Louis de Broglie's "double solution" a promising but unfinished theory

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ABSTRACT. A brief account is given of the ideas underlying L. de Broglie's conceptions of wave particle dualism, based on a close association between particle and wave, both being objective physical objects. Some later developments by his pupils are also given.

RÉSUMÉ. On donne un bref résumé des idées qui sont à la base de la conception du dualisme onde corpuscule de L. de Broglie. Elles reposent sur une association étroite de l'onde et de la particule, toutes deux considérées comme des objets physiques objectifs. Quelques développements ultérieurs de ces idées sont également présentés.

1 Introduction

From 1923 to 1927 L. de Broglie worked on a theory where both particle and wave existed and were closely associated. But after the 1927 Solvay Congress and the objections which this theory received, and in face of its mathematical difficulties he gave up and devoted its time to standard quantum theory[1]. He returned only after 1951 to his first ideas and developed along the same lines what is called the "double solution theory" which unfortunately remains up to now more a program, although with some striking results, than a complete, effective theory.

2 Phase harmony (1923-1924)

Inspired by Einstein's relativity theory and photon concept, L. de Broglie assumed in his first papers [3, 4] that a particle moving with

[1] For a complete list of L. de Broglie's publications, see [1]

velocity v behaves as a small clock $e^{i\nu_0 t_0}$, with frequency ν_0 such that $W = M_0 c^2 = h\nu_0$ in its rest frame B.

In the laboratory frame A, the frequency of this clock becomes

$$\nu_1 = \nu_0 \sqrt{1-\beta^2} \tag{1}$$

but $M = M_0/\sqrt{1-\beta^2}$ and the relation between M_0 and ν_0 is lost.

L. de Broglie's idea was that in B there exists an associated standing wave with the same frequency ν_0. In A this wave becomes

$$e^{i\nu_0 t_0} = e^{i\frac{\nu_0}{\sqrt{1-\beta^2}}(t - vx/c^2)} \tag{2}$$

and now the relation between M and ν is conserved :

$$W = Mc^2 = h\nu \tag{3}$$

The phase velocity V is given by $vV = c^2$ and this shows that the phase of the clock remains equal to the phase of the wave *in the point where the particle is located at time t*. This is the L. de Broglie's principle of phase harmony. As is well known this gives also $\lambda = h/p$.

Almost a century later, this conjecture has been tested experimentally [5] using a beam of electrons travelling in a cristal. The observations are compatible with the above relation.

3 Double solution, first version, linear theory

Looking for an explanation of interference phenomena for light quanta, L. de Broglie came already in 1924 [6] to the idea of a wave comprising a singularity which represents the corpuscle and which moves along the rays of the wave. The bright fringes were bright simply because the rays concentrate in the corresponding regions and the number of corpuscles is thus higher.

3.1 Principles

In his 1927 paper [7] L. de Broglie sums up his first version of the double solution theory. He considers the association of two waves. The "physical wave" named u is also called a "phase wave" because its important part is its phase ϕ :

$$u = a(x,y,z,t).exp\left(\frac{i}{\hbar}\phi(x,y,z,t)\right) \qquad \hbar = h/2\pi \tag{4}$$

satisfying for instance in the case of Klein-Gordon equation in a constant exterior electric field of potential V the *linear* equation

$$\Box u - \frac{2i}{\hbar} \cdot \frac{eV}{c^2} \cdot \frac{\partial u}{\partial t} + \frac{1}{\hbar^2} \left(m_0^2 c^2 - \frac{e^2}{c^2} V^2 \right) u = 0 \tag{5}$$

which can be split into a "Jacobi" equation and a conservation equation

$$\frac{1}{c^2} \left(\frac{\partial \phi}{\partial t} - eV \right)^2 - \sum \left(\frac{\partial \phi}{\partial x} \right)^2 = m_0^2 c^2 + \hbar^2 \frac{\Box a}{a} = M_0^2 c^2 \tag{J}$$

$$\frac{1}{c^2} \left(\frac{\partial \phi}{\partial t} - eV \right) \frac{\partial a}{\partial t} - \sum \left(\frac{\partial \phi}{\partial x} \right) \frac{\partial a}{\partial x} + \frac{a}{2} \Box \phi = 0 \tag{C}$$

This wave u is supposed to contain a singularity representing the particle. Assuming that the singularity of a is of a pole type leads to the relativistic guidance formula as a *consequence* of eq. (C) :

$$\vec{v} = -c^2 \frac{\nabla \phi}{\partial \phi / \partial t - eV} \qquad \text{non-relativistic limit} \quad \vec{v} = -\frac{1}{m} \nabla \phi \tag{6}$$

L. de Broglie then shows that a "cloud of material points" described by a set of such singular solutions may be represented by a continuous solution ψ of the same equation, having the same phase, such that the cloud density is given by the (already accepted) formula

$$\rho = \text{const.} \times a^2 = \text{const.} \times \psi \psi^*. \tag{7}$$

Hence the name of "double solution theory" which associates two waves u and ψ sharing the same phase (principle of phase harmony).

3.2 Pilot wave theory

In 1927 L. de Broglie was only able to give some examples of these singular solutions u, and struck by the mathematical difficulties involved he simplified his ideas for the Solvay Congress where he presented a restricted theory, the "pilot wave theory" where the material point and the continuous wave are considered as *distinct physical realities*, and the particle motion is defined by the guidance formula (6) acting as a *postulate* where ϕ is the phase of the continuous solution ψ.

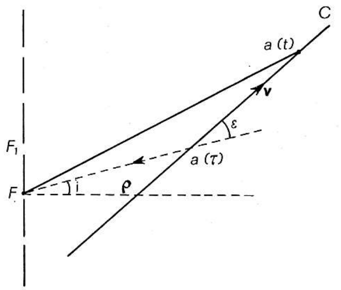
3.3 Exact singular solutions

Many years later, it was shown that a rigorous mathematical construction is possible as was given by F. Fer (for Klein-Gordon equation) [9, 10], and later M. Thiounn (for particle with spin $\neq 0$) [11]. These singular solutions have the required properties predicted by L. de Broglie :

a. the singularity follows a trajectory obeying the guidance formula, the phase ϕ of the singular solution u remaining regular everywhere
b. the singular solution can be connected with a regular solution, according to the principle of phase harmony.

These solutions are able to account for interference phenomena as L. de Broglie had conjectured in his 1924 paper [6] as may be seen in the following example given by F. Fer [9, 10].

3.4 Young's slits



For L. de Broglie's photon equation $\Box u - ku = 0$, the singular solutions read (they are analogous to Liénard-Wiechert potentials)

$$u(x) = \frac{\omega(\tau)}{\rho(c - v_\rho)}$$

Under standard conditions of interference, slits in phase $\forall t$ (u periodic on the screen), observation made at infinity, the solutions are such that the singularity necessarily moves only in the directions of the emerging rays given by the usual interference theory.

According to this theory, the particle may be as small as one wants, interferences appear even in single photon experiments.

4 Madelung's hydrodynamic interpretation

In 1926 Madelung [12] decomposed Schrödinger's equation

$$i\hbar\,\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\,\Delta\psi + U.\psi \quad \text{with} \quad \psi = a.\exp(i\phi/\hbar) \tag{8}$$

into

$$\frac{\partial \phi}{\partial t} + \frac{1}{2m} (\nabla \phi)^2 + U = \frac{\hbar^2}{2m} \frac{\Delta a}{a} = Q \tag{9}$$

$$\frac{\partial (a^2)}{\partial t} + \frac{1}{m} . \operatorname{div}(a^2 \nabla \phi) = 0 \tag{10}$$

which are the usual equations of an irrotational fluid of density $\rho = a^2$ and velocity potential ϕ, apart from the stress tensor which is replaced by the *quantum potential* Q. And the relation between the velocity and its potential $\vec{v} = \nabla \phi$ is essentially the same as the guidance formula (6) in the non-relativistic case.

These results, known to L. de Broglie in 1927, look formally similar to his theory, but with a very different interpretation as Madelung considers only a continuous fluid and the density and velocity of the flow are regular functions in every point of the medium.

5 Double solution, non linear equation

Starting in 1952, L. de Broglie's growing insatisfaction from QM measurement theory was expressed in his lectures [13, 15] unpublished at the time, for he changed his mind while delivering them as shown by small notes inserted in his manuscript. This and Bohm rediscovery of the pilot wave theory [14], led him to revive his first attempts and develop anew his double solution theory, based on the assumption of a non linear equation describing the behavior of matter waves.

The "true", objective, wave u obeys a (up to now unknown) non-linear equation, admitting solutions as bunched waves (solitons) representing the association of a particle and a wave. But, lacking information about the non-linear equation, L. de Broglie splits this wave in a low amplitude regular part v obeying the linear wave equation and a part u_0 carrying the particle which may be approximated by a singular solution as seen before

$$u = u_0 + v \quad \text{with} \quad u_0 \gg v \quad \text{inside the particle,} \quad u_0 \ll v \quad \text{outside} \tag{11}$$

these waves sharing the same value of the phase in the vicinity of the particle.

The guidance formula given in 1927 may then be given a more precise signification : it describes the motion of the bump which represents the particle, *as a consequence of the equations.* One of the derivations is the following. If $\xi(x_0, t)$ is any flow line of Madelung's fluid

$$\partial_t \xi(x_0, t) = \frac{1}{m} \nabla \phi(\xi(x_0, t), t) \implies \tag{12}$$

$$a^2(\xi(x_0, t), t) = a^2(x_0, 0) \exp(-\int_0^t \frac{\Delta\phi(\xi(x_0, t), t)}{m} dt) \tag{13}$$

if ϕ is quasi-constant over the singular region, $\exp(.) \approx 1$ and the bump stays on the flow line, hence the meaning of the guidance formula : the particle follows the guidance (flow) lines.

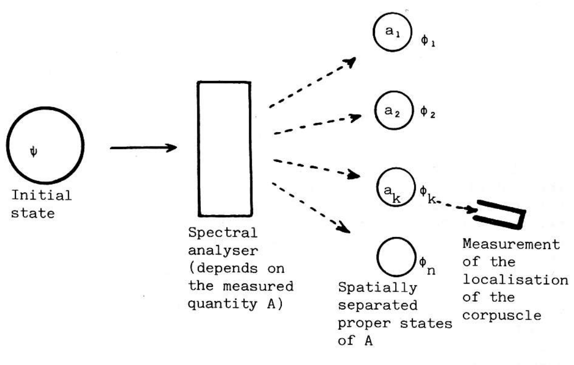
In L. de Broglie's views [16, 29], the guidance rule may be used to recover the probability density, not only by a simple analogy with a fluid, but by the superposition of a random element, a kind of brownian motion which induces jumps of the particle from one guidance line to another. This comes from the interaction with a hidden heat reservoir, the "subquantum medium" as first proposed by D. Bohm and J-P Vigier [17]. Its consequence is that, outside the singular region of the wave, one has

$$\psi = C\, v \quad \text{with} \quad \int |\psi|^2 = 1 \tag{14}$$

where ψ is the standard wave function and $|\psi|^2$ gives the position probability density.

5.1 Measurement theory

L. de Broglie's theory of measurement is based on the remark that in actual experiments, *only one actual probability density, position* is used as shown in the diagram given by G. Lochak [4]



L. de Broglie's general scheme of a measurement process

In free states experiments, this scheme introduces a distinction between present, predicted and hidden probabilities, allowing to solve many of the paradoxes involved in measurement experiments [18, 4].

5.2 Subquantum medium and Hidden thermodynamics

The interaction of the particle with a hidden medium (akin to Boltzmann's molecular chaos) together with the introduction of the quantum potential have led to a new type of thermodynamics called by L. de Broglie Thermodynamics of the single (isolated) particle [19, 29]. Its starting point is a relativistic dynamics where the proper mass M_0 of the particle is variable due to the value of the quantum potential.

The energy of a particle is :

$$\frac{M_0 c^2}{\sqrt{1-\beta^2}} = Q + E_t = M_0 c^2 \sqrt{1-\beta^2} + \frac{M_0 v^2}{\sqrt{1-\beta^2}} \tag{15}$$

where the internal heat Q is

$$Q = Q_0 \sqrt{1-\beta^2} = M_0 c^2 \sqrt{1-\beta^2} = h\nu_0 \sqrt{1-\beta^2} \tag{16}$$

The motion of this particle is governed by a least action principle with

$$A = \int_0^T -M_0 c^2 \sqrt{1-\beta^2} dt \approx -M_0 c^2/\nu_0$$

with $1/T = \nu = \nu_0 \sqrt{1-\beta^2}$ period of the internal clock. If v is given by the guidance formula, this implies (as it should) phase harmony between the particle and its associated wave.

L. de Broglie also shows that in this theory, action and entropy are proportional

$$S/k = A/h \tag{17}$$

which reduces the least action principle to the entropy one, linking the two most important of physics principles.

6 F. Fer charged quantum fluid

6.1 Steady states

The guidance formula in the fundamental state of the hydrogen atom has the unacceptable consequence that the electron is at rest. There are two ways of addressing this issue. The first one is the hypothesis of the subquantum medium where the electron jumps from one place to the other. But there exists a second one, by assuming that the electron is spread in the bound states and point like only in free states.

Consider with F. Fer [21] a model of an electron, extension of Madelung's fluid with *internal* electromagnetic fields and a cohesion potential G to counterbalance electrostatic repulsion (an idea already suggested by Poincaré [23]). Forgetting retardation effects the equations governing the behaviour of this fluid are (u being the wave function)

$$\begin{aligned} i\hbar\,\partial_t\psi = \Omega\,\psi &= -\frac{\hbar^2}{2m}\,\Delta\psi + (U+V).\psi + G.\psi \\ \text{with} \quad V &= \frac{e}{mK_0}\int \frac{\psi(x')\psi^*(x')}{|x-x'|}d^3x' \\ \text{and} \quad G &= \int \psi(x')\psi^*(x')g(|x-x'|)d^3x' \end{aligned} \tag{18}$$

A more general equation would introduce the vector potential, together with V, given by retarded integrals.

The wave function being written as in (8), steady states of this fluid obey

$$\Omega\,a = Fa \quad \text{with} \quad F = \int (aa^*\frac{v^2}{2} + G - Q) + \frac{e}{m}aa^*(U+V))d^3x \tag{19}$$

Existence of at least one solution results from the coercive character of the hamiltonian

$$H = \int aa^*\left(\frac{v^2}{2} - Q + \frac{1}{2}G + \frac{e}{2m}V + \frac{e}{m}U\right)d^3x \tag{20}$$

provided G is not too strong. If G and V almost cancel the eigenvalues F_i are close to Schrödinger's ones [24].

6.2 Quantum transitions

In 1960-62 a few physicists close to L. de Broglie studied a model of dynamical system with limit cycles [25]. It shows the possibility of one-way transitions of one limit cycle to another. This is the case also for Fer's quantum fluid.

A detailed analysis of the evolution of the quantum fluid when a steady state is perturbed shows that [26]

– the transient motion is oscillatory
– transition frequencies are given by Bohr's relation :

$$F_i - F_k = h(\nu_{ik} + \delta\nu) \tag{21}$$

– the oscillations are exponentially increasing when starting towards a lower energy level, and damped in the opposite case.

7 Conclusion

This work has taken many years, and is largely unfinished, as the set of non linear equations describing the association of wave and particle remains largely unknown. But it shows that there are no fundamental obstacles to its achievement, on the contrary many results already gained show that the road is open and leads in the right direction.

It is also important not to forget the results obtained by L. de Broglie during his "Copenhagen" years, his "fusion method" [28], a way to deal with compound particles made up of Dirac spin 1/2 particles, e.g. photons and Maxwell equations, gravitons. For him, Dirac equation plays a fundamental role and spin may be the key to the internal clock, and the whole of wave mechanics.

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