Analyzing a suggested derivation of the perihelion precession of planets from quantum potential

Mahbubeh Amini[a], Mahdi Atiq[b]

[a,b] Department of Physics, University of Qom, Qom, Iran
[b] Foundations of Physics Group, School of Physics, Institute for Research in Fundamental Sciences (IPM), Tehran 19395-5531, Iran
[a] email: qm.amini@gmail.com
[b]email: mma_atiq@yahoo.com

RÉSUMÉ. Depuis l'application du potentiel quantique à la théorie des quanta de "De Broglie -Bohm ",et en prenant en considération que le potentiel quantique ne diminue pas en éloignant ,certains chercheurs tâchent de trouver les signes de tel propriété à l'échelle macroscopique .Par exemple A.Shojai et F.Shojai dans un article intitulé "De Broglie-Bohm quantum theory and perihelion precession" tâchent d'expliquer la précession du périastre grâce à cette idée .Dans le présent article nous voulons montrer que l'extension du potentiel quantique à l'échelle macroscopique n'est pas aussi simple que ça; car pour réaliser une telle extension il faut considérer la fonction du temps et la densité probable minutieusement .De plus pour la résolution des équations nous nous proposons une méthode précise .Finalement nous démontrons que la démarche de A.Shojai et F.Shojai pour expliquer le mouvement de la précession du périastre en fonction du potentiel quantique n'est pas juste

ABSTRACT. Since the introduction of quantum potential in the de Broglie-Bohm interpretation of the quantum theory and taking into consideration that the quantum potential does not fall with distance, some researchers have tried to find signs of this property at large scales. For example A.Shojai and F.Shojai in an article entitled "De Broglie-Bohm quantum theory and perihelion precession" strive to explain the perihelion precession of planets with this idea. In this article, we show that the extension of quantum potential on the macroscopic scale is not as simple as expressed in that work, and to use quantum potential in the planetary motion, the time dependence of the probability density needs to be considered more carefully. We also consider the method of solving the equations and the assumptions made for the their solution. The conclusion is that A.Shojai and F.Shojai's approach to explain the perihelion precession of planets in terms of quantum potential is not correct.

Keywords: Bohmian Mechanics, Quantum Potential, Hamilton-Jacobi, Gereral Relativity, Perihelion Precession

P.A.C.S.: 03.65.Ge, 04.20.-q

1 introduction

The de Broglie-Bohm interpretation of quantum mechanics is a causal theory for the description of quantum phenomena. In this interpretation, the physical system consists of a wave $\psi(\mathbf{x}, t)(= R \exp iS/\hbar)$ propagating in spacetime (in general, in configuration space) and a particle moving continuously under the guidance of that wave. Although, in this interpretation, the path of the particle is fully determined by the guidance equation $\mathbf{p} = \nabla S$, where $\mathbf{p}$ is the momentum of particle, the non-classical behavior of the quantum particle can be attributed to the quantum potential

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} \tag{1}$$

exerted on the particle by the wave. Putting this potential beside the classical potential $V(x)$,in the equation of motion, provides an explanation for non-classical behaviors of particles in the quantum domain.

Since Q does not necessarily fall with on decrease in the amplitude R and therefore does not necessarily fall with distance, one might suggest that, just like the way it causes quantum phenomena, it can also have some effects on large-scale systems and it can possibly explain those phenomena that are not explainable by Newtonian mechanics.

One of the problems considered in this respect is the anomalous perihelion precession of planets which has been explained accurately by general relativity. As we know, the planets in the solar system move on elliptic orbits around the sun and the orientation of those ellipses are not fixed in the orbital plane. Such a precession of the orbit in the orbital plane has been predicted in the Newtonian mechanics and is caused by the gravitational pull of other planets upon a specific planet. However, for the inner planets, the calculated precession does not match the observation. For example, for Mercury, the observed precession deviates from the predicted value by 43 arc second per century. The theory of general relativity has succeeded in explaining this exact deviation from the Newtonian prediction. Shojai and Shojai [1] have asked, and answered positively, the question of whether this deviation can also be explained by quantum potential. They claim that using the de Broglie-Bohm theory, the radial dependence of the quantum potential is exactly what we need for the perihelion precession of planets. However, it seems that using the quantum potential for that problem according to this way has serious problems and we can not consider it as a way of explaining the precession problem. We first review that suggested approach, and then turn to analyzing and disproving the claim.

2 Shojai-Shojai approach to the preihelion precession problem using quantum potential

In the approach suggested by Shojai and Shojai [1], the aim is to solve the de Broglie-Bohm equations for a central gravitational force with a slight deviation from the Newtonian solution. The classical potential energy for a two-body system is

$$V(x_1, x_2) = -\frac{Gm_1m_2}{|x_1 - x_2|} \equiv -\frac{k}{r}. \tag{2}$$

To solve the problem using the de Broglie-Bohm pilot wave theory, we must solve the Schrodinger equation or equally consider the equation of motion

$$m_i\frac{\mathrm{d}^2r_i}{\mathrm{d}t^2} = -\nabla(V + Q);\ i = 1, 2 \tag{3}$$

alongside the continuity equation

$$\frac{\partial\rho}{\partial t} + \sum_{i=1}^{2}\nabla.(\rho\frac{\mathrm{d}r_i}{\mathrm{d}t}) = 0. \tag{4}$$

Since the potential (2) is a function of the relative distance r, one solution of the Schrodinger equation for ψ can be such that $Q = Q(r)$. However, this is not general and infact ψ and Q can have θ and ϕ dependencies. To solve the problem, we take the plane $\theta = \frac{\pi}{2}$ as the orbital plane so that Q does not depend on θ. Also, it is assumed that the quantum potential has a small dependency on ϕ, i.e., $Q = Q_1(r) + \varepsilon Q_2(r)$. By imposing these conditions and solving the equations of motion, the orbital equation is found as

$$\frac{\mathrm{d}^2 u}{\mathrm{d}\phi^2} + u \simeq -\frac{\mu}{\bar{l}^2}\frac{\mathrm{d}}{\mathrm{d}u}\Big(V(\frac{1}{u}) + Q(\frac{1}{u})\Big) \tag{5}$$

where $u = \frac{1}{r}$, and $\bar{l}$ is the average value of angular momentum. One must note that this equation is approximate since the ϕ dependence of the quantum potential has been assumed small. Then, the continuity equation must be solved. Shojai and Shojai assume that $\frac{\partial \rho}{\partial t} = 0$, i.e., the probability density does not have time dependency. From the continuity equation, the following equation

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(\frac{|\psi|^2}{r}\Big) \simeq= 0 \tag{6}$$

has been derived. Then, it is assumed that $|\psi|$ has a solution of the form

$$|\psi| = r^n e^{-\alpha r}. \tag{7}$$

Such a solution satisfies the boundary conditions. The solution (7) for the equation (6) puts the extremum points of $|\psi|$ approximately at

$$r \simeq \frac{2n-1}{2\alpha}. \tag{8}$$

From (7), the quantum potential takes the form

$$Q(r) = -\frac{\hbar^2}{2\mu}\Big(\frac{n(n-1)}{r^2} - \frac{2n\alpha}{r} + \alpha^2\Big). \tag{9}$$

This potential contains a constant term which is similar to the potential energy of Newtonian gravity and a term proportional to $\frac{1}{r^2}$. By this quantum potential, the orbit equation (5) has a solution

$$u = \frac{1}{r} = \frac{1}{r_0}(1 + e\cos(\omega\phi)), \tag{10}$$

where

$$r_0 = \frac{\bar{l}^2 - \hbar^2 n(n-1)}{k\mu - \hbar^2 \alpha n}, \tag{11}$$

$$\omega = \sqrt{1 - \frac{\hbar^2}{\bar{l}^2 n(n-1)}}, \tag{12}$$

and e is the eccentricity of the orbit. Since e is small, if we set

$$\alpha = \frac{k\mu}{2\bar{l}^2} \frac{2n-1}{1+\hbar^2 \frac{n}{2\bar{l}^2}}, \tag{13}$$

the equations (8) and (11) can be made compatible. The orbit equation (10) is the equation of an ellipse with a precession

$$\delta\phi = \frac{\pi\hbar^2}{\bar{l}^2} n(n-1) \tag{14}$$

per revolution. Since

$$r_0 \simeq r_{classical} \equiv \frac{\bar{l}^2}{k\mu}, \tag{15}$$

we have

$$\alpha \simeq \frac{n}{r_{classical}}. \tag{16}$$

So, the wave function satisfying (7) represents a wave packet localized at the $r_{classical}$. The required value of precession can be fixed by choosing an appropriate n. By this approach, for the precession of Mercury, the appropriate selection is

$$n = 34 \times 10^{68}, \tag{17}$$

or

$$\alpha = 5.9 \times 10^{68} \mathrm{m}^{-1}, \tag{18}$$

which yield the value 41 arc second per century for the precession.

So, according to Shojai-Shojai [1], by choosing a localized wave packet, the quantum potentiall produces a force proportional to $\frac{1}{r^2}$, which causes the precession of the orbit in accordance with observations and general relativity.

3 Analyzing the Shojai-Shojai approach to the problem of perihelion precession

In the previous section, we described briefly the explanation of the perihelion precession of Mercury using de Broglie-Bohm quantum theory and quantum potential according to the reference [1]. It seems that some important points have not been considered in that derivation, and thus, it is necessary to consider the problem more carefully. Due to the problems of that approach, it seems that using the de Broglie-Bohm theory to explain the perihelion precession problem is not justified.

3.1 The continuity equation and the shape of the orbit

In the de Broglie-Bohm quantum theory, the equation governing the evolution of system is the Schrodinger equation or the combination of the equation of motion and the continuity equation, but the latter form was used in the previous section. However, we can ask whether application of the continuity equation, as used in the quantum theory, is correct for macroscopic systems, or whether considering *conservative states* ($\partial\rho/\partial t = 0$) problem is correct for this problem.

About the first question we can say that the continuity equation in the de Broglie-Bohm theory is obtained from the Schrodinger equation and that equation governs the motions of particles at atomic scales, not for large scales. So, it seems that application of the continuity equation for large scales is not a correct assumption.

Now, we consider the section question. The major problem with the Shojai-Shojai approach refers to their assumption of a time-independent ρ. (This assumption is clear from the equation (7).) In reference [2], it has been proved that the assumption of time-independent ρ in the presence of quantum potential and continuity equation leads definitely to exact circular orbits, and consequently, there is no room for an elliptic orbit with precession. Since the classical potential used in [2] is in the form $-\frac{k}{r}$, the discussion in reference [2] applies here as well. So, we reiterate briefly the main points of that reference.

In the previous section, conservative states, i.e.

$$\frac{\partial\rho}{\partial t} = 0, \tag{19}$$

was considered and therefore the continuity equation was reduced to

$$\nabla.\left(\rho\frac{\nabla S}{m}\right) = 0, \tag{20}$$

where $p = \nabla S$ is the momentum and S is the phase of the wavefunction ψ.

Now, we consider the general case $R^2 = |\psi|^2 = \rho(r,\theta,\phi)$. Due to the spherical symmetry of the potential $V(r)$, we can decompose R in the form of

$$R(r,\theta,\phi) = R_r(r)R_\theta(\theta)R_\phi(\phi). \tag{21}$$

Then the quantum potential takes the form

$$Q(r,\theta,\phi) = Q_r(r) + \frac{Q_\theta(\theta)}{r^2} + \frac{Q_\phi(\phi)}{r^2\sin^2\theta}, \tag{22}$$

where

$$Q_r(r) = -\frac{\hbar^2}{2m}\frac{1}{R_r}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R_r}{\mathrm{d}r}\right) \tag{23}$$

$$Q_\theta(\theta) = -\frac{\hbar^2}{2m}\frac{1}{R_\theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}R_\theta}{\mathrm{d}\theta}\right) \tag{24}$$

$$Q_\phi(\phi) = -\frac{\hbar^2}{2m}\frac{1}{R_\phi}\frac{\mathrm{d}^2}{\mathrm{d}\phi^2}R_\phi \tag{25}$$

From the classical Hamilton-Jacobi theory we know that whenever the potential takes the form

$$V(r,\theta,\phi) = V_r(r) + \frac{V_\theta(\theta)}{r^2} + \frac{V_\phi(\phi)}{r^2\sin^2\theta}, \tag{26}$$

the principal function S decomposes as

$$S(r,\theta,\phi,t) = W_r(r) + W_\theta(\theta) + W_\phi(\phi) - Et. \tag{27}$$

Our effective potential $V + Q$ here has the exact form of (26). So, (27) applies to this problem as well. Then, the continuity equation becomes

$$0=\frac{r^2}{R^2}\nabla.\Big(R^2\nabla S\Big)=f_r+f_\theta+\frac{f_\phi}{\sin^2\theta}, \tag{28}$$

where

$$f_r=\frac{1}{R_r^2}\frac{\mathrm{d}}{\mathrm{d}r}\Big(r^2R_r^2\frac{\mathrm{d}W_r}{\mathrm{d}r}\Big) \tag{29}$$

$$f_\theta=\frac{1}{R_\theta^2}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\Big(\sin\theta R_\theta^2\frac{\mathrm{d}W_\theta}{\mathrm{d}\theta}\Big) \tag{30}$$

$$f_\phi=\frac{1}{R_\phi^2}\frac{\mathrm{d}}{\mathrm{d}\phi}\Big(R_\phi^2\frac{\mathrm{d}W_\phi}{\mathrm{d}\phi}\Big). \tag{31}$$

The equation (28) shows that for some constants c_θ and c_ϕ we have

$$f_\phi(\phi)=c_\phi, \tag{32}$$

$$f_\theta(\theta)+\frac{c_\phi}{\sin^2\theta}=c_\theta, \tag{33}$$

and

$$f_r(r)=-c_\theta. \tag{34}$$

Now if the wavefunction is such that $\dot{r}$ is identically zero, according to (29) and (34), f_r vanishes and we have $c_\theta=0$. But if the orbit of the particle is such that $\dot{r}$ is not always zero, then in a bound system, there are necessarily at least one $r_{\min}$ and one $r_{\max}$ in the orbit of particle. Then, if we integrate (34) from one $r_{\min}$ to its subsequent $r_{\max}$, we obtain

$$\Big(r^2R_r^2\frac{\mathrm{d}W_r}{\mathrm{d}r}\Big)_{r_{\max}}-\Big(r^2R_r^2\frac{\mathrm{d}W_r}{\mathrm{d}r}\Big)_{r_{\min}}=-c_\theta\int_{r_{\min}}^{r_{\max}}R_r^2\mathrm{d}r. \tag{35}$$

However, of r_{min} and r_{max}, we have $p_r=\frac{\mathrm{d}W_r}{\mathrm{d}r}=0$, and so, the left hand side of the equality vanishes. On the other hand the integral in the right hand side is non-negative, and since $\dot{r}$ is not identically zero, we have $r_{\max}\neq r_{\min}$. Therefore, the equality holds only when $c_\theta=0$. So, c_θ is zero anyway.

Now, from (34), we obtain

$$r^2 R_r^2 \frac{\mathrm{d}W_r}{\mathrm{d}r} = r^2 R_r^2 m\dot{r} = \lambda_r, \tag{36}$$

where, λ_r is a constant. If this constant is nonzero, we have for the full path of particle: $\dot{r} \neq 0$. This means that there are no turning points on the path of the particle. The particle moves continuously toward the center or away from it. Obviously this can not be a stable bound state. So, for a bound system, we must have $\lambda_r = 0$, and consequently we have identically $\dot{r} = 0$.

Therefore, as we observe, the continuity equation for conservative states of the central potential leads to $\dot{r} = 0$. This means that the orbit of particle can not be elliptic at all. But, we can proceed further.

We apply $c_\theta = 0$ and consider the θ coordinate. Similar to the above argument, if $\dot{\theta}$ is identically zero, we conclude from (33) that $c_\phi = 0$. However, if $\dot{\theta}$ is not identically zero, then there are at least one $\theta_{\min}$ and one $\theta_{\max}$ on the trajectory of particle. By applying $c_\theta = 0$ to the equation (33) and integrating from one $\theta_{\min}$ to the subsequent $\theta_{\max}$ we obtain

$$\begin{aligned}\left(\sin\theta R_\theta^2 \frac{\mathrm{d}W_\theta}{\mathrm{d}\theta}\right)_{\theta_{\max}} &- \left(\sin\theta R_\theta^2 \frac{\mathrm{d}W_\theta}{\mathrm{d}\theta}\right)_{\theta_{\min}} \\ &= -c_\phi \int_{\theta_{\min}}^{\theta_{\min}} \frac{R_\theta^2}{\sin\theta}\mathrm{d}\theta.\end{aligned} \tag{37}$$

By an argument similar to the one made for the vanishing of c_θ, we also conclude that $c_\phi = 0$. This means that we have from (33) that

$$\sin\theta R_\theta^2 \frac{\mathrm{d}W_\theta}{\mathrm{d}\theta} = \sin\theta R_\theta^2 m r^2 \dot{\theta} = \lambda_\theta, \tag{38}$$

where, λ_θ is a constant. Now, if λ_θ is nonzero, $\dot{\theta}$ will never be zero. Then, θ will always be increasing or decreasing, meaning that somewhere on the trajectory it must exceed the boundaries 0 and π, which is impossible, according to the definition of that coordinate. So, the continuation of the motion requires that we have somewhere $\dot{\theta} = 0$, which contradicts our assumption. Therefore, we must have $\lambda_\theta = 0$, and because $\sin\theta$ or R_θ are not identically zero, we must have $\dot{\theta} = 0$.

Now that both $\dot{r}$ and $\dot{\theta}$ are zero, the orbit of particle in the conservative state ($\partial\rho/\partial t = 0$) of the central potential is strictly a circle.

Even if we take the *ad hoc* assumption that R does not have θ and ϕ dependencies, as in the Shojai-Shojai approach, which means that we set $R_\theta = R_\phi = 1$, the vanishing of c_θ and f_r occurs once again and again the orbit will not be elliptic.

So, the careful analysis shows that the application of conservative states ($\partial\rho/\partial t = 0$) in this problem, which implies elliptic orbits for planets, is unjustified and that those states do not allow ellipses at all, let alone advancing ones.

3.2 Solution of the continuity equation for the conservative states

Although we stated in the previous section that "conservative states" can not provide the correct answer for the determination of the orbit for the central potential at large-scales, but in this section, just for the sake of argument, we assume that such a state is valid and we show that even by this assumption, the continuity equation does not take the form

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{|\psi|^2}{r}\right) \simeq 0 \tag{39}$$

used in the Shojai-Shojai approach.

Since we have assumed $\partial\rho/\partial t = 0$, we obtain from the continuity equation that

$$\nabla.\left(\rho\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}\right) = 0, \tag{40}$$

where $\mathrm{d}\vec{r} = \nabla S/m$. Then, assuming that $\theta = \pi/2$, we have

$$\begin{gathered}\nabla.\Big(\,|\psi|^2\,\nabla W\Big) = \frac{1}{r^2}\Big[\\ \frac{\mathrm{d}}{\mathrm{d}r}\Big(\,|\psi|^2\,\frac{\mathrm{d}W_r}{\mathrm{d}r}r^2\Big) + \frac{\mathrm{d}}{\mathrm{d}\phi}\Big(\,|\psi|^2\,\frac{\mathrm{d}W_\phi}{\mathrm{d}\phi}\Big)\Big] = 0,\end{gathered} \tag{41}$$

or

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}\,|\psi|^2 + \frac{2}{r}\,|\psi|^2\right)\frac{\mathrm{d}W_r}{\mathrm{d}r} + \left(\frac{\mathrm{d}^2W_r}{\mathrm{d}r^2} + \frac{1}{r^2}\,\frac{\mathrm{d}^2W_\phi}{\mathrm{d}\phi^2}\right)|\psi|^2 = 0. \tag{42}$$

However, equation (39) yields

$$\frac{\mathrm{d}}{\mathrm{d}r}\,|\psi|^2 - |\psi|^2\,\frac{1}{r} \simeq 0. \tag{43}$$

Comparing the last two equations, we do not observe any similarity (even approximately) between the them.

3.3 Objection to the selected probability density

Another objection to the Shojai-Shojai approach refers to the *ad hoc* selection of $|\psi|$ in the form of

$$|\psi| = r^n e^{-\alpha r}. \tag{44}$$

Logically, to determine $|\psi|$ one must solve Hamilton-Jacobi and continuity equations. If we accept the assumption of this approach, i.e. $\frac{\partial}{\partial r}(\frac{|\psi|^2}{r}) \simeq 0$, then this equation must yield solution for $|\psi|$, but in this approach, to determine $|\psi|$ this equation is not solved but a solution is assumed, a solution that can yield the additional potential required for the description of the complete planetary motion. In fact, we can say that the assumed $|\psi|$ is an *ad hoc* solution that can add a potential proportional to $1/r^2$ to the central potential.

On the other hand, no clear reason for the selection of the value 34×10^{68} for the free parameter n has been provided, except that it is required for yielding the assumed value of the Mercury's perihelion precession.

Considering these basic objections and *ad hoc* assumptions, this approach can not be considered as a suitable approach for the description of the perihelion advances of planets' orbits.

4 Conclusion

According to the discussions of the previous sections, we can say that application of the quantum potential for the description of the perihelion advances of planets according to the reference [1] is not an appropriate approach. In fact, to describe the problem using the de Broglie-Bohm theory, not only we must consider the radial dependence of the quantum potential but before that the time dependence of the probability density is the most basic assumption that must be considered. Even if the quantum potential, because of not falling with distance, can provide a description for the large-scale phenomena that are not explainable by Newtonian mechanics, we need to analyze the conditions of the considered system more appropriately and avoid *ad hoc* assumptions. Therefore, by using these authors' approach we can not claim that there is a connection between quantum potential and classically-unexplained large-scale phenomena. Although, we can not deny the existence of a connection between gravity and quantum mechanics, but we can not establish such a connection by using such approaches.

References

[1] A. Shojai, F. Shojai, "De Broglie-Bohm Quantum Theory and Perihelion Precession", Annales. Fond. Broglie, **25** (2000), 293.

[2] M. Atiq, M. Karamian, M. Golshani, "A Quasi-Newtonian Approach to Bohmian Mechanics II: Inherent Quantization", *Annales. Fond. Broglie*, **34**, 2 (2009), 165.

[3] H. Goldstein, C. Poole, J. Safko, *Classical Mechanics* (Addison Wesley, 2000).

(Manuscrit reçu le 24 avril 2015)