome 2 Les interactions entre les photons et la matière, Hermann, Paris 1942.

L. de Broglie : *Théorie générale des particules à spin (méthode de fusion)*, Gauthier-Villars, Paris 1954

L. de Broglie : *Ondes électromagnétiques et photons*, Gauthier-Villars, Paris 1968

[5] G. Lochak : *Sur la présence d'un second photon dans la théorie de la lumière de Louis de Broglie*, Annales de la Fondation Louis de Broglie, **20**, 1995, p. 111.

G. Lochak : *"Photons électriques" et "photons magnétiques" dans la théorie du photon de de Broglie (un renouvellement possible de la théorie du champ unitaire d'Einstein)*, Annales de la Fondation Louis de Broglie **29**, 2004, p. 297 and **33**, 2008, p. 107.

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