On variation in spin rate of bodies in a variable gravity field

Yuriy A. Portnov

Moscow State University of Printing Arts,
Russia, 127550, Moscow, Pryanishnikova st., 2A
E-mail: portnovyura@yandex.ru

ABSTRACT. This article is aimed at studying implications of the gravitation theory subject to the idea of seven-dimensional Universe proposed by the author hereof. Using body-orienting Eulerian coordinates as coordinates of seven-dimensional space we can derive the same body motion equations as result from standard equations of classical dynamics of solid bodies. The most attractive implication of this idea is that the notion of force can be excluded from mechanics, as well as the notion of moment of force, and replaced with geometric representations even in case with motion of non-point bodies. Moreover, the seven-dimensional Universe theory allows varying in spin rate of body subject to variation in the external gravity potential, which is actually discussed herein.

P.A.C.S.: 04.80.Cc

1 Introduction

The principal result of using the equivalence principle consists in exclusion of the metaphysical notion of force from the gravitation theory. According to Einstein, motion of a material point in the gravity field is determined by a simple condition meaning that the spatiotemporal curve which represents motion is geodetic. The success of this fine interpretation of the gravity field resulted in attempts to completely exclude the notion of force from physics and replace it with notions of metric geometry. However, this was associated with a significant complication. The general relativity theory having been developed for material points is not capable to describe motion of dimensional bodies due to the fact that any body can move translationally and spin about three independent axes. Accordingly, a task hereof is to show that my recent ideas of seven-dimensional space-time allow deriving an equation of motion for both material points and dimensional bodies capable to rotate about random axes.

At first, I will give the general definition [1]: seven-dimensional space-time coordinates shall mean time x^0, three body center coordinates x^1, x^2, x^3, self-rotation angle x^4, precession angle x^5, nutation angle x^6; three latter coordinates are three Euler angles orienting a body in space. To simplify further analysis I will assume that dimensional and angular coordinates are fully equivalent.

Let us consider motion of a globoid with proper weight m and moment of inertia with respect to axes over the center of masses J in the gravity field set by the metric form

$$ds^2 = g_{ik} dx^i dx^k,$$

wherein the conventional rule of summation over repeating characters from 0 to 6 is accepted. In the absence of gravity source the metric tensor shall look as follows:

$$\begin{aligned} g_{11} = g_{22} = g_{33} = -g_{00} = -1, \\ g_{44} = g_{55} = g_{66} = -\frac{J}{m}, \\ g_{45} = g_{54} = -\frac{J\cos(x^6)}{m}. \end{aligned} \tag{1}$$

Paradoxicality of the record of the metric tensor which in spite of four-dimensional metric tensors of the general relativity theory is a function of proofmass parameters (inertia-to-mass ratio) suggests that space is not absolute and the gravity field depends on the proofmass within it. However, the only gravity detection method available so far is curvature of geodetic lines along which proofmasses are moving. Therefore, the gravity presence or absence can even be only discussed from the point of view of a moving proofmass. Such a concept results in rethinking of the relativity notion: motion becomes relative as well as space-time itself depends on the particular proofmass.

By using metrics (1) for calculation of coherence symbols:

$$\Gamma^i_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{lm}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right),$$

which shall be further inserted in motion equations:

$$\frac{d^2x^i}{dt^2} = -\Gamma^i_{kl}\frac{dx^k}{dt}\frac{dx^l}{dt}. \tag{2}$$

Zero component equation results in null equation; accelerations for the first, the second and the third components are equal to zero:

$$\begin{aligned} a^1 &= 0, \\ a^2 &= 0, \\ a^3 &= 0, \end{aligned} \tag{3}$$

because in the absence of gravity in a closed-loop system accelerations are equal to zero. The remaining angular acceleration components result in the following:

$$\begin{aligned} \varepsilon^4 &= \frac{\omega^5\omega^6}{\sin(x^6)} - \frac{\cos(x^6)}{\sin(x^6)}\omega^4\omega^6, \\ \varepsilon^5 &= \frac{\omega^4\omega^6}{\sin(x^6)} - \frac{\cos(x^6)}{\sin(x^6)}\omega^5\omega^6, \\ \varepsilon^6 &= -\omega^4\omega^5\sin(x^6), \end{aligned} \tag{4}$$

where ω^k and ε^k are angular velocity and angular acceleration of body. Analysis of equations (4) shows that they are body rotation equations of classical mechanics. For instance, by multiplying the left-hand side and the right-hand side of the third equation of system (4) by inertia moment J, the well-known gyro equation can be easily derived:

$$M_\theta = -J[\omega_\varphi \times \omega_\psi],$$

where M_θ is moment of force applied to gyroscope axis, ω_φ is spin rate of gyroscope, ω_ψ is angular rate of precession.

It is obvious that the right-hand side of motion equation (2) becomes the force applied to the body when multiplied by mass for components $i = 1, 2, 3$, or moment of force when multiplied by inertia moment for components $i = 4, 5, 6$ (see [2]). Such an approach allows completely excluding the notions of force and moment of force from the gravitation theory.

Thus, on the assumption of seven-dimensional Universe existence the following concept can be formulated: world line of any body is geodetic in seven-dimensional space-time.

2 Deriving field equations

When developing a geometrized theory of gravitation a situation occurred in four dimensions when field equations look as follows:

$$R_{im} = æ\left(T_{im} - \frac{1}{2}g_{im}T\right), \tag{5}$$

$$R_{im} - \frac{1}{2}g_{im}R = æT_{im}. \tag{6}$$

Einstein, in his studies of the years 1916 and 1917, derived equation (5) while in the same years D. Gilbert derived equation (6) from the variation principle (see [3]). These equations can also be derived from each other by means of index convolution but in four-dimensional space-time only.

If space is highly dimensional, e.g., seven-dimensional like herein, a situation occurs where the equations should be recorded either as

$$R_{im} - \frac{1}{2}g_{im}R = æT_{im}, \tag{7}$$
$$R_{im} = æ\left(T_{im} - \frac{1}{5}g_{im}T\right),$$

or as

$$R_{im} - \frac{1}{5}g_{im}R = æT_{im}, \tag{8}$$
$$R_{im} = æ\left(T_{im} - \frac{1}{2}g_{im}T\right),$$

As a criterion [4] for choosing the right system of equations we shall use two experiments and check equations (7) and (8) above for agreement with experiments: by Keplerian motion of bodies according to Newton's law of gravitation and by Gravity Probe B experiment results.

Let us consider the general gravitation equation:

$$R_{im} = æ\left(T_{im} - \alpha g_{im}T\right), \tag{9}$$

where α is a constant to be defined later. Expanding equation (9) in powers of velocity we shall obtain the following equations:

$$\frac{1}{2}\nabla^2 g_{00}^{(2)} = æ(1-\alpha)T^{(0)00}, \tag{10}$$

$$\frac{1}{2}\nabla^2 g^{(3)}_{\alpha 0} = æ g^{(0)}_{\alpha\lambda} T^{(1)\lambda 0}, \tag{11}$$

$$\frac{1}{2}\nabla^2 g^{(2)}_{\alpha\beta} = -æ\alpha g^{(0)}_{\alpha\beta} T^{(0)00}, \tag{12}$$

where $A^{(N)}$ means N term of A series expansion in order of powers of V^N. Let us introduce the notion of potentials

$$g^{(2)}_{00} = -2\phi, \tag{13}$$

$$g^{(3)}_{\alpha 0} = -g^{(0)}_{\alpha\lambda}\zeta^{\lambda}, \tag{14}$$

$$g^{(2)}_{\alpha\beta} = \frac{2\alpha}{1-\alpha} g^{(0)}_{\alpha\beta}\phi, \tag{15}$$

By solving equations (10) to (12) using metric components (13) to (15) we shall obtain

$$\phi = \frac{æ(1-\alpha)}{4\pi}\int \frac{T^{(0)00}}{|x^1 - x|} d^3x, \tag{16}$$

$$\zeta^{\alpha} = \frac{æ}{2\pi}\int \frac{T^{(1)\alpha 0}}{|x^1 - x|} d^3x. \tag{17}$$

By inserting energy-momentum tensor into potential equation (16) and metric-to-potential relation (13) we shall obtain

$$g^{(2)}_{00} = -\frac{æ(1-\alpha)c^2}{2\pi}\int \frac{\varepsilon(x)}{|x^1 - x|} d^3x$$

whence it follows that

$$g^{(2)}_{00} = -\frac{æ(1-\alpha)c^2}{2\pi}\frac{M}{x^1}. \tag{18}$$

There can be relation of acceleration along radial component to metric tensor derived from the motion equations

$$a^1 = -\frac{c^2}{2}\frac{\partial g^{(2)}_{00}}{\partial x^1}. \tag{19}$$

By inserting the resulting value of metric tensor (18) in motion equation (19) we shall find that test particle acceleration is

$$a^1 = -\frac{æ(1-\alpha)c^4 M}{4\pi(x^1)^2}. \tag{20}$$

Alternatively, according to Newton's law of gravitation we have

$$a^1 = -\frac{kM}{(x^1)^2}. \tag{21}$$

By comparing acceleration equations (20) and (21) we find that the gravitational constant is equal to

$$æ = \frac{4\pi k}{(1-\alpha)c^4}. \tag{22}$$

By inserting gravitational constant (22) in formulas (16) and (17) we obtain the potentials

$$\phi = \frac{k}{c^4}\int \frac{T^{(0)00}}{|x^1 - x|} d^3x, \tag{23}$$

$$\zeta^\alpha = \frac{2k}{(1-\alpha)c^4}\int \frac{T^{(1)\alpha 0}}{|x^1 - x|} d^3x. \tag{24}$$

A special attention must be paid to the fact that the gravitational potential in equation (23) is the same as in the classical theory. Hence, Newton's law of gravitation is fulfilled with any value of constant α.

Let us derive gyroscope axis precession equations for Gravity Probe B experiment. To this effect we shall calculate the following Christoffel symbol components

$$\Gamma^{(2)0}{}_{\alpha 0} = -\frac{\partial \phi}{\partial x^\alpha}, \tag{25}$$

$$\Gamma^{(3)\beta}{}_{\alpha 0} = \frac{1}{2} g^{(0)\beta\lambda}\left(\frac{\partial \zeta_\alpha}{\partial x^\lambda} - \frac{\partial \zeta_\lambda}{\partial x^\alpha}\right) + \frac{\alpha}{1-\alpha}\delta^\beta_\alpha \frac{\partial \phi}{c\partial t}, \tag{26}$$

$$\Gamma^{(2)\beta}{}_{\alpha\gamma} = \frac{\alpha}{1-\alpha}\left(\delta^\beta_\alpha \frac{\partial \phi}{\partial x^\gamma} + \delta^\beta_\gamma \frac{\partial \phi}{\partial x^\alpha} - g^{(0)\beta\lambda} g^{(0)}_{\alpha\gamma}\frac{\partial \phi}{\partial x^\lambda}\right). \tag{27}$$

By inserting the values of Christoffel symbols (25) to (27) in the gyroscope axis precession equation

$$\frac{d\omega'_\alpha}{dt} = \left(\Gamma^{(3)\beta}_{\ \ \alpha 0} c - \Gamma^{(2)0}_{\ \ \alpha 0} u^\beta + \Gamma^{(2)\beta}_{\ \ \alpha\gamma} u^\gamma\right) \omega'_\beta,$$

we shall obtain

$$\frac{d\vec{\omega}'}{dt} = -\frac{c}{2}\,\vec{\omega}' \times (\vec{\nabla} \times \vec{\zeta}) + \frac{c\alpha}{1-\alpha}\,\vec{\omega}'\frac{\partial\phi}{\partial t} - \\ -\frac{1}{1-\alpha}\vec{\nabla}\phi \cdot (\vec{u} \cdot \vec{\omega}') + \frac{\alpha}{1-\alpha}\,\vec{u} \cdot (\vec{\omega}' \cdot \vec{\nabla}\phi). \tag{28}$$

Let us introduce a new symbol for angular rate

$$\vec{\omega} = \vec{\omega}' - \frac{c\alpha}{1-\alpha}\,\phi \cdot \vec{\omega}' - \frac{1}{2}\,\vec{u} \cdot (\vec{u} \cdot \vec{\omega}'). \tag{29}$$

We shall differentiate angular rate (29) with respect to time and using the resulting equation (28) we shall obtain

$$\frac{d\vec{\omega}}{dt} = -\frac{c}{2}\,\vec{\omega} \times (\vec{\nabla} \times \vec{\zeta}) - \frac{1+\alpha}{2(1-\alpha)}\vec{\nabla}\phi \cdot (\vec{u} \cdot \vec{\omega}) + \\ +\frac{1+\alpha}{2(1-\alpha)}\,\vec{u} \cdot (\vec{\omega} \cdot \vec{\nabla}\phi). \tag{30}$$

Using the rules of vector multiplication for convolution of two double scalar products we shall obtain

$$\frac{d\vec{\omega}}{dt} = \frac{c}{2}\,(\vec{\nabla} \times \vec{\zeta}) \times \vec{\omega} - \frac{1+\alpha}{2(1-\alpha)}(\vec{u} \times \vec{\nabla}\phi) \times \vec{\omega}.$$

or, alternatively

$$\frac{d\vec{\omega}}{dt} = \vec{\Omega} \times \vec{\omega}, \tag{31}$$

where

$$\vec{\Omega} = \frac{c}{2}\,(\vec{\nabla} \times \vec{\zeta}) - \frac{1+\alpha}{2(1-\alpha)}(\vec{u} \times \vec{\nabla}\phi). \tag{32}$$

Equation (31) shows that vector $\vec{\omega}$ being constant in magnitude is only precessing at the rate of Ω. Addend (32) is called geodetic precession.

The first term (32) corresponds to the interaction between spin and orbital angular moments.

Let us analyze the resulting rate of precession (32). According to Gravity Probe B experiment we shall find that inconsistency between the experiment

$$\vec{\Omega}_e = \frac{c}{2}\,(\vec{\nabla} \times \vec{\zeta}) - \frac{3}{2}(\vec{u} \times \vec{\nabla}\phi)$$

and equation (32) comes out only in the single case when

$$\alpha = \frac{1}{2}. \tag{33}$$

If follows from constant (33) that the right recording format of gravitation equations (8) is as follows

$$R_{im} - \frac{1}{5} g_{im} R = æ T_{im},$$
$$R_{im} = æ\left(T_{im} - \frac{1}{2} g_{im} T\right).$$

In case there is no matter that would generate the gravity field $T_{im} = 0$, the metric tensor (1) should be fulfilled for any proofmass. In such case we find that all curvature tensor components R_{im} go to zero. In order to ensure equality in equations (8) we shall introduce additional tensor Λ_{im} in the right-hand side of the equations which components for a spherically symmetric proofmass are:

$$\Lambda_{44} = \Lambda_{55} = \Lambda_{66} = \frac{1}{2}, \tag{34}$$

$$\Lambda_{45} = \Lambda_{54} = \frac{1}{2}\cos(x^6). \tag{35}$$

Then, the final gravity field equations for seven-dimensional space-time shall look as follows:

$$R_{im} - \frac{1}{5} g_{im} R = æ T_{im} + \Lambda_{im} - \frac{1}{5} g_{im} \Lambda, \tag{36}$$

$$R_{im} = æ\left(T_{im} - \frac{1}{2} g_{im} T\right) + \Lambda_{im}. \tag{37}$$

3 Spin rate versus gravitational potential characteristic

Let us analyze in which conditions spin rate variation occurs. To this effect we shall write rotational motion equations and write out their components

$$\varepsilon^{\lambda} = -\Gamma^{\lambda}_{00}c^2 - 2\Gamma^{\lambda}_{0\alpha}cu^{\alpha} - 2\Gamma^{\lambda}_{0\mu}c\omega^{\mu} - \\ -\Gamma^{\lambda}_{\alpha\beta}u^{\alpha}u^{\beta} - 2\Gamma^{\lambda}_{\alpha\mu}u^{\alpha}\omega^{\mu} - \Gamma^{\lambda}_{\mu\nu}\omega^{\mu}\omega^{\nu},$$

where $\alpha, \beta = 1, 2, 3$ are translational degrees of freedom, $\lambda, \mu, \nu = 4, 5, 6$ are rotational degrees of freedom, u^{α} are translational center-of-mass velocities. Regarding expansion of connected components in max third order of smallness we shall obtain only two non-zero terms

$$\varepsilon^{\lambda} = -2\Gamma^{\lambda}_{0\mu}c\omega^{\mu} - \Gamma^{\lambda}_{\mu\nu}\omega^{\mu}\omega^{\nu},$$

where

$$\Gamma^{\lambda}_{0\mu} = \delta^{\nu}_{\mu}\frac{\partial\phi}{c\partial t},$$

the value which is a function of gravitational potential while the non-zero addend is defined by metric tensor equations (1). As a result we shall obtain three rotational motion equations

$$\varepsilon^4 = \frac{\omega^5\omega^6}{\sin(x^6)} - \frac{\cos(x^6)}{\sin(x^6)}\omega^4\omega^6 - 2\frac{\partial\phi}{\partial t}\omega^4, \tag{38}$$

$$\varepsilon^5 = \frac{\omega^4\omega^6}{\sin(x^6)} - \frac{\cos(x^6)}{\sin(x^6)}\omega^5\omega^6 - 2\frac{\partial\phi}{\partial t}\omega^5, \tag{39}$$

$$\varepsilon^6 = -\omega^4\omega^5\sin(x^6) - 2\frac{\partial\phi}{\partial t}\omega^6. \tag{40}$$

Assuming that body is pivoting and is not subject to precession $\omega^5 = 0$ and nutation $\omega^6 = 0$, then the equations (39) and (40) shall go to zero while equation (38) will become as follows:

$$\varepsilon^4 = -2\frac{\partial\phi}{\partial t}\omega^4, \tag{41}$$

It is seen from equation (41) that if a variation in the gravitational potential occurs at the gyroscope point, the non-zero angular acceleration $\varepsilon^4 \neq 0$ will occur with non-zero spin rate $\omega^4 \neq 0$ and ω^4 will change.

Let us consider equation (41) having it written as follows:

$$\frac{d\omega^4}{\omega^4} = -2d\phi.$$

By integrating the resulting differential equation we shall obtain

$$\omega^4 = \omega_0^4 \exp(-2\Delta\phi), \tag{42}$$

where ω_0^4 is initial angular velocity, $\phi = \phi(x^\alpha)$ is gravitational potential at point x^α.

Gravitational potential of the Sun may be recorded as

$$\phi = \frac{k}{c^2}\frac{M_{sol}}{x^1}, \tag{43}$$

where k is Newton's gravitation constant, M_{sol} is mass of the Sun, x^1 is the distance from the Sun center to the point in question.

By inserting formula (43) in equation (42) we obtain the characteristic of body spin rate versus the distance to the Sun center:

$$\frac{\omega^4}{\omega_0^4} = \exp\left(-\frac{2kM_{sol}}{c^2}\left(\frac{1}{r} - \frac{1}{r_0}\right)\right), \tag{44}$$

where r_0 is initial distance to the Sun center, r is end distance to the Sun center. Therefore, with variation in distance to the Sun center pivoting body spin rate will vary. Spin rate of a body near the Sun shall be lower while spin rate of a body far from the Sun shall be higher.

We shall perform a numerical calculation using the following values: $M_{sol} = 1.9891 \cdot 10^{30}$ kg, $k = 6.67384 \cdot 10^{-11}$ m^{3}s^{-2}kg^{-1}, $c = 299792458$ m/s, $r_0 = 5.79 \cdot 10^6$ km, $r = 1.43 \cdot 10^9$ km. The relation of angular velocities (44) with the given values shall be

$$\frac{\omega^4}{\omega_0^4} = 1.000000508. \tag{45}$$

It follows from (45) that when a pivoting gyroscope is moving away from point r_0 to point r when the initial rotating velocity is 10 rpm the angular velocity will increase by 13 rotations per month which can be observed experimentally.

4 Conclusion

To summarize, let us note that the following main conclusions hereof:

1. A model of seven-dimensional space-time can be used to completely exclude the notions of force and moment of force applied to proofmass from the gravitation theory.

2. There have been gravitation equations obtained for seven-dimensional space-time.

3. There are no doubts that a close dependence exists between a proofmass rotation velocity and the gravitational potential variation (42). Such link discloses a new, experimentally observable effect that relates rotation and gravitation.

References

[1] Yu. A. Portnov, *Gravitation and Cosmology* **17**, 152-160 (2011).

[2] Yu. A. Portnov, The field equations in a seven-dimensional space-time *(Moscow State University of Printing Arts, Moscow, 2013)*, p. 154.

[3] I. E. Tamma, Ya. A. Smorodinskiy and B. G. Kuznecova, in: Work on the theory of relativity 1905-1920, edited by S. I. Larin, Albert Einstein, Collected scientific works, 1 *(Publishing House "Nauka", Moscow, 1965)*, p. 700.

[4] Yu. A. Portnov, *Engineering Physics* **5**, 03-06 (2013).

(Manuscrit reçu le 28 October 2013)