Annales de la Fondation Louis de Broglie, Volume 37, 2012

Clock effect and operational definitions of time

Claudio Borghi

Liceo Scientifico Belfiore, via Tione 2, Mantova, Italy

ABSTRACT. This paper shows the need to deduce different operational definitions of time from the different clocks that we can use to quantify time intervals. A new operational definition of thermodynamic (irreversible) time, as different from Newtonian and Einsteinian, is proposed. As a measuring instrument of irreversible time we can use a radioactive clock, which quantifies the increasing product of decay and allows us to measure the increase in the amount of change and transformation that occurs in the system to which it belongs.

1 Introduction

This paper explores the problem of the physical measurement of time. We discuss the nature of physical time and we propose the hypothesis of reality of thermodynamic time. We can deduce the operational definition of thermodynamic time from radioactive clocks that could quantify different durations, related to the same world line between two fixed points, compared to those quantified by atomic clocks. The distinction between the concept of proper time, an external characterization of a mechanical system related to the spacetime curvature, and that of internal time, which could be quantified by radioactive clocks, is also suggested.

2 Clock effect

In 1907 Albert Einstein tried to generalize his special theory of relativity to non-inertial reference frames. In the first place he observed that in an accelerated reference frame some apparent forces appear, having in common with the gravitational force the property of being proportional to the mass of the body on which they act. From this observation Einstein deduced the well-known principle of equivalence, according to which an apparent force in an accelerated reference frame is physically indistinguishable from a gravitational force. Since the generalization of the theory of relativity to non-inertial reference frames would lead to a new theory of gravitation, the following options were possible:

- he could write all the laws of physics in a valid form in an arbitrary coordinate system (principle of general covariance), as the transformation of coordinates between reference frames in arbitrary motion is more general than that between inertial reference frames (Lorentz transformations);
- he could consider that, in the presence of gravitational fields, spacetime has no longer the metric of Lorentz-Minkowski valid in special relativity: since the gravitational fields are generated by masses, Einstein formulated the hypothesis that the presence of masses curves the spacetime and he set out to find the laws of this curvature (Einstein's equations of gravitational field, interpreted as a spacetime curvature).

Elio Fabri [1] argues that general relativity, developed from these intentions of its creator, became a theory that has been interpreted in different ways. Some see as its hallmark the adoption of an arbitrary coordinate system and of an arbitrary reference frame, but some believe that being Lorentzian spacetime or not marks the boundary between the territories of the two theories of relativity. According to the second interpretation, we are not using the general theory if we study physical situations in which spacetime is flat (not curved), whatever the coordinates we choose to use: the choice of coordinates is a simply mathematical fact, without physical meaning. If we accept instead the first interpretation, even if spacetime is flat, the simple description of phenomena in an accelerated reference frame implies that we are working outside the bounds of the special theory, being impossible to use the Lorentz transformations. It should be noted (this is not inconsiderable) that the experiments in an accelerated reference frame show phenomena not observable in an inertial reference frame, such as the inability to synchronize clocks located in different places or the non rectilinear propagation of light. If we share the second interpretation, while not denying the effects mentioned above, we deduce that the physical properties of spacetime are not affected by the adoption of an accelerated reference frame, nor are they influenced by the choice of one coordinate system rather than another. We describe now ideally what happens in an experiment in which are compared the measurements of two clocks: as clock A (initially synchronized with B) moves in uniform rectilinear motion (with respect to a suitable inertial reference frame), B leaves and then rejoins A. We can imagine A and B on two spacecrafts side by side: while A has power off for the whole experiment, B lights the rockets, moves away in to space and rejoins A. Now if we compare the clocks, we find that A is ahead of B. The well known clock paradox is based on: a) the Newtonian conception of absolute time, so it looks absurd that two clocks can record different times; b) the apparent paradox of symmetry, so if it is true that clock B is moved with respect to clock A, and we accept that being in motion it measures less time, if we see things from the perspective of B we can reverse the argument. In this second case, B is still and A departs from and returns to it, so A should measure less time. Because the two perspectives cannot both be true, the only way out is to admit that both are false, namely that the two times are equal (this argument was supported by Herbert Dingle [2]). While the paradox in its first formulation is (easily) removed by eliminating the psychological consequences of the interpretation of Newtonian absolute time, the second argument is more subtle and at first sight irrefutable: it appears to open a hole in the logical building of relativity, undermining the foundations of the concepts of relativity of motion and equivalence of inertial reference frames. The solution of the controversy is that the two spacecrafts are not both inertial reference frames, so the supposed equivalence does not exist, because at least one is accelerated. If we choose to interpret the accelerated reference frames in the theory starting from the concept of proper time, we can deduce that each body has its proper time, closely related to the metric of spacetime. Proper time is the length of the world line described by the material point in spacetime (the world line always exists, whether the body is moving or not, because time passes). If we refer to Hafele-Keating experiment [3] and we consider the clocks on the airplane and on the ground, we see them describe two different world lines (with the same initial and end points) having different lengths, that we can calculate using Minkowski's theory. Since it is shown that, among all lines joining two fixed points in spacetime, the straight line segment has the maximum length, it follows, according to Einstein, that this length should be the time measured by the clock, so clock A, which moves in uniform rectilinear motion (its world line is a straight line) should record a time longer than any other: the conditional is a must, since the theory does not clearly explain what it is either a clock or what it means, physically, the experimental fact that time passes and clocks provide different measures according to the world lines they describe.

3 Clock effect and gravitational potential

Ilaria Bonizzoni and Giuseppe Giuliani [4] argue that in atomic clocks atoms behave as microscopic locking devices of frequency of the microwave absorbed by atoms themselves, the block being guaranteed by the matching between the energy $h\nu$ of the microwave and the energy difference ΔE between the two quantum states of the transition. Since in experiments on the gravitational red-shift the energy difference ΔE, and thus the energy difference of the photons emitted or absorbed by atoms or nuclei, depends on the potential associated with the gravity or the pseudopotential due to acceleration, it follows that the fundamental period of the atomic clocks depends on the potential as the energy absorbed by the atoms or nuclei. If we refer to an experiment on the relativistic measurement of proper time, the processing of experimental data can be obtained from the law:

$$\Delta E = \Delta E_0 \sqrt{1 + \frac{2\chi}{c^2}} \tag{1}$$

that provides the energy difference ΔE between two quantum states of an atom at rest in a gravitational potential χ as a function of energy difference ΔE_0 at zero potential. Since the fundamental period of an atomic clock in a zero gravitational potential is expressed by the relation:

$$T_0 = \frac{h}{\Delta E_0} \tag{2}$$

we obtain from (1) the fundamental period $T(R)$ of a clock at rest at the equator:

$$T(R) = T_0 \left(1 - \frac{1}{c^2}\left[\phi_G(R) + \phi_A(R)\right]\right) \tag{3}$$

where $\phi_G(R)$ is the gravitational potential of the Earth at the equator and $\phi_A(R)$ is the pseudogravitational potential at the equator due to centripetal acceleration. It follows that:

$$T(R) = T_0 \left(1 + \frac{1}{c^2} \left[\frac{GM}{R} + \frac{1}{2} \Omega^2 R^2 \right] \right) \tag{4}$$

where M, R and Ω are respectively the mass, the radius and the angular speed of the Earth. For clocks in flight on a circle of radius $R + h$ we obtain instead:

$$T(R+h) = T_0 \left(1 + \frac{1}{c^2} \left[\frac{GM}{R+h} + \frac{1}{2} \left[v + (R+h)\Omega \right]^2 \right] \right) \tag{5}$$

where v is the speed of a moving body with respect to the ground, positive for the eastward motion, negative for the westward. From the above analysis we can argue that Hafele-Keating experiment (considered as one of the most important verifications of time dilation tested on macroscopic clocks) is only a verification of the effect of gravitational or pseudogravitational potential on the energy of photons emitted or absorbed by atoms inside the atomic clocks. The question if this effect is observable in all kind of clocks is open.

4 Internal and external time

A careful analysis shows that in physics the concept of time is used in two different ways: as an external attribute of motion or as an implicit variable that measures the internal evolution of a system. The first one is explicitly used in mechanics, the second, implicitly, in thermodynamics. Since in thermodynamics the variable t in practice does not appear in the definition of the physical quantities, we naively think that the concept of time introduced in mechanics can be used everywhere. Although it may sound a simplification, it is immediately clear that, as the mechanical evolution is related to the change in the position of a body with respect to others, the thermodynamic evolution of a system is linked to processes that involve it internally and might not have relationships with the environment, thus with the external space of relations. In the first case time is like a label attached to the system, in the second it is a quantity that informs us of its intrinsic evolution. In Einstein's mechanics clocks are objects ideally linked to bodies that measure different intervals of time, related to their location or to their state of motion. The difference between the theories of Newton and Einstein is the mathematical structure arising from their different conceptions of space and time. According to Newton, space is a container and time flows in a parallel dimension as an absolute. According to Einstein, space does not exist as a container, only fields and interactions are real (matter can be thought of as a particular concentration of field), while time flows in a dimension which, although inaccessible from space, forms a continuum with it. The structure of this continuum is described by mathematical equations, from which it follows that clocks should measure their proper times depending on the different world lines they describe. While Newton does not require that clocks measure absolute time (which by definition cannot be truly quantified, being a simple mathematical abstraction), Einstein requires that properly constructed clocks measure proper times, as observers probing the mathematical laws that form the logical structure of his theory. Proper time depends on spacetime curvature, determined by the distribution of matter and energy in which the observer clock is located in or moving, therefore is a proper time external to the clock, whose measure is influenced by the relational structure of the fields in which it describes its world line. Only atomic clocks give answers which concord with the theory of relativity, for reasons due to quantum phenomena that determine the link of their period with the gravitational or pseudogravitational potential. If we build clocks that quantify durations through different microscopic phenomena (an example is radioactive decay) with respect to those that determine the operation of an atomic clock, we can analyze the measurements obtained with these instruments and verify if they quantify proper time as expected by Einstein's theory. If the quantified durations are not comparable to those of the atomic clocks, we can conclude that relativistic time is a concept that applies only within the theory of relativity, and exporting it creates conceptual overlaps. In the next section we propose an analysis of the radioactive clocks as examples of instruments that measure internal proper time, whose operational definition is different from that of relativistic proper time measured by atomic clocks.

5 Thermodynamic time and radioactive clocks

Unlike the Leibnizian monads, without doors or windows, atomic clocks are influenced by the distribution of matter and energy in which they are immersed. These tools reduce the measure of time to a counting of ideally reversible periodic oscillations. What is the relationship between this quantity and irreversible time, an intrinsic variable that measures the thermodynamic evolution of a system, implicitly linked to the real degradation of matter and energy? As a measuring instrument of irreversible time we can use a radioactive clock, which quantifies the increasing product of decay and allows us to measure the increase in the amount of change and transformation that occurs in the system (if isolated) to which it belongs. We know that radioactive decay is an irreversible process, consisting in the production of a daughter from a parent substance. Let us consider the law:

$$N = N_0 \left(1 - e^{-\frac{t}{\tau}}\right) \tag{6}$$

where N expresses the quantity of the substance produced by the decay as a strictly increasing function of time t. We observe that the value of N can be considered a direct measure of thermodynamic time: it quantifies, in units of N_0, the duration of the decay. If we solve equation (6), we can calculate the corresponding value of t: the quantification of t is not necessary, it is simply useful. From equation (6) we obtain:

$$t = \tau \ln \left(\frac{N_0}{N_0 - N}\right) \tag{7}$$

If we measure (in units of N_0) the duration N of a phenomenon locally simultaneous to the decay (for example the duration of the trip of a plane), we can then calculate the duration itself in units of τ using equation (7). We can also convert t in seconds using the known value of the average lifetime of the substance. This procedure has an important physical and conceptual meaning: it makes us understand that this clock measures internal proper time using a non periodic and irreversible phenomenon. By varying the radioactive substance (the dynamic heart of the instrument) we get another clock with a different τ, which determines the rate of decay. The incommensurability of the measures of durations obtained by different radioactive clocks prevents us from concluding that they can be used to measure absolute time. Nor can we conclude that radioactive clocks can be relativistic clocks, as they are inadequate for quantifying the effects expected from Einstein's theory. Newtonian and Einsteinian time is reversible, physically different than what we measure with a radioactive clock. Radioactive clocks are clear examples of thermodynamic devices. In fact, radioactive decay involves billions of atoms and there is no sense in asking if they all obey the law (6): from microscopic chaos emerges the mathematical order as an average behavior of a population of particles which boils down to a simple formula. The thermodynamic concept of time comes out of molecular chaos, from microscopic disorder. Equation (6) implicitly says that a population of N_0 atoms decays into a daughter substance, producing a measurable amount of decay. The quantification of this product is the experimental consequence of the increase of entropy: the progressive increase of N is a proof. It could be argued that a radioactive clock is not necessarily an isolated system: the principle of increasing entropy applies to isolated systems or more generally to systems and environments, therefore to the whole universe as an isolated system. However, if we experimentally test that it is not influenced on the fields in which it is immersed nor on acceleration, we can conclude that this clock forms an isolated system and then it measures internal proper time. In agreement with the working hypotheses (the rules of the game), every theoretical formalization implies a conceptual framework within which the operational definition of physical time is given, thus it is necessary to clearly distinguish the specific areas of investigation of the different theories. Thermodynamic time, measured by clocks inside of which periodic phenomena are not produced, but energy transformations associated with irreversible processes, quantifies the real degradation of matter and energy. The irreducibility of thermodynamic time to absolute or relativistic time implies that in any specific area specific clocks should be designed. Two not identically constructed clocks quantify different physical processes and furnish different measurements. So the results of experiments on the measure of time intervals do not express the properties of time itself, but of the objects or phenomena that these instruments are called upon to investigate.

6 Conclusions

In Newton's theory, absolute, true and mathematical time, of itself, and from its own nature, flows equably without regard to anything external, and by another name is called duration. Newtonian clocks are devices that provide some sensible and external (whether accurate or unequable) measure of duration by the means of motion, which is commonly used instead of true time. In Einstein's theory time forms a continuum with space and the observer's clocks obtain different values for the durations depending on the reference frame. The analysis of the experiments seems to prove unequivocally that the key to open the door of the operational definition of physical time is the Einstein's theory, as that of Newton is inadequate to interpret experimental data: Newtonian time appears to be a simple mathematical variable, that cannot be quantified by real clocks. The experimental measures on particles in flight are in accordance with the Einsteinian law

$$\tau = \frac{\tau_0}{\sqrt{1-\beta^2}} \tag{8}$$

where τ_0 is the proper lifetime of the particle (measured by an observer in the same state of motion) and τ is the non proper lifetime (measured in a reference frame by which the particle is observed in flight). The above law of time dilation, verified in several experiments (muons in cosmic rays [5, 6] or in a storage ring [7]), implies that lifetime changes if measured by two observers in different states of motion. We argue that this relativistic variation is not interpretable as a different behavior of clocks: the only experiment in which were compared time intervals measured by real clocks initially synchronized, then separated along different world lines and finally rejoined, is that of Hafele and Keating [8]. Proper times measured by relativistic clocks depend from the length of the world lines they described. We know that atomic clocks obtain measures of relativistic time in accordance with theoretical predictions because the atoms, behaving as microscopic locking devices of frequency of the microwave absorbed by atoms themselves, measure variations of gravitational or pseudo gravitational potential. The correspondence between the proper period of the instrument and the potential seems actually to be a property that only applies to atomic clocks. We wonder: if clocks of different construction, in which are not produced quantum transitions that make possible the measurement of relativistic time, describe the same world line but provide measures not comparable with those of the atomic clocks, what consequences can we deduce about the reality of physical time? Starting from the above analysis, we suggest that a radioactive clock, by which a time interval is measured quantifying the decay product of a given quantity of muons, probably will not provide measurements in accordance with the relativistic predictions. The postulate that relativistic effects must be verified by every clock, then by clocks of different construction than the atomic, implies therefore a clear epistemological error. If radioactive or biological clocks are not equivalent to atomic clocks, we have to admit that in physics does not exist a unitary operational definition of time. The observation of the behavior of biological clocks is however of difficult actualization. We propose to observe the behavior of radioactive clocks in the same conditions in which in the past was observed the behavior of atomic clocks. We remember that, if the positive proofs of the relativistic law of time dilation refer to measures of average lifetime of particles in inertial flight (in the atmosphere or in vacuum), a reliable comparison of the proper times measured by clocks of different construction with respect to the atomic clocks has not yet been attempted. If, as suggested by I. Bonizzoni and G. Giuliani [4], there is also the possibility that very large accelerations may modify in some way the internal constitution of particles, no such effects, in so far as they affect the particle lifetime, are seen in experiments where the transverse acceleration is $10^{18}\ g$. For instance, since the lifetime of muons does not depend on acceleration and, therefore, from gravitational potential, it may be argued that two muons-based clocks should read the same after a Hafele-Keating trip of one of them. If the quantity of decayed muons in a clock on a plane is equal to the quantity decayed in a clock on Earth, we can conclude that radioactive clocks do not measure different time intervals relative to two different world lines between the same points of departure and arrival. A new interpretation of the reality of physical time is necessary, not simply based on the naive assumption that time exists and flows, but on the measure of the amount of transformation that an instrument can quantify. The reality of physical time is therefore not explainable referring it to metaphysical assumptions as those of Newton nor to relativistic hypotheses like those of Einstein, because, while the ones anchor time to an absolute duration inaccessible to the instruments, the others need special clocks that can only simulate its flow. The new operational definition of physical time provided in this work, as a measure of the amount of transformation given by a radioactive clock, opens the possibility that thermodynamic time is a quantity of different nature with respect to Newtonian and Einsteinian time. It clearly opens new perspectives of experimental and theoretical investigations.

Acknowledgements. I would like to thank Silvio Bergia (Dipartimento di Fisica, Università di Bologna, Italy) for the fruitful discussions on my ideas and the critical reading of the manuscript of this work.

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(Manuscrit reçu le 7 mai 2012)