Are mechanical clocks relativistic clocks?

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ABSTRACT. This work completes a research about the theoretical and experimental issue of relativistic time. We analyze the behavior of mechanical clocks, particularly pendulums and spring clocks, and we deduce that their proper period is not in agreement with the predictions of general relativity. We remark the importance of the concept of effective mass and its relationship with the measurements obtained by clocks whose proper period depends on the clock mass. Some consequences about the measure of relativistic time are discussed in the conclusive section.

1 Introduction

This paper is structured in three parts. The first one (section 2) summarizes the ideas of Albert Einstein about the concept of relativistic time and the measurement of durations in special relativity. The second one (sections 3 and 4) provides a synthesis of the characteristics of relativistic clocks according to general relativity and shows that atomic clocks behave as relativistic clocks since their proper period correctly depends on gravitational and pseudo gravitational potential. The third one (sections 5 and 6) analyzes the behavior of pendulums, balance wheel and mass-spring clocks and shows that they do not behave as relativistic clocks. While pendulums were already recognized by Einstein himself not to behave as relativistic clocks, we point out the absolute novelty of the probable disagreement between the measurements obtained by spring clocks and the relativistic predictions, also remembering that in 1905's paper they were indicated as possible witness of the relativistic law of time dilation. Since the experiments have currently verified the clock effect only on particular (cesium, optical, at resonant cavity, etc.) clocks, in the conclusive section we suggest to reinterpret the relativistic theory of time starting from an accurate survey about the operation of real clocks.

2 Relativistic time

In 1905's work on the electrodynamics of moving bodies [1], Albert Einstein set the problem of the quantification of different temporal durations by clocks of identical construction in different states of motion. After introducing the definition of simultaneity between events, of which he detects the relative nature, he obtains the Lorentz transformations from the c-invariance principle and the principle of relativity extended to electromagnetic phenomena. Einstein then explores the physical meaning of the equations, with particular regard to the behavior of rigid rods and clocks in motion. In relation to the operations of measure it should be observed that, while the quantification of the length of a rigid rod can be made with a straight edge or a metric rope (in the case where the rod is in motion the observer must determine, by means of suitably synchronized clocks at rest located in the system at rest, in which points of this system are the beginning and the end of the rod to be measured at a given instant t), the quantification of a time duration must be made with an appropriate instrument, of which, for reasons of consistency, the internal structure should be investigated. Einstein shows little attention to this operational aspect and admits, without delving into the issue, that a duration can be measured by any clock (excluding pendulums and hourglasses, for reasons that will be later clarified). If we suppose that one of them marks the time t when it is at rest with respect to the inertial system K and the time *t'* when it is at rest in the origin of the system K', in uniform rectilinear motion with respect to K, Einstein asks what marks the clock in the origin of K' if observed from K. From the Lorentz transformation of time:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

and from the relation $x = vt$, that quantifies the distance traveled by the clock in motion, he concludes that the clock located in the origin of K', with respect to the observer in K, will be delayed, for every second, of $(1-\sqrt{1-\frac{v^2}{c^2}})$ seconds, i.e., less than quantities of the fourth order or higher, of $\frac{1}{2}\frac{v^2}{c^2}$ seconds. It follows that, if in the points A and B of K there are clocks at rest that, observed from the system at rest, are synchronized, and if the clock in A is moved with velocity v along a line that joins it with B, when it arrives in B the two clocks will be no longer synchronized, but the clock moved from A to B will be delayed of $\frac{1}{2}\frac{v^2}{c^2}\ t$ seconds (neglecting effects of fourth order or higher), where $t=\frac{x}{v}$ is time taken by the clock in the trip from A to B, measured by the clock that has not changed its speed. It may be immediately seen, says Einstein, that this result is also true when the clock moves from A to B along an arbitrary polygonal line, especially when the points A and B coincide. Assuming that the result proved for a polygonal line is also valid in the case of a continuously curved line, given in A two synchronized clocks, if one of them moves along a closed path with constant velocity (neglecting accelerations and decelerations) until it returns in A, this latter, when it arrives in A, is delayed, compared to that which has not been moved, of $\frac{1}{2}\frac{v^2}{c^2}\ t$ seconds. Einstein concludes that a balance wheel clock located at the equator will have an infinitesimally slowed rate, due to the greater tangential velocity linked to Earth's rotation, with respect to a clock of identical construction, subjected to the same conditions, located at a pole. In summary, in the analysis of the consequences of the coordinate transformations drawn in the quoted paragraph: a) the coordinate t implicitly corresponds to a clock, that is supposed to be able to measure it, and it is deduced that if it moves with velocity v along a line toward another clock initially synchronized with it, when rejoined they no longer march in step; b) it is suggested that the effect of loss of synchronization can be verified through a balance wheel clock, that at the equator must slow down compared to an identical clock placed at a pole, given the different tangential velocities due to Earth's rotation: the extent of delay is a direct consequence of the axiomatic assumptions underlying the theory. The Einsteinian reasoning clearly shows that the delay is only caused by the speed. In the following developments of theory, thanks to the four-dimensional spacetime model introduced by Minkowski in 1908, the delay appears to be linked to the reduced length of the world line: the issue of loss of synchronization, still in embryonic form in 1905's work, will be explicitly resolved, not without leaving, at a careful analysis, room for doubt or discussion about the interpretation of experiments in which the measures of real clocks are compared, in situations similar to those described in the above mentioned article. Limiting to this paper, we can conclude that the c-invariance postulate and the principle of relativity lead to new equations of transformation of the spatiotemporal coordinates of events, whereby clocks in different states of motion must provide different measures of durations, given the same extreme events. Experimental verifications on real clocks, says explicitly Einstein, can confirm whether or not the delay effect can be applied to the instruments of measure.

3 Relativistic clocks

Simple theoretical considerations [2] developed within the conceptual framework of general relativity, starting from the relationship between the metric coefficient g_{00} associated to the coordinate ct and the potential φ [1] in a static and weak gravitational and/or pseudo gravitational field:

$$g_{00} = 1 + 2\frac{\varphi}{c^2} \tag{2}$$

and from the relationship between the proper period of a clock and the metric coefficient:

$$T = \frac{T_0}{\sqrt{g_{00}}} \tag{3}$$

(where T_0 is the period outside the field), lead to deduce that the period of a relativistic clock has to satisfy the approximate relation:

$$T(\varphi) = T_0(1 - \frac{\varphi}{c^2}) \tag{4}$$

currently tested on atomic, optical, based on the use of maser or at resonant cavity clocks, and that should also characterize all the other clocks. It is fundamental to stress that a relativistic clock requires an internal process that does not cease to occur in free fall.

[1] The gravitational potential is assumed equal to the Newtonian potential $-\frac{Gm}{r}$. In the case of rotation, with angular velocity Ω, of the reference frame with respect to an inertial system, the pseudo gravitational potential, measured in a point by a co-rotating observer, is given by the centrifugal potential $-\frac{1}{2}\Omega^2 r^2$, where r is the distance from the point to the center of the rotating reference. In the case of a translational motion of the reference, at a constant acceleration a_l with respect to an inertial system, the pseudo gravitational potential, measured in a point by an observer in the accelerated frame, is given by $-a_l h$, where h is the distance from the point to an appropriately prefixed zero level.

4 Why atomic clocks are relativistic clocks

The original Einstein's intention of testing relativistic effects on real clocks proved to be very problematic. The issue of the experimental disagreement between the behavior of some real clocks and the theoretical predictions cannot in fact be simply attributed to the impossibility of building ideals clocks or tending to an ideal model (as noted by Ludvik Kostro [3], which underlines the need that relativistic clocks be practically point-like, since any real clock, having an extension and a mass, will always be far from the model to which it should be close), but to the fact that in nature there are clocks that cannot give measures in accordance with relativistic predictions. We ask: why some clocks agree and others do not agree with general relativity? Atomic clocks provide measurements in accordance with theory because the energy difference ΔE between two given quantum levels of an emitting atom, observed at rest, depends on the gravitational potential φ according to the equation [4, 5, 6]:

$$\Delta E = \Delta E_0 \sqrt{1 + 2\frac{\varphi}{c^2}} \tag{5}$$

where ΔE_0 is the energy difference in a null potential. This dependence has been verified also with atoms and nuclei at rest (with respect to a corotating observer) on a disc that describes a uniform circular motion, then in a pseudo gravitational potential. Since the proper period of an atomic clock depends on the energy difference between two given quantum states according to the equation:

$$T = \frac{h}{\Delta E} \tag{6}$$

we obtain:

$$\frac{T(\varphi)}{T_0} = \frac{\Delta E_0}{\Delta E} = \frac{1}{\sqrt{1 + 2\frac{\varphi}{c^2}}} \approx 1 - \frac{\varphi}{c^2} \tag{7}$$

(where T_0 is the proper period in a null potential), that is clearly in agreement with equation (4). The different measures of a duration, between two extreme events, given by atomic clocks in different gravitational or pseudogravitational potentials are therefore a consequence of their different proper period, as related to the energy of a given quantum transition [2]. As we will explain in detail in the following sections, pendulums and hourglasses, when working in references not in free fall, reduce their frequency with increasing altitude while, according to theory, they should increase it. We will also observe that spring clocks, in the presence of a gravitational or pseudo gravitational potential, do not change their proper period or they change it non in accordance with law (4), showing a clearly different behavior from that of cesium clocks.

5 Pendulums, balance wheel and mass-spring clocks

The only gravitational clocks that can be used (neglecting the hourglasses, for reasons of practicality) to test the Einstein theory are pendulums, that behave in clear disagreement with the predictions, as it can be deduced from the known Newton's classical law (for small oscillations):

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{8}$$

Going up in altitude (increasing the gravitational potential) they reduce their proper frequency of oscillation, while, according to (4), they should increase it, as indeed cesium clocks do. Einstein himself, in a footnote of his 1905's work, discarded the possibility of using pendulum clocks [3], as devices that do not work in zero gravity (for example, in free fall), since the oscillations are present only in gravitational or acceleration fields. Discarded therefore pendulums and hourglasses, we believe it is interesting to observe if there are other mechanical clocks that behave as relativistic clocks: we limit in this context to spring clocks, i.e. balance wheel and mass-spring clocks. The mechanism of a balance wheel clock is constituted by a swinging wheel, rotating around an axis, by a spiral spring that develops an elastic force, and by an anchor (the system said escapement) that gives the wheel small thrusts at the right time. Given the complexity of the balance wheel mechanism [4], we can study the

[2] The analysis given for cesium clocks can also be applied, with little differences in the theoretical approach, to optical clocks, to clocks based on the use of maser and to clocks at resonant cavity.

[3] "physically a system to which the Earth belongs"

[4] Since the proper period of a balance wheel clock is given by $T = 2\pi\sqrt{\frac{I}{k}}$, where I is the wheel's moment of inertia with respect to the axis and k is the spring constant, we believe that the following considerations about the independence of the proper period of a mass-spring clock on the gravitational and pseudogravitational potential can also be applied to a balance wheel clock.

motion of a linear harmonic oscillator, a mass-spring system. In relation to (4) it is interesting to know how this kind of clock behaves in the presence of a potential linked to a gravitational field or to the acceleration of the reference frame. A classical dynamical analysis leads to conclude that the fundamental period, expressed by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{9}$$

remains unchanged both in the case in which the system oscillates [5] in an horizontal plane, with respect to any inertial reference, and vertically, in absence of gravity, for example in a free fall reference: this behavior, unlike that of a pendulum, allows to consider a similar device suitable for measuring relativistic durations, so it could belong to the set of relativistic clocks. In fact, if it oscillates vertically in a (supposed inertial) terrestrial laboratory, it is observed, compared to the experiment carried out in a free fall reference frame, that both the center of oscillation and the amplitude [6] change, but not the period, still given by (9), since it is independent on the position of the center and on the amplitude. If the system vertically oscillates in an elevator that rises, for example, with acceleration equal to $\frac{g}{2}$, an observer in the accelerated system detects that the mass hanging from the spring (neglecting the mass of the latter) will be subjected to an apparent weight (the sum of the weight and the apparent force) equal to 1,5 times the weight quantified by an inertial observer: the effect is an harmonic oscillation with a different center of oscillation and a different amplitude compared to the previous case, but without variations of the period, that maintains the value given by (9). We conclude that: 1) an oscillator-clock, relative to the observer in the accelerated elevator, over that a gravitational also feels a pseudo gravitational potential due to the acceleration of the reference, but it maintains unchanged its proper period, given by (9); 2) the same period is measured by an inertial observer on a clock in the laboratory. We remember that, in the light of Minkowski's theory, the different measures given by two clocks in different states of motion are caused by the different length of the world lines they ideally describe, so that, if the

[5] The initial speed in all the following cases is assumed to be null.

[6] In the case of horizontal oscillation, for example in a terrestrial laboratory, or of vertical oscillation in a free fall reference, the amplitude is always arbitrary, as determined by the position, relative to the center of oscillation, from which the system is started, while, in the vertical case in a reference not in free fall, it is assumed that the oscillation starts at the point where the mass is hung on the spring.

gravitational potential is the same, the two clocks must count a different number of seconds without altering their proper period. However, in the case where the variation of the measurement is linked to a variation of the proper period of clock as a function of the potential (as it happens for cesium clocks [7]), the above given analysis of the forces acting on a mechanical oscillating system does not lead to deduce a law for the period in agreement with Einstein's theory. Limiting to this classical dynamical analysis, therefore, pendulum clocks, whose proper period depends on gravitational potential, reduce their proper frequency when the potential increases, against the Einsteinian predictions, but they cannot be considered relativistic clocks since they stop working in free fall references; spring clocks, on the contrary, work in free fall references, but they cannot be considered relativistic clocks because their proper period is independent on the potential. With increasing altitude, the first ones slow down their rate while the second ones keep it unchanged, whereas both they should increase it. These elementary and seemingly obvious considerations generate an undeniable interest in relation to the operational definition of time in physics and to the concept of relativistic clock. We must therefore deepen the issue in the framework of general relativity.

6 Spring clocks in general relativity. Effective mass

Time dilation, in special relativity, implies a different quantification of a duration with respect to inertial observers in different states of motion, according to whom a phenomenon occur at rest at the same point in space, or in movement, whereby the initial and final events spatially do not coincide. In this theoretical framework the different duration of the phenomenon is an effect of the relative motion of the observers, verifiable without the need of comparing real clocks. Though often confused and overlapped with the previous one, in general relativity an effect of different nature is observed, linked to the physical process that characterizes the operation of the device used for measuring time intervals. A theoretical problem rises, linked to the behavior of the different instru-

[7]Both in Hafele-Keating (1971) and in Alley *et al* (1979) experiment were compared the measurements of durations obtained by atomic clocks on a plane with those obtained by atomic clocks remained on the ground, with which they were initially synchronized. According to the analysis developed in [7], such experiments, in particular that of Hafele and Keating, can be interpreted in the light of the dependence of the period of atomic clocks on the gravitational and pseudo gravitational potential.

ments, in particular of spring clocks [8], whose proper period, as above shown, should be in agreement with equation (4). Through a theoretical analysis developed inside the framework of general relativity, J.K. Ghose and P. Kumar [8] have deduced that if a body, having mass m and rest energy $E = mc^2$ outside a gravitational or pseudo gravitational field, is brought inside the field, it is attracted from the source of the field and so it increases its kinetic energy, which is radiated out in different forms of energies if it is brought to rest. From (2) we can derive, in the approximation to static and weak fields, the law:

$$E(\varphi) = mc^2\sqrt{1 + \frac{2\varphi}{c^2}} \tag{10}$$

which expresses the rest energy of the mass m as a function of the gravitational and/or pseudo gravitational potential φ. ¿From equation (10) we deduce the law:

$$m_e = m\sqrt{1 + \frac{2\varphi}{c^2}} \tag{11}$$

which expresses the mass of the body within the field, that Ghose and Kumar (limiting to the Newtonian potential $\varphi = -\frac{Gm}{r}$) call effective mass [9]. Law (11) implicitly quantifies a mass defect for a body within the field compared to that measured outside. This implies a decrease of the proper period of an harmonic oscillator (considering negligible the variation of the elastic constant of spring) in a gravitational or pseudo gravitational field, clearly in disagreement with law (4), that provides an increase of the proper period of relativistic clocks inside the field with respect to the same period measured outside. In the light of the concept of mass defect theorized by Ghose and Kumar and of the concept of effective mass by them introduced, spring clocks (particularly balance wheel clocks, for which equivalent conclusions can be deduced) would therefore not be relativistic clocks. Since all mechanical clocks seem

[8]We yet refer to the theoretical simplification of a mass-spring linear oscillator to explore the issue of the dependence of the proper period on the potential.

[9]Equation (11) significantly differs from the law $m_e = m(1 + \frac{GM}{rc^2})$, that A. Grishaev [9] applies to a classical oscillator in the Newtonian gravitational potential generated by a body of mass M, where r is the distance from the oscillator to the center of M. We point out that Grishaev (which in turn makes use of the concept of effective mass) in the quoted work does not provide any bibliographic reference to justify this law, whereby we believe he introduces it as necessary to equate harmonic oscillators to relativistic clocks.

not to behave as relativistic clocks, the same operational definition of relativistic time, based on the implicit need for an homogeneous behavior of all clocks, would be obviously compromised.

7 Conclusions

In special relativity the c-invariance and relativity principles lead to Lorentz transformations and to Minkowski's model of spacetime, whereby a time duration should be differently quantified from observers in motion along not equipollent world lines between two given extreme events. The logical-operational leap from theory to instruments of measure (yet in dubitative form in 1905's work) occurs therefore when these times are not indirectly calculated, but directly measured through real clocks that, initially synchronized, ideally describe different world lines and finally rejoin. In general relativity the behavior of real clocks is correctly predicted only if their proper period agrees equation (4). Since, as above remarked, only the period of atomic and other electromagnetic clocks agree this equation, but not that of gravitational (hourglasses, pendulums) or mechanical (mass-spring, balance wheel) clocks, we believe that relativistic time is a quantity that does not allow to measure durations through clocks of different construction. Which is the reason for this different behavior? Atomic clocks work well because the energy difference between two fixed levels in an atom, observed at rest, depends on the gravitational or pseudo gravitational potential, while the above mentioned proper period of mechanical clocks is not in agreement with law (4), that expresses the relationship between the proper period and the potential. We stress as fundamental the issue of the effective mass, since the measurements obtained by instruments whose proper period depends on the device mass should decrease when they are in a gravitational or pseudo gravitational potential as a consequence of the mass defect theorized by Ghose and Kumar, in explicit contradiction with equation (4), that provides an increase of the proper period of a relativistic clock inside a gravitational or pseudo gravitational field. In fact, the Einstein theory is a mathematical device that, about the measure of durations, implies the agreement between the proper period of all clocks and law (4), whereby the experimental tests on real clocks give an important validity check not only of the consistency of the relativistic theory of time, but also of the dynamical consequences of the spatiotemporal continuum theory. In the light of the remarks here proposed, we believe that at least the relativistic theory of time should be reinterpreted [10]. The possibility exists that clocks of different construction measure times of different physical nature, whereby the problem of time can be resolved only starting from an accurate analysis of their possible different behavior in the same experimental situations.

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