Superconductivity with Weber's Electrodynamics: the London Moment and the Meissner Effect

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ABSTRACT. Weber's electrodynamics is utilized to model superconductivity. We show that it successfully reproduces well known superconducting effects like the London moment, the Meissner effect and the London penetration depth. The calculations presented here have the advantage to include the mass of the free electrons where they belong, namely, in Newton's second law of motion.

Key Words: Superconductivity, Weber's electrodynamics, the London moment, the Meissner effect.

PACS: 41.20.-q (applied classical electromagnetism), 74.20.-z (theories and models of superconducting state).

1 Introduction

In this work we utilize Weber's electrodynamics to model superconductivity. As there is no resistance in superconductors, free electrons will be accelerated under the action of electromagnetic forces. After the external force is removed, any acquired velocity will continue indefinitely with its constant magnitude.

We first consider the London moment, namely, the magnetic moment acquired by a rotating superconductor. When we rotate a superconducting body relative to an inertial frame of reference with an angular velocity $\vec{\Omega}$, a magnetic field $\vec{B} = 2m\vec{\Omega}/e$ is developed throughout its interior, where $m = 9.1 \times 10^{-31}\ kg > 0$ and $e = 1.6 \times 10^{-19}\ C > 0$ represent the magnitude of the electron's mass and charge, respectively.

We then consider the Meissner effect [1]. Experiments show that the net magnetic field inside superconductors in the presence of an external applied magnetic field decreases exponentially from the surface inwards with a characteristic length called London penetration depth, λ_L. A surface current is induced in the superconductor material creating an induced magnetic field inside it which completely opposes the external applied magnetic field. This phenomenon is called the Meissner effect.

It is shown here how to deduce these two effects, the London moment and the Meissner effect, from Weber's force coupled with Newton's second law of motion. The usual theoretical treatment of these subjects is based on the works of Becker, London and co-authors [2, 3, 4, 5]. In these earlier works the mass of the free electrons was introduced *ad hoc* in purely electrodynamic equations describing the magnetic field of the superconducting material. In the model presented in this work, on the other hand, the mass of the test particle appears where it really belongs, namely, in Newton's second law of motion. This is the main advantage of the present calculations.

The existence or not of an electric field penetration depth in superconductors will not be considered in the present work [6, 7]. It should be emphasized that the London equations and the Meissner effect can also be derived in purely classical terms based on the principle of minimum action [8, 9].

2 Weber's Force

Consider an inertial frame of reference S with origin O and a point particle 1 electrified with charge q_1. Let $\vec{r}_1 = x_1\hat{x} + y_1\hat{y} + z_1\hat{z}$ be its position vector relative to the origin O of S, while $\vec{r}_2 = x_2\hat{x} + y_2\hat{y} + z_2\hat{z}$ is the position vector of another point particle 2 electrified with charge q_2. The velocities and accelerations of these charges in S are given by, respectively: $\vec{v}_1 = d\vec{r}_1/dt$, $\vec{v}_2 = d\vec{r}_2/dt$, $\vec{a}_1 = d\vec{v}_1/dt = d^2\vec{r}_1/dt^2$ and $\vec{a}_2 = d\vec{v}_2/dt = d^2\vec{r}_2/dt^2$.

The position vector pointing from q_2 to q_1 will be defined by $\vec{r}_{12} \equiv \vec{r}_1 - \vec{r}_2 \equiv \vec{r}$. We also define in this reference frame the relative vector velocity $\vec{v}_{12}$ and the relative vector acceleration $\vec{a}_{12}$ by the following expressions: $\vec{v}_{12} \equiv \vec{v}_1 - \vec{v}_2$ and $\vec{a}_{12} \equiv \vec{a}_1 - \vec{a}_2$. The charges are separated by a distance $r_{12} \equiv r \equiv \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$. The unit vector $\hat{r}$ pointing from 2 to 1 can be written as $\hat{r}_{12} \equiv \hat{r} \equiv (\vec{r}_1 - \vec{r}_2)/r$.

In the International System of Units and in vector notation Weber's force $\vec{F}_{21}$ exerted by particle 2 on particle 1 is given by [10, 11, 12, 13, 14, 15]:

$$\vec{F}_{21} = -\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}}{r^2}\left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\ddot{r}}{c^2}\right)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}}{r^2}\left[1 + \frac{1}{c^2}\left(\vec{v}_{12}\cdot\vec{v}_{12} - \frac{3}{2}\left(\hat{r}\cdot\vec{v}_{12}\right)^2 + \vec{r}_{12}\cdot\vec{a}_{12}\right)\right] . \qquad (1)$$

Here $\vec{F}_{12}$ is the force exerted by q_1 on q_2, $\dot{r} \equiv dr/dt$ is the relative *radial* velocity between them, while $\ddot{r} \equiv d\dot{r}/dt = d^2r/dt^2$ is the relative *radial* acceleration between the charges. In vector notation these magnitudes can be written as: $\dot{r} = \hat{r}\cdot\vec{v}_{12}$ and $\ddot{r} = \left[\vec{v}_{12}\cdot\vec{v}_{12} - (\hat{r}\cdot\vec{v}_{12})^2 + \vec{r}_{12}\cdot\vec{a}_{12}\right]/r$. The constant $c \equiv 1/\sqrt{\mu_o\varepsilon_o} = 2.998\times 10^8\ m/s$ is the ratio of electromagnetic and electrostatic units of charge. Its experimental value was first determined by W. Weber and R. Kohlrausch. Its value is the same as light velocity in vacuum.

In this work we will be dealing with neutral materials, so that the electrostatic or coulombian component of equation (1), $q_1q_2\hat{r}/(4\pi\varepsilon_o r^2)$, will not need to be considered in the calculations. For the London moment, the superconductor material is rotated mechanically relative to the laboratory with velocities of the order of meters per second. The conduction electrons will acquire velocities of the same order of magnitude, so that $v_1 \ll c$ and $v_2 \ll c$, where $v_1 \equiv |\vec{v}_1|$ and $v_2 \equiv |\vec{v}_2|$. Therefore, the velocity components of Weber's force (1) will be very small compared to light velocity and will be neglected in the following calculations. The only remaining term of Weber's force which will need to be considered here is the last component depending on the accelerations $\vec{a}_1$ and $\vec{a}_2$, namely:

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}}{r^2}\frac{\vec{r}_{12}\cdot\vec{a}_{12}}{c^2} = \frac{\mu_o q_1 q_2}{4\pi}\frac{\hat{r}}{r}\left(\hat{r}_{12}\cdot\vec{a}_{12}\right) . \qquad (2)$$

3 The London Moment

The London moment was predicted by Becker, London and others considering two cases: (I) a superconducting body set into rotation relative to an inertial frame of reference, and (II) a rotating normal metal cooled into the superconducting state while rotating [2], [5, pp. 78-83] and [16]. In this work we consider case (I) with two geometries, namely, a rotating cylindrical shell and a rotating spherical shell.

3.1 Rotating Cylindrical Shell

We first consider a superconducting cylindrical shell of radius R_2 and infinite length with its axis along the z axis. We suppose it is composed of a single monoatomic layer of superconducting material. This assumption is utilized to illustrate clearly what are the source charges producing the effect, the net forces they exert on the test charges and the resulting motion of these free electrons. Arguments have already been presented to show that if the London penetration depth (usually tens of nanometers) is much larger than the thickness of the shell (monoatomic in this first model, that is, of the order of one ångström), then there will be no London moment [17]. In Subsections 4.2, 5.1 and 5.2 we will expand the monoatomic layer model to a model with many layers and to a more realistic model of a continuous superconducting body with a finite thickness.

The cylindrical shell is assumed to be electrically neutral, being composed of a positive lattice with surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$ and a set of free electrons with surface charge density $\sigma_{2-} = -\sigma_2$. The positive lattice will be identified with the macroscopic superconducting sample, so that when we say that the superconductor is rotating with an angular velocity $\vec{\Omega}_2$, the lattice is assumed to rotate with the same velocity, namely, $\vec{\Omega}_{2+} \equiv \vec{\Omega}_2$. We assume that the positive cylindrical shell and the conduction electrons are initially at rest relative to the inertial frame of reference S. In the time interval from $t = 0$ to the final value $t = t_f$ the superconducting material is rotated mechanically around the z axis with a variable and given angular velocity $\vec{\Omega}_{2+}(t) = \Omega_{2+}(t)\hat{z}$, until it reaches a final and constant angular velocity $\vec{\Omega}_{2+f} = \Omega_{2+f}\hat{z}$. Our goal is to calculate for $0 < t < t_f$ Weber's force exerted by this positive cylindrical shell acting on a test free electron located at a distance ρ from the axis of the shell, figure 1. We then apply Newton's second law of motion in order to deduce the corresponding motion induced in the conduction electron. That is, we assume that when we rotate the superconducting material, only part of the body (lattice plus bound electrons) does indeed rotate with the material. The free electrons, on the other hand, don't rotate together with the lattice because they don't feel any friction. In our model the free electrons will be dragged behind the positive lattice due to the action of an induced Weber-force exerted by the lattice.

Consider an element of source charge dq_2 of the cylindrical shell having a surface charge density σ_2 and area da_2. In cylindrical coor-

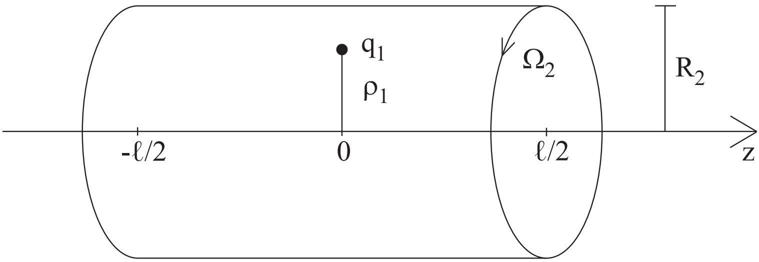

Figure 1: Particle with charge q_1 at a distance ρ_1 from the axis of a cylindrical shell of length ℓ and radius R_2 rotating relative to the inertial frame of reference S with an angular velocity $\Omega_2(t)$ around the z axis.

dinates $da_2 = R_2 d\varphi_2 dz_2$, where φ is the azimuthal angle. Therefore, $dq_2 = \sigma_2 da_2 = \sigma_2 R_2 d\varphi_2 dz_2$. When the cylindrical shell is rotating with angular velocity $\Omega_2(t)$ around the z axis, the position vector, velocity and acceleration of this element of charge are given by, respectively: $\vec{r}_2 = R_2\hat{\rho}_2 + z_2\hat{z}$, $\vec{v}_2 = R_2\Omega_2\hat{\varphi}_2$ and $\vec{a}_2 = -R_2\Omega_2^2\hat{\rho}_2 + R_2\dot{\Omega}_2\hat{\varphi}_2$, where $\hat{\rho}_2$, $\hat{\varphi}_2$ and $\hat{z}$ are the unit vectors of cylindrical coordinates at the location of dq_2, while $\dot{\Omega} \equiv d\Omega/dt$. The test charge q_1 will be a conduction electron belonging to this cylindrical shell. It may also have centripetal and tangential components of its acceleration. We assume that contact forces maintain the conduction electrons at a constant distance $\rho_1 = R_2$ from the axis of the cylinder, so that $\dot{\rho}_1 = 0$ and $\ddot{\rho}_1 = 0$. We consider the test charge located at $z_1 = 0$. We assume that it will move along the tangential direction φ with an angular velocity $\omega_1(t)$. Its position vector, velocity and acceleration are then given by, respectively: $\vec{r}_1 = \rho_1\hat{\rho}_1$, $\vec{v}_1 = \rho_1\omega_1\hat{\varphi}_1$ and $\vec{a}_1 = -\rho_1\omega_1^2\hat{\rho}_1 + \rho_1\dot{\omega}_1\hat{\varphi}_1$, where $\dot{\omega}_1 \equiv d\omega_1/dt$. Equation (2) yields the force exerted by the source charge dq_2 acting on the conduction electron q_1.

Integrating equation (2) over the surface of the cylindrical shell yields the following net force acting on the test electron along the tangential or azimuthal direction $\hat{\varphi}_1$:

$$\vec{F} = \int_{z_2=-\infty}^{\infty}\int_{\varphi_2=0}^{2\pi} \frac{\mu_o q_1 dq_2}{4\pi}\frac{\hat{r}}{r}\left(\hat{r}_{12}\cdot\vec{a}_{12}\right)$$

$$= \begin{cases} -\mu_o q_1 \sigma_2 R_2 (\dot{\Omega}_2 - \dot{\omega}_1) \rho_1 \hat{\varphi}_1 / 2 \ , & \text{if } \rho_1 < R_2 \ , \\ -\mu_o q_1 \sigma_2 R_2 (\dot{\Omega}_2 - \dot{\omega}_1) R_2 \hat{\varphi}_1 / 2 \ , & \text{if } \rho_1 = R_2 \ , \\ -\mu_o q_1 \sigma_2 R_2 (\dot{\Omega}_2 - \dot{\omega}_1) R_2^2 \hat{\varphi}_1 / (2\rho_1) \ , & \text{if } \rho_1 > R_2 \ . \end{cases} \quad (3)$$

The calculation with $\rho_1 = R_2$ and $\dot{\Omega}_2 = 0$ had already been presented before [18].

We now apply Newton's second law of motion to a conduction electron located at $\rho_1 = R_2$ and moving only along the azimuthal direction, namely:

$$\vec{F} = m_1 \vec{a}_1 = m R_2 \dot{\omega}_1 \hat{\varphi}_1 \ . \quad (4)$$

Here $\vec{F}$ represents the total force acting on the test electron with inertial mass $m_1 \equiv m = 9.1 \times 10^{-31}\ kg$. We are considering only the azimuthal component of this force along the φ direction, as contact forces prevent the electron from moving along the radial ρ direction.

There are two sets of charges exerting forces on any specific conduction electron, namely, (a) the positive lattice rotating around the z axis with angular velocity $\Omega_{2+}(t)$; and (b) all the other conduction electrons rotating around the z axis with angular velocity $\omega_{1-}(t)$. These forces exerted by the positive and negative charges of the cylindrical shell and acting on any specific test electron with charge $q_1 = -e < 0$ will be represented by $\vec{F}_{2+,-e}$ and $\vec{F}_{2-,-e}$, respectively. As all conduction electrons rotate together around the z axis, we have $\dot{r} = 0$ and $\ddot{r} = 0$ for any pair of electrons. Therefore, there will be no net component along the azimuthal direction acting on any specific conduction electron due to all the other conduction electrons, so that $\vec{F}_{2-,-e} = \vec{0}$. The net force $\vec{F}$ acting on any conduction electron will be then due only to the rotating positive lattice of the shell, that is, $\vec{F} = \vec{F}_{2+,-e} + \vec{F}_{2-,-e} = \vec{F}_{2+,-e}$.

We now combine Newton's second law of motion, that is, equation (4), with equation (3) for $\rho_1 = R_2$. Considering the force of the positive lattice acting on a conduction electron we obtain the following equation of motion:

$$-\frac{\mu_o q_1 \sigma_2 R_2 (\dot{\Omega}_{2+} - \dot{\omega}_{1-}) R_2}{2} \hat{\varphi}_1 = m R_2 \dot{\omega}_{1-} \hat{\varphi}_1 \ , \quad (5)$$

or

$$\dot{\omega}_{1-} = \frac{|m_{Wc}|}{m + |m_{Wc}|}\dot{\Omega}_{2+} \ , \tag{6}$$

where $q_1 \equiv -e < 0$. Here $e = 1.6\times10^{-19}\ C > 0$ represents the magnitude of the charge of the electron, while $|m_{Wc}| \equiv \mu_o e \sigma_2 R_2/2 > 0$ represents the magnitude of the so-called weberian electromagnetic mass for this cylindrical geometry [18, 19].

Equation (6) shows that $\dot{\omega}_{1-}$ is proportional to $\dot{\Omega}_{2+}$ at any time t during the time interval $0 < t < t_f$. Therefore, $\omega_{1-}(t)$ will also be proportional to $\Omega_{2+}(t)$, no matter how Ω_{2+} changes with time. Assuming that the shell and the test electrons begin at rest, $\Omega_{2+}(0) = \omega_{1-}(0) = 0$, then the final value of ω_{1-}, represented by $\omega_{1-}(t_f) \equiv \omega_{1-f}$, will be proportional to the final angular velocity of the positive lattice represented by $\Omega_{2+}(t_f) \equiv \Omega_{2+f}$, that is:

$$\omega_{1-f} = \frac{|m_{Wc}|}{m + |m_{Wc}|}\Omega_{2+f} \ . \tag{7}$$

This equation is valid no matter how fast the superconducting shell reaches its final angular velocity. It can reach Ω_{2+f} in 1 second or in 1 hour. It will also be valid if the angular velocity of the shell grows linearly, sinusoidally or with any other function of time. That is, the final value of the angular velocity of the conduction electrons will be independent of the history of how the rotating shell reached its final value. The final state, namely, the final angular velocity of the conduction electrons ω_{1-f}, will not depend on the history to get there. In conclusion, ω_{1-f} will not depend on how $\Omega_{2+}(t)$ changed with time. It will only depend on the final value Ω_{2+f} acquired by the superconductor material.

In any macroscopic material we have $|m_{Wc}| \gg m$. For instance, in the case of a niobium cylindrical shell of radius $R_2 = 0.1\ m$ and surface charge density $\sigma_2 \approx 2\ C/m^2$, we have $|m_{Wc}| \approx 2 \times 10^{-25}\ kg$, which is some five orders of magnitude larger than the inertial mass of a free electron. With this condition equation (7) can be approximated to

$$\omega_{1-f} \approx \left(1 - \frac{m}{|m_{Wc}|}\right)\Omega_{2+f} \ , \tag{8}$$

where $m/|m_{Wc}| \ll 1$.

Therefore, according to Weber's electrodynamics, when we rotate a superconducting shell clockwise, the positive charges of the lattice will exert a clockwise force on the conduction electrons making them move in the same direction as the lattice. However, the inertial mass of the electrons will make them lag slightly behind the positive lattice, figure 2 (a).

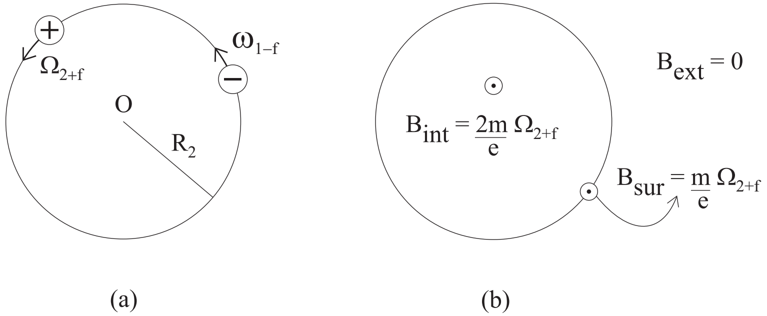

Figure 2: (a) When the positive lattice of the superconducting cylindrical shell is rotated around the z axis with an angular velocity Ω_{2+f}, it causes the set of conduction electrons to rotate in the same sense with a slightly smaller angular velocity $\omega_{1-f} = (1 - m/|m_{Wc}|)\Omega_{2+f}$. (b) Magnetic field $\vec{B}$ inside, at the surface and outside a rotating superconducting cylindrical shell.

As is well known, the magnetic field $\vec{B}(\rho)$ at a distance ρ from the axis of a cylindrical shell of radius R and surface charge density σ rotating uniformly around the z axis with an angular velocity $\Omega\hat{z}$ is given by:

$$\left.\begin{array}{l} \vec{B}(\rho < R) \equiv \vec{B}_{int} = \mu_o R\sigma\Omega\hat{z} \ , \\ \vec{B}(\rho = R) \equiv \vec{B}_{sur} = \mu_o R\sigma\Omega\hat{z}/2 \ , \\ \vec{B}(\rho > R) \equiv \vec{B}_{ext} = \vec{0} \ . \end{array}\right\} \quad (9)$$

Here $\vec{B}_{int}$ is the internal magnetic field at $\rho < R$, $\vec{B}_{sur}$ is the magnetic field at the surface $\rho = R$, while $\vec{B}_{ext}$ is the external magnetic field at $\rho > R$.

We can now calculate the magnetic field of the rotating superconducting cylindrical shell of radius R_2 composed of a positive lattice with surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$ rotating around the z axis with an angular velocity Ω_{2+f}, combined with a set of free electrons having a negative surface charge density $\sigma_{2-} = -\sigma_2$ and rotating around the z

axis with an angular velocity $\omega_{1-f} = (1 - m/|m_{Wc}|)\Omega_{2+f}$. For $\rho < R_2$ equation (9) yields:

$$\vec{B}(\rho < R_2) \equiv \vec{B}_{int} = \mu_o R_2 \sigma_2 \Omega_{2+f} \hat{z} - \mu_o R_2 \sigma_2 \omega_{1-f} \hat{z}$$

$$= \mu_o R_2 \sigma_2 \Omega_{2+f} \hat{z} - \mu_o R_2 \sigma_2 \left(1 - \frac{m}{|m_{Wc}|}\right) \Omega_{2+f} \hat{z}$$

$$= \frac{\mu_o R_2 \sigma_2 m}{|m_{Wc}|} \Omega_{2+f} \hat{z} = \frac{2m}{e} \Omega_{2+f} \hat{z} \ . \tag{10}$$

Similar calculations yield the magnetic field at $\rho = R_2$ as given by $\vec{B}(\rho = R_2) \equiv \vec{B}_{sur} = m\Omega_{2+f}\hat{z}/e$, while the external magnetic field goes to zero for $\rho > R_2$, that is, $\vec{B}(\rho > R_2) \equiv \vec{B}_{ext} = \vec{0}$, figure 2 (b). It is normally assumed that the magnetic field throughout a superconductor has the value $\vec{B} = 2m\vec{\Omega}/e$. Our new result indicates that in the cylindrical shell model with a single monolayer this value of the magnetic field will be valid only inside the superconductor, being zero outside it and having half of this value at its surface. In the next model of a superconducting spherical shell we will show that the radial component of the magnetic field is continuous at the surface of the material, while only the component parallel to the surface will be discontinuous.

Equation (10) is exactly the London moment, which has been experimentally verified many times [20]. It indicates that the magnetic field points in the same sense as the angular rotation of the superconducting material. That is, the produced magnetic field is parallel to the angular velocity of the positive lattice.

3.2 Rotating Spherical Shell

We now consider the same problem in the configuration of a superconducting spherical shell of radius R_2 centered on the origin of the coordinate system with the shell initially at rest in the inertial frame of reference S. The positive lattice of this shell has a surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$, while the negative conduction electrons have an equal and opposite surface charge density, namely, $\sigma_{2-} = -\sigma_2$. The conduction electrons at this shell are also considered initially at rest. We then rotate the positive lattice in the time interval $0 < t < t_f$ around the z axis until it reaches the final and constant angular velocity $\vec{\Omega}_{2+f} = \Omega_{2+f}\hat{z}$. The test particle with charge q_1 will be located at

$\vec{r}_1 = r_1\hat{r}_1 = r_1 (\sin\theta_1 \cos\varphi_1\hat{x} + \sin\theta_1 \sin\varphi_1\hat{y} + \cos\theta_1\hat{z})$. Here we are utilizing spherical coordinates $(r,\ \theta,\ \varphi)$ with the angular velocity $\dot{\varphi} \equiv \omega$, figure 3.

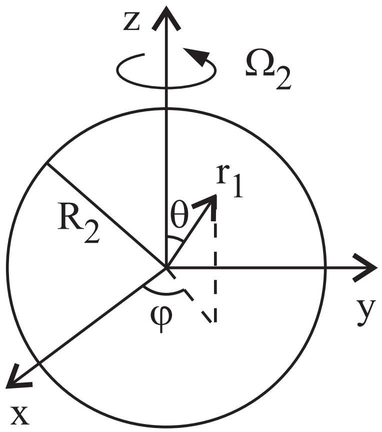

Figure 3: Particle with charge q_1 at the position vector $\vec{r}_1$ relative to the origin of a spherical shell of radius R_2 which rotates around the z axis with an angular velocity Ω_2.

We assume that contact forces will keep the mobile conduction electron at a constant distance $r_1 = R_2$ from the center of the shell, so that $\dot{r}_1 = 0$ and $\ddot{r}_1 = 0$. When we rotate the shell relative to the inertial frame of reference S, there will arise centrifugal forces pointing away from the z axis of rotation, analogous to the forces acting on the Earth while it is spinning daily around its axis relative to the frame of fixed stars and causing its flattening at the poles. This effect will cause a redistribution of charges of the spherical shell around the polar θ direction, with free electrons accumulating around the equator. This equatorial accumulation of charges will stop when the coulombian forces generated by this excess of negative charges at the equator balance the centrifugal force generated by the rotation of the shell. In this stable configuration there will be no motion of the conduction electrons along the polar θ direction, so that $\dot{\theta}_1 = 0$ and $\ddot{\theta}_1 = 0$. It can be shown that this redistribution of charges is negligible due to the large value of the electrostatic force when compared with the centrifugal force. Therefore we will not consider this effect in this work, assuming that the surface density of free electrons in the stable configuration will be equal and opposite the surface charge density of the positive lattice. That is, we assume that the surface charge density of the conduction electrons will be given by $-\sigma_2$, no matter the value of the polar angle θ. We will concentrate our analysis in the motion of the test electrons along the azimuthal φ direction.

The velocity and acceleration of the test particle relative to the inertial frame of reference S are then given by, respectively, $\vec{v}_1 = r_1\omega_1 \sin\theta_1 \hat{\varphi}_1$ and $\vec{a}_1 = -r_1\omega_1^2 \sin^2\theta_1 \hat{r}_1 - r_1\omega_1^2 \sin\theta_1 \cos\theta_1 \hat{\theta}_1 + r_1\dot{\omega}_1 \sin\theta_1 \hat{\varphi}_1$. An element of source charge dq_2 of the spherical shell with surface charge density σ_2 and area da_2 is given by $dq_2 = \sigma_2 da_2 = \sigma_2 R_2^2 \sin\theta_2 d\theta_2 d\varphi_2$. Its position vector, velocity and acceleration are given by, respectively:

$$\vec{r}_2 = R_2\hat{r}_2 = R_2\left(\sin\theta_2\cos\varphi_2\hat{x} + \sin\theta_2\sin\varphi_2\hat{y} + \cos\theta_2\hat{z}\right) , \qquad (11)$$

$$\vec{v}_2 = R_2\Omega_2\sin\theta_2\hat{\varphi}_2 , \qquad (12)$$

and

$$\vec{a}_2 = -R_2\Omega_2^2\sin^2\theta_2\hat{r}_2 - R_2\Omega_2^2\sin\theta_2\cos\theta_2\hat{\theta}_2 + R_2\dot{\Omega}_2\sin\theta_2\hat{\varphi}_2 . \qquad (13)$$

We integrate equation (2) over the surface of the spherical shell in order to obtain the force exerted by the shell on a test particle with charge q_1. This integration yields [21] [15, Appendix B]:

$$\vec{F}(r_1 \leq R_2) = \int_{\theta_2=0}^{\pi}\int_{\varphi_2=0}^{2\pi} \frac{\mu_o q_1 dq_2}{4\pi}\frac{\hat{r}}{r}\left(\hat{r}_{12}\cdot\vec{a}_{12}\right)$$
$$= \frac{\mu_o q_1\sigma_2 R_2}{3}\left[\vec{a}_1 + \vec{\Omega}_2\times\left(\vec{\Omega}_2\times\vec{r}_1\right) - \frac{d\vec{\Omega}_2}{dt}\times\vec{r}_1\right] , \qquad (14)$$

$$\vec{F}(r_1 > R_2) = \mu_o q_1\sigma_2\left\{\frac{R_2^2}{r_1^2}\left(\vec{r}_1\cdot\vec{a}_1\right)\hat{r}_1 + \frac{R_2^4}{r_1^4}\left[\frac{r_1\vec{a}_1}{3} - \left(\vec{r}_1\cdot\vec{a}_1\right)\hat{r}_1\right.\right.$$
$$\left.\left. + \frac{r_1}{3}\left(\vec{\Omega}_2\cdot\vec{r}_1\right)\vec{\Omega}_2 + \frac{r_1^2\Omega_2^2}{6}\hat{r}_1 - \frac{\left(\vec{r}_1\cdot\vec{\Omega}_2\right)^2}{2}\hat{r}_1 + \frac{r_1}{3}\left(\vec{r}_1\times\frac{d\vec{\Omega}_2}{dt}\right)\right]\right\} . \qquad (15)$$

The azimuthal components of equations (14) and (15) along the $\hat{\varphi}_1$ direction are given by, respectively:

$$\vec{F} = \begin{cases} -\mu_o q_1 \sigma_2 R_2 r_1 \sin\theta_1 (\dot{\Omega}_2 - \dot{\omega}_1)\hat{\varphi}_1/3 \ , & \text{if } \rho_1 \leq R_2 \ , \\ -\mu_o q_1 \sigma_2 R_2^4 \sin\theta_1 (\dot{\Omega}_2 - \dot{\omega}_1)\hat{\varphi}_1/(3r_1^2) \ , & \text{if } \rho_1 > R_2 \ . \end{cases} \tag{16}$$

We now apply Newton's second law of motion to a conduction electron with mass $m_1 = m > 0$ and charge $q_1 = -e < 0$ located at $r_1 = R_2$ and moving only along the azimuthal direction φ with angular velocity ω_{1-}. The azimuthal component of this equation can be written as:

$$\vec{F} = m_1 \vec{a}_1 = m R_2 \dot{\omega}_{1-} \sin\theta_1 \hat{\varphi}_1 \ , \tag{17}$$

where $\vec{F}$ represents the net force acting on the conduction electron. Once more there are two sets of charges exerting forces on any conduction electron, namely, (a) the positive lattice rotating around the z axis with angular velocity $\Omega_{2+}(t)$; and (b) the remaining free electrons spread over the spherical shell and rotating together with angular velocity $\omega_{1-}(t)$. As all electrons move together around the z axis, we have $\dot{r} = 0$ and $\ddot{r} = 0$ for any pair of electrons, so that there will be no net component of the force along the azimuthal direction acting on any specific conduction electron due to all the other conduction electrons. The only remaining force acting on any conduction electron will be then the force due to the rotating positive lattice of the shell.

Combining Newton's second law of motion with equation (14) for $r_1 = R_2$ and considering the azimuthal component of the force exerted by the positive lattice acting on a conduction electron located at the polar angle θ_1 yields:

$$-\frac{\mu_o q_1 \sigma_2 R_2^2 \sin\theta_1}{3}(\dot{\Omega}_{2+} - \dot{\omega}_{1-})\hat{\varphi}_1 = m R_2 \dot{\omega}_{1-} \sin\theta_1 \hat{\varphi}_1 \ , \tag{18}$$

or

$$\dot{\omega}_{1-} = \frac{|m_{Ws}|}{m + |m_{Ws}|}\dot{\Omega}_{2+} \ , \tag{19}$$

where the magnitude of the so-called weberian electromagnetic mass for this spherical geometry is given by $|m_{Ws}| \equiv \mu_o e \sigma_2 R_2/3 > 0$. This equation shows that $\dot{\omega}_{1-}$ will be always proportional to $\dot{\Omega}_{2+}$, no matter the value of the polar angle θ. Therefore, all conduction electrons will undergo the same azimuthal acceleration around the z axis, rotating together as a rigid body.

Once more we have $|m_{Ws}| \gg m$ for macroscopic situations. We can then integrate equation (19) from the stationary initial value $\Omega_{2+}(0) = 0$ up to the final and constant angular velocity $\Omega_{2+f}(t_f) = \Omega_{2+f}$ yielding the final approximate result for the angular velocity of the conduction electron as given by:

$$\omega_{1-f} \approx \left(1 - \frac{m}{|m_{Ws}|}\right) \Omega_{2+f} \ , \tag{20}$$

where $m/|m_{Ws}| \ll 1$.

For this spherical geometry we arrived at essentially the same result obtained earlier in the configuration of the cylindrical geometry, namely, equation (8). That is, by rotating the positive lattice, the positive ions of the superconductor will generate an azimuthal force on the conduction electrons inducing them to move in the same direction as the lattice. The inertial mass of the electrons will make them lag slightly behind the positive lattice.

Consider a spherical shell of radius R centered at the origin of a coordinate system and uniformly charged with a surface charge density σ. When this shell rotates uniformly around the z axis with an angular velocity Ω, it produces a magnetic field given by [22, exercise 14-6, pp. 14-3 and 14-4], [23, pp. 61 and 250] and [24, pp. 229-230]:

$$\left.\begin{array}{l}
\vec{B}(r < R) \equiv \vec{B}_{int} = (2\mu_o R\sigma\Omega/3)(\cos\theta\hat{r} - \sin\theta\hat{\theta}) = (2\mu_o R\sigma\Omega/3)\hat{z} \ , \\
\vec{B}(R) \equiv \vec{B}_{sur} = \mu_o R\sigma\Omega(4\cos\theta\hat{r} - \sin\theta\hat{\theta})/6 \ , \\
\vec{B}(r > R) \equiv \vec{B}_{ext} = \mu_o R^4\sigma\Omega(2\cos\theta\hat{r} + \sin\theta\hat{\theta})/(3r^3) \ .
\end{array}\right\} \tag{21}$$

Here $\vec{B}_{int}$ is the internal magnetic field at $r < R$, $\vec{B}_{sur}$ is the surface magnetic field at $r = R$, while $\vec{B}_{ext}$ is the external magnetic field at $r > R$.

The internal magnetic field is uniform, having the same magnitude and direction anywhere inside the shell. Outside the shell we have a dipolar magnetic field. The radial component of the magnetic field along $\hat{r}$ is continuous at the surface of the material. On the other hand, the poloidal component along $\hat{\theta}$, parallel to the surface, will be discontinuous at $r = R$.

We can now calculate the magnetic field inside a rotating superconducting spherical shell of radius R_2 composed of a positive lattice with surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$ rotating around the z axis with an angular velocity Ω_{2+f}, combined with a set of free electrons having a negative charge density $\sigma_{2-} = -\sigma_2$ and rotating around the z axis with an angular velocity $\omega_{1-f} = (1 - m/|m_{Ws}|)\Omega_{2+f}$. From equations (20) and (21) we obtain:

$$\vec{B}(r < R_2) \equiv \vec{B}_{int} = \frac{2}{3}\mu_o R_2 \sigma_2 \Omega_{2+f}\hat{z} - \frac{2}{3}\mu_o R_2 \sigma_2 \left(1 - \frac{m}{|m_{Ws}|}\right)\Omega_{2+f}\hat{z}$$

$$= \frac{2m}{e}\Omega_{2+f}\hat{z} \ . \tag{22}$$

Once more we obtained the London moment from Weber's electrodynamics combined with Newton's second law of motion. This effect was now obtained by rotating a superconducting spherical shell around its axis. We showed that the magnetic field anywhere inside the rotating superconductor will be given by $2m\vec{\Omega}_{2+f}/e$.

4 The Meissner Effect

The Meissner effect was discovered by Meissner and Ochsenfeld in 1933 [1]. The net magnetic field inside a superconductor in the presence of an external applied magnetic field goes to zero due to surface currents induced in the material. This phenomenon has since then been observed in two cases, namely, (I) applying a magnetic field in the presence of a superconducting body, and (II) a normal metal cooled into the superconducting state in the presence of an applied magnetic field. In this work we consider case (I) from the point of view of Weber's electrodynamics with two geometries, namely, cylindrical and spherical shells.

4.1 Two Cylindrical Shells

The first geometry to be considered here is the configuration with two neutral cylindrical shells of infinite lengths and radii R_1 and $R_2 > R_1$ concentric along the z axis. The outer shell is a normal conductor with positive surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$ and negative surface charge density $\sigma_{2-} = -\sigma_2$, while the inner shell is superconducting with positive surface charge density $\sigma_{1+} \equiv \sigma_1 > 0$ and negative surface charge density $\sigma_{1-} = -\sigma_1$. All these charges are supposed initially at rest relative to the inertial frame of reference S. Our goal is to calculate the motion induced in the conduction electrons of the inner shell when an external azimuthal current is applied to the outer shell. We will also calculate the net magnetic field produced inside the inner shell. We will assume that the positive lattices of both shells remain stationary during the whole process, as each atom has a mass much greater than that of an electron. The conduction electrons of the outer shell will be accelerated by an external source along the azimuthal direction during the time interval $0 < t < t_f$, moving around the z axis with a variable and given angular velocity $\vec{\Omega}_{2-}(t) = \Omega_{2-}(t)\hat{z}$. They begin at rest and at the end of this time interval they will be moving with the final and constant angular velocity $\Omega_{2-f}\hat{z}$.

As both shells are electrically neutral, the electrostatic or coulombian component of equation (1), $q_1 q_2 \hat{r}/(4\pi\varepsilon_o r^2)$, will not need to be considered in the calculations. In London moment we rotated mechanically a superconductor. In the Meissner effect, on the other hand, the conductor producing the applied magnetic field and the superconducting material placed in this magnetic field remain at rest in the laboratory or move with small velocities. The drifting velocities of the conduction electrons in normal conductors have the order of magnitude of millimeters per second. In superconducting materials the conduction electrons move relative to the lattice with velocities v larger than this drifting velocity of normal conductors, but still much smaller than light velocity. We can then neglect the velocity components of Weber's force (1). Therefore, as it happened with the London moment, the only remaining component of Weber's force which will need to be considered here is the last component depending on the accelerations $\vec{a}_1$ and $\vec{a}_2$, namely, equation (2).

The test charge will be a conduction electron of the inner superconducting shell located at $(\rho_1,\ \varphi_1,\ z_1) = (R_1,\ \varphi_1,\ 0)$ which will rotate around the z axis in the time interval $0 < t < t_f$ with a variable angular velocity $d\varphi_1/dt \equiv \omega_{1-}(t)$ which needs to be calculated. There are four sets of charges which might exert a force on any conduction electron of the inner shell in the time interval $0 < t < t_f$, namely: (a) the stationary positive lattice of the outer shell with surface charge density σ_2 and zero angular velocity $\Omega_{2+}(t) = 0$; (b) the set of negative conduction electrons of the outer shell with surface charge density $-\sigma_2$ moving with an angular velocity $\vec{\Omega}_{2-}(t) = \Omega_{2-}(t)\hat{z}$; (c) the stationary positive lattice of the inner shell with surface charge density σ_1 and zero angular velocity $\Omega_{1+}(t) = 0$; and (d) the set of remaining negative conduction electrons of the inner shell with surface charge density $-\sigma_1$ and angular velocity $\omega_{1-}(t)$. These four forces acting on a test electron with mass $m_1 \equiv m > 0$ and charge $q_1 \equiv -e < 0$ will be represented by, respectively, $\vec{F}_{2+,-e}$, $\vec{F}_{2-,-e}$, $\vec{F}_{1+,-e}$ and $\vec{F}_{1-,-e}$. As all negative charges of the inner shell move together with the test electron around the z axis, there will be no component of Weber's force acting on the test charge due to the other electrons of the same shell. That is, equation (2) goes to zero for this fourth set of charges represented by letter (d) when acting on any specific conduction electron of the superconducting inner shell, so that, $\vec{F}_{1-,-e} = \vec{0}$.

We assume that contact forces maintain any conduction electron at a constant distance $\rho_1 = R_1$ from the axis of the cylinder, so that $\dot{\rho}_1 = 0$ and $\ddot{\rho}_1 = 0$. We will be interested only in the motions along the azimuthal φ direction. We include here only force components along this direction. Newton's second law of motion as applied to the test electron of the inner shell can be written as:

$$\vec{F} = m_1\vec{a}_1 = mR_1\dot{\omega}_{1-}\hat{\varphi}_1 \ , \tag{23}$$

where $\vec{F}$ represents the total force.

The total force $\vec{F}$ will be given by equation (3) for all sets of charges, namely, (a), (b), (c) and (d) above, acting on the conduction electron. Combining equations (3) and (23) for these four sets of charges acting on the test electron yields:

$$\vec{F}_{2+,-e} + \vec{F}_{2-,-e} + \vec{F}_{1+,-e} + \vec{F}_{1-,-e}$$

$$= -\frac{\mu_o}{2}q_1\rho_1\left[\sigma_2R_2(0-\dot{\omega}_{1-}) - \sigma_2R_2(\dot{\Omega}_{2-}-\dot{\omega}_{1-}) + \sigma_1\rho_1\left(0-\dot{\omega}_{1-}\right) - 0\right]\hat{\varphi}_1$$

$$= m_1\rho_1\dot{\omega}_{1-}\hat{\varphi}_1 \ . \tag{24}$$

Utilizing $m_1 \equiv m = 9.1\times10^{-31}\ kg > 0$, $q_1 = -e = -1.6\times10^{-19}\ C < 0$ and $\rho_1 = R_1$ in equation (24) yields:

$$\dot{\omega}_{1-} = -\frac{R_2\sigma_2}{R_1\sigma_1}\frac{\dot{\Omega}_{2-}}{1+m/|m_{Wc}|} \ , \tag{25}$$

where $|m_{Wc}| \equiv \mu_o e \sigma_1 R_1/2 > 0$ is the magnitude of the weberian electromagnetic mass for this cylindrical geometry. Utilizing that $|m_{Wc}| \gg m$ and integrating equation (25) from $t = 0$ to $t = t_f$ yields the approximate final result:

$$\omega_{1-f} \approx -\frac{R_2\sigma_2}{R_1\sigma_1}\left(1 - \frac{m}{|m_{Wc}|}\right)\Omega_{2-f} \ , \tag{26}$$

where $m/|m_{Wc}| \ll 1$. This equation indicates that the negative electrons of the inner shell will move in the opposite sense of the motion of the negative charges of the outer shell. The magnitude of ω_{1-f} will be generally larger than that of Ω_{2-f} in this example of a single monoatomic conducting layer. When, for instance, $\sigma_2 = \sigma_1$ and $R_2 = 2R_1$, equation (26) indicates that $\omega_{1-f} \approx -2\Omega_{2-f}$, figure 4 (a).

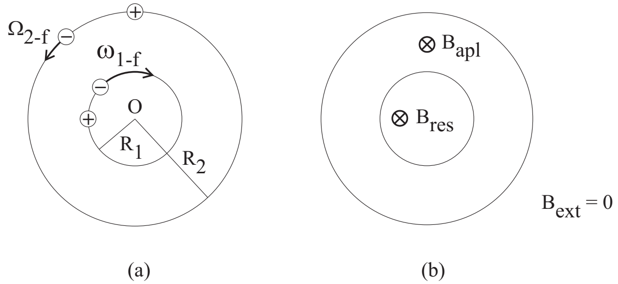

Figure 4: (a) When the negative charges of the outer shell are accelerated tangentially up to a final angular velocity Ω_{2-f}, the negative charges of the inner superconducting shell will reach a final angular velocity ω_{1-f} given by equation (26) which has the opposite sense from that of Ω_{2-f} and a larger magnitude. (b) Magnetic field produced by these rotating negative charges.

Utilizing equation (9) we can obtain the magnetic field produced by these two negative rotating cylindrical shells. To this end we need to add the magnetic field produced by the outer shell R_2 when its negative charges are rotating around the z axis with the angular velocity $\Omega_{2-f}\hat{z}$, with the magnetic field produced by the inner shell R_1 when its negative charges are rotating with angular velocity $\omega_{1-f}\hat{z}$, where ω_{1-f} is given by equation (26). According to Weber's electrodynamics this net magnetic field will be given by:

$$\left.\begin{array}{l}\vec{B}(\rho < R_1) \equiv \vec{B}_{res} \equiv -2(R_2\sigma_2/(R_1\sigma_1))(m/e)\Omega_{2-f}\hat{z} = (m/|m_{Wc}|)\vec{B}_{apl} \ , \\ \vec{B}(R_1 < \rho < R_2) \equiv \vec{B}_{apl} \equiv -\mu_o R_2\sigma_2\Omega_{2-f}\hat{z} \ , \\ \vec{B}(\rho > R_2) \equiv \vec{B}_{ext} = \vec{0} \ . \end{array}\right\} \tag{27}$$

Here $\vec{B}_{apl} \equiv -\mu_o R_2\sigma_2\Omega_{2-f}\hat{z}$ is the magnetic field produced by the external source, $\vec{B}_{res}$ is the residual magnetic field which will remain inside the inner shell, while $\vec{B}_{ext}$ is the external magnetic field at $\rho > R_2$, figure 4 (b).

The magnetic field inside a single monoatomic superconducting shell will not be exactly zero, although its order of magnitude will be much smaller than the applied magnetic field. In the previous example of a niobium cylindrical shell of radius $R_1 = 0.1\ m$ and surface charge density $\sigma_1 \approx 2\ C/m^2$, we had $|m_{Wc}| \approx 2 \times 10^{-25}\ kg$, so that $|\vec{B}_{res}|/|\vec{B}_{apl}| = m/m_{Wc} \approx 4.5 \times 10^{-6} \ll 1$. These orders of magnitude are compatible with the Meissner effect. In the next Subsection we show that if the superconductor is composed of three or more shells, the magnetic field goes to zero inside it.

4.2 Four Cylindrical Shells

To illustrate the penetration depths of currents and magnetic fields in the Meissner effect with Weber's electrodynamics, we first generalize the configuration of figure 4. The superconductor is now replaced by a set of three infinite cylindrical shells of radii $R_{1a} < R_{1b} < R_{1c}$ centered along the z axis. Each superconducting cylindrical shell is supposed to consist of a single monoatomic layer composed of positive and negative charges. These cylindrical shells are surrounded by an external resistive cylindrical shell of radius $R_2 > R_{1c}$. The positive surface charge densities of the cylindrical shells $1a$, $1b$, $1c$ and 2 will be represented by $\sigma_{1+a} > 0$, $\sigma_{1+b} > 0$, $\sigma_{1+c} > 0$ and $\sigma_{2+} \equiv \sigma_2 > 0$, respectively. The conduction electrons of the outer resistive shell R_2 will be accelerated by an external source along the azimuthal direction during the time interval $0 < t < t_f$, moving around the z axis with a variable and given angular velocity $\vec{\Omega}_{2-}(t) = \Omega_{2-}(t)\hat{z}$. They begin at rest and are accelerated until they reach the final and constant angular velocity $\Omega_{2-f}\hat{z}$ at $t = t_f$. We will assume that the positive charges of shells $1a$, $1b$, $1c$ and 2 remain at rest during the whole process. We need to obtain the angular velocities ω_{1-a}, ω_{1-b} and ω_{1-c} of the conduction electrons of shells $1a$, $1b$ and $1c$

as a function of the given angular velocity Ω_{2-} of the electrons of the resistive shell 2.

Consider, for instance, a specific conduction electron $q_1 = -e < 0$ of shell $1b$. There are eight sets of charges exerting forces on it, namely, the positive and negative charges of shells $1a$, $1b$, $1c$ and 2. We can utilize equation (3) to express the azimuthal components of these eight forces by taking care if this test electron of shell $1b$ is inside or outside the cylindrical shell of the source charges which are exerting forces on it. We assume the charge neutrality of each shell such that their negative surface charge densities will be equal and opposite the corresponding positive surface charge densities, namely, $\sigma_{1-a} = -\sigma_{1+a}$, $\sigma_{1-b} = -\sigma_{1+b}$, $\sigma_{1-c} = -\sigma_{1+c}$ and $\sigma_{2-} = -\sigma_2$. We are interested only in the azimuthal component of Newton's second law of motion along the φ direction. In each line of the next equation, from top to bottom, we represent the forces exerted by the positive and negative charges of shells $1a$, $1b$, $1c$ and 2 acting on a conduction electron of shell $1b$. With $q_1 = -e$, the azimuthal component of Newton's second law of motion for this test electron can then be written as:

$$\frac{\mu_o e}{2}\left[\sigma_{1a}\frac{R_{1a}^3}{R_{1b}}(0-\dot{\omega}_{1-b}) - \sigma_{1a}\frac{R_{1a}^3}{R_{1b}}(\dot{\omega}_{1-a}-\dot{\omega}_{1-b})\right.$$

$$+ \sigma_{1b}R_{1b}^2(0-\dot{\omega}_{1-b}) - \sigma_{1b}R_{1b}^2(\dot{\omega}_{1-b}-\dot{\omega}_{1-b})$$

$$+ \sigma_{1c}R_{1b}R_{1c}(0-\dot{\omega}_{1-b}) - \sigma_{1c}R_{1b}R_{1c}(\dot{\omega}_{1-c}-\dot{\omega}_{1-b})$$

$$\left. + \sigma_2 R_{1b}R_2(0-\dot{\omega}_{1-b}) - \sigma_2 R_{1b}R_2(\dot{\Omega}_{2-}-\dot{\omega}_{1-b})\right] = mR_{1b}\dot{\omega}_{1-b} \ . \quad (28)$$

By writing similar equations for a test electron of shells $1a$ and $1c$ we will have three equations with three unknowns, namely, $\dot{\omega}_{1-a}$, $\dot{\omega}_{1-b}$ and $\dot{\omega}_{1-c}$. By solving this set of three equations we obtain that $\dot{\omega}_{1-a}$, $\dot{\omega}_{1-b}$ and $\dot{\omega}_{1-c}$ will be proportional to $\dot{\Omega}_{2-}$. We can then integrate in time the final solutions, obtaining the final values ω_{1-af}, ω_{1-bf} and ω_{1-cf}. The solution of this set of three equations, approximated up to first order in $m/|m_{Wc}| \ll 1$ for each shell, yields the following integrated final values of ω_{1-a}, ω_{1-b} and ω_{1-c}, respectively:

$$\omega_{1-af} = 0 \ , \tag{29}$$

$$\omega_{1-bf} = -\frac{m}{|m_{Wc2}|}\frac{\sigma_2}{\sigma_{1b}}\frac{R_{1c}^2 R_2}{R_{1b}(R_{1c}^2 - R_{1b}^2)}\Omega_{2-f} \ , \tag{30}$$

$$\omega_{1-cf} = -\frac{\sigma_2}{\sigma_{1c}}\frac{R_2}{R_{1c}}\left(1 - \frac{m}{|m_{Wc1}|}\frac{R_{1c}^2}{R_{1c}^2 - R_{1b}^2}\right)\Omega_{2-f} \ . \tag{31}$$

In these equations $|m_{Wc1}| \equiv \mu_o e\sigma_{1c}R_{1c}/2 > 0$ and $|m_{Wc2}| \equiv \mu_o e\sigma_2 R_2/2 > 0$ are the magnitudes of the weberian electromagnetic masses for the electrons of the cylindrical shells of radii R_{1c} and R_2, respectively.

As $m/|m_{Wc2}| \ll 1$ and $m/|m_{Wc1}| \ll 1$ we have $\omega_{1-cf} \gg \omega_{1-bf} \gg \omega_{1-af} = 0$. That is, the angular velocities of these shells decrease rapidly as we go from the external to the internal shell.

The magnetic field for each region can be obtained from equation (9) by taking into account the azimuthal motion of the electrons not only in the three superconducting shells $1a$, $1b$ and $1c$, but also in the external shell 2. The net magnetic field in each region is then found to be given by:

$$\left.\begin{array}{l}
\vec{B}(\rho < R_{1b}) \equiv \vec{B}_{int} = \vec{0} \ , \\
\vec{B}(R_{1b} < \rho < R_{1c}) \equiv \vec{B}_{res} \equiv -2m\sigma_2 R_{1c}R_2\Omega_{2-f}\hat{z}/(e\sigma_{1c}(R_{1c}^2 - R_{1b}^2)) \ , \\
\vec{B}(R_{1c} < \rho < R_2) \equiv \vec{B}_{apl} \equiv -\mu_o R_2\sigma_2\Omega_{2-f}\hat{z} \ .
\end{array}\right\} \tag{32}$$

Once more the residual magnetic field $B_{res} \equiv |\vec{B}_{res}|$ is much smaller than the applied magnetic field $B_{apl} \equiv |\vec{B}_{apl}|$. Moreover, the magnetic field decreases quickly to zero for $\rho < R_{1b}$, that is, $\vec{B}(\rho < R_{1b}) \equiv \vec{B}_{int} = \vec{0}$.

The qualitative behavior of the angular velocity of the conduction electrons at each shell and the net magnetic field in each region are represented in figure 5.

4.3 Two Spherical Shells

We now perform a similar calculation considering two concentric spherical shells of radii R_1 and $R_2 > R_1$ centered at the origin of the coordinate system. The outer resistive shell has a stationary positive charge density

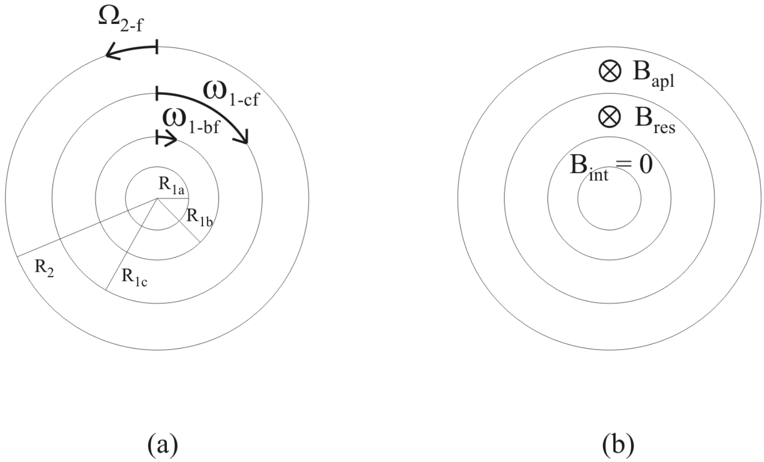

Figure 5: (a) Qualitative representation of the angular velocities of the conduction electrons of each shell. (b) Magnetic field in each region between the shells.

$\sigma_2 > 0$ and a negative charge density $-\sigma_2$. These negative charges will be considered initially at rest, being accelerated azimuthally by an external source around the z axis during the time interval $0 < t < t_f$. They will move around the z axis with a given angular velocity $\vec{\Omega}_{2-}(t) = \Omega_{2-}(t)\hat{z}$, until they reach the final and constant angular velocity $\Omega_{2-f}\hat{z}$. The inner superconducting shell has a stationary positive charge density $\sigma_1 > 0$ and a negative charge density $-\sigma_1$. These negative charges will also be considered initially at rest. Due to the azimuthal forces exerted by the other charges, they will begin to rotate around the z axis with an angular velocity $\vec{\omega}_{1-}(t) = \omega_{1-}(t)\hat{z}$, until they reach the final and constant angular velocity $\omega_{1-f}\hat{z}$. Our goal is to calculate ω_{1-f} as a function of the given Ω_{2-f}. We will also calculate the magnetic field inside the inner shell.

The test charge will be a conduction electron of the inner superconducting shell located at $(r_1, \theta_1, \varphi_1) = (R_1, \theta_1, \varphi_1)$. Contact forces will keep it at a constant distance from the center of the shell, so that $\dot{r}_1 = 0$ and $\ddot{r}_1 = 0$. We will be interested only in its motion along the azimuthal direction φ with an angular velocity $d\varphi_1/dt = \omega_{1-}$, so that $\dot{\theta}_1 = 0$ and $\ddot{\theta}_1 = 0$. The velocity and acceleration of the test charge along the azimuthal direction are then given by $\vec{v}_1 = R_1\omega_{1-}\sin\theta_1\hat{\varphi}_1$ and $\vec{a}_1 = R_1\sin\theta_1\dot{\omega}_{1-}\hat{\varphi}_1$, where $\dot{\omega}_{1-} \equiv d\omega_{1-}/dt$.

As in the Subsection 4.1, there are four sets of charges acting on any conduction electron of the inner shell during the time interval $0 < t < t_f$, namely: (a) the stationary positive charges of outer shell with radius R_2 and surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$; (b) the negative charges of the outer shell with radius R_2 and surface charge density $\sigma_{2-} = -\sigma_2$ moving with a given angular velocity $\Omega_{2-}(t)\hat{z}$; (c) the stationary positive charges of the inner shell with radius R_1 and surface charge density $\sigma_{1+} \equiv \sigma_1 > 0$; and (d) the negative charges of the inner shell with radius R_1 and surface charge density $\sigma_{1-} = -\sigma_1$ moving with angular velocity $\omega_{1-}(t)\hat{z}$. Each one of these forces has already been calculated, being given by equations (14) and (16), with the appropriate values of σ, R and Ω. They will be represented by $\vec{F}_{2+,-e}$, $\vec{F}_{2-,-e}$, $\vec{F}_{1+,-e}$, and $\vec{F}_{1-,-e}$. We utilize the azimuthal component of Newton's second law of motion $\vec{F} = m_1\vec{a}_1 = m_1R_1\sin\theta_1\dot{\omega}_{1-}\hat{\varphi}_1$ with $\vec{F} = \vec{F}_{2+,-e} + \vec{F}_{2-,-e} + \vec{F}_{1+,-e} + \vec{F}_{1-,-e}$. Utilizing $m_1 = m$, $q_1 = -e < 0$ and $r_1 = R_1$ we obtain:

$$\vec{F}_{2+,-e} + \vec{F}_{2-,-e} + \vec{F}_{1+,-e} + \vec{F}_{1-,-e}$$

$$= -\frac{\mu_o q_1\sigma_2R_2R_1\sin\theta_1}{3}(0 - \dot{\omega}_{1-})\hat{\varphi}_1 + \frac{\mu_o q_1\sigma_2R_2R_1\sin\theta_1}{3}(\dot{\Omega}_{2-} - \dot{\omega}_{1-})\hat{\varphi}_1$$

$$- \frac{\mu_o q_1\sigma_1R_1^2\sin\theta_1}{3}(0 - \dot{\omega}_{1-})\hat{\varphi}_1 + \vec{0}$$

$$= mR_1\sin\theta_1\dot{\omega}_{1-}\hat{\varphi}_1 \ . \qquad (33)$$

This equation can also be written as:

$$\dot{\omega}_{1-} = -\frac{\sigma_2R_2}{\sigma_1R_1}\frac{|m_{Ws}|}{m + |m_{Ws}|}\dot{\Omega}_{2-} \approx -\frac{\sigma_2R_2}{\sigma_1R_1}\left(1 - \frac{m}{|m_{Ws}|}\right)\dot{\Omega}_{2-} \ , \qquad (34)$$

where $|m_{Ws}| \equiv \mu_o e\sigma_1R_1/3 > 0$ is the magnitude of the weberian electromagnetic mass for this spherical geometry. As before, usually we have $|m_{Ws}| \gg m$. This equation shows that $\dot{\omega}_{1-}$ will be always proportional to $\dot{\Omega}_{2-}$, no matter the value of the polar angle θ. Therefore, all conduction electrons will undergo the same azimuthal acceleration, rotating together as a rigid body around the z axis.

Integrating this equation from $t = 0$ to $t = t_f$ yields the final result given by:

$$\omega_{1-f} = -\frac{\sigma_2 R_2}{\sigma_1 R_1} \frac{|m_{Ws}|}{m + |m_{Ws}|} \Omega_{2-f} \approx -\frac{\sigma_2 R_2}{\sigma_1 R_1} \left(1 - \frac{m}{|m_{Ws}|} \right) \Omega_{2-f} \ , \quad (35)$$

where $m/|m_{Ws}| \ll 1$.

These equations indicate that the conduction electrons of the inner superconducting shell will be accelerated in the opposite direction of the acceleration of the electrons of the outer resistive shell. Moreover, the magnitude of the angular velocity acquired by the inner electrons will be usually larger than the angular velocity of the electrons of the outer shell. If, for instance, $\sigma_1 = \sigma_2$ and $R_2 = 2R_1$, then $\omega_{1-f} \approx -2\Omega_{2-f}$.

Utilizing equation (21) we can obtain the magnetic field due to these two spherical shells. To this end we need to add the magnetic field produced by two sets of charges, namely, (a) the outer shell R_2 when its negative charges with surface density $-\sigma_2$ are rotating around the z axis with the angular velocity $\Omega_{2-f}\hat{z}$; and (b) the inner shell R_1 when its negative charges with surface density $-\sigma_1$ are rotating with angular velocity $\omega_{1-f}\hat{z}$ given by equation (35). According to Weber's electrodynamics, this net magnetic field inside the inner superconducting shell will be given by:

$$\vec{B}_{res}(r < R_1) = -\frac{2}{3}\mu_o R_2 \sigma_2 \Omega_{2-f}\hat{z} - \frac{2}{3}\mu_o R_1 \sigma_1 \omega_{1-f}\hat{z}$$

$$= -2\frac{\sigma_2 R_2}{\sigma_1 R_1}\frac{m}{e}\Omega_{2-f}\hat{z} = \frac{m}{|m_{Ws}|}\vec{B}_{apl} \ , \quad (36)$$

where $\vec{B}_{apl} \equiv -(2\mu_o R_2 \sigma_2 \Omega_{2-f}/3)\hat{z}$ is the applied magnetic field in this region and $|m_{Ws}| = \mu_o e \sigma_1 R_1/3 > 0$ is the magnitude of the weberian electromagnetic mass in this spherical geometry. The applied magnetic field would be the net magnetic field in the internal region $r < R_1$ if the inner superconducting spherical shell were not present. As $m \ll |m_{Ws}|$, we have $B_{res} \ll B_{apl}$, indicating that there will remain a small residual magnetic field in the internal region, as it happened in the case of two cylindrical shells.

In the region $R_1 \leq r < R_2$ we need to add the uniform magnetic field due to the outer shell with the dipolar magnetic field due to the inner shell. In this case we can neglect m compared with $|m_{Ws}|$ in equation (35) as $m/|m_{Ws}| \ll 1$, so that $\omega_{1-f} \approx -\sigma_2 R_2 \Omega_{2-f}/(\sigma_1 R_1)$. Utilizing that $\hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$, the net magnetic field in this region can then be expressed as:

$$\vec{B}(R_1) = -\frac{1}{2}\left[\frac{2}{3}\mu_o R_2\sigma_2\Omega_{2-f}\hat{z} + \lim_{r\to R_1}\frac{\mu_o R_1^4\sigma_1\omega_{1-f}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{3r^3}\right]$$

$$= \frac{\mu_o R_2\sigma_2\Omega_{2-f}\sin\theta}{2}\hat{\theta} - \frac{1}{6}\mu_o R_2\sigma_2(2\cos\theta\hat{r} + \sin\theta\hat{\theta})\frac{m}{|m_{Ws}|}\Omega_{2-f}$$

$$\approx \frac{\mu_o R_2\sigma_2\Omega_{2-f}\sin\theta}{2}\hat{\theta}\ , \tag{37}$$

and

$$\vec{B}(R_1 < r < R_2) = -\frac{2}{3}\mu_o R_2\sigma_2\Omega_{2-f}\hat{z} - \frac{\mu_o R_1^4\sigma_1\omega_{1-f}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{3r^3}$$

$$\approx -\frac{2}{3}\mu_o R_2\sigma_2\Omega_{2-f}(\cos\theta\hat{r} - \sin\theta\hat{\theta}) + \frac{\mu_o R_1^3 R_2\sigma_2\Omega_{2-f}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{3r^3}\ . \tag{38}$$

The present situation will be equivalent to the configuration of figure 4 with one difference. In the situation of two ideal cylindrical shells of infinite lengths carrying azimuthal currents, the magnetic field between the shells will always be the same, no matter the value of the current in the inner shell. In the more realistic situation of two spherical shells, on the other hand, the magnetic field between them will be the uniform magnetic field produced by the outer shell, combined with the dipolar magnetic field produced by the inner superconducting shell. They have the same orders of magnitude, as can be seen from equation (38). This alteration of the magnetic field around the superconductor is always shown in experiments related to the Meissner effect, due to the fact that any real superconducting sample has a finite volume.

Equation (38) indicates that the magnitude of the magnetic field at $\theta = \pi/2$ *rad* just outside the superconducting spherical shell is larger than the magnitude of the applied magnetic field $\vec{B}_{apl} \equiv -(2\mu_o R_2 \sigma_2 \Omega_{2-f}/3)\hat{z}$. This behavior is typical in the Meissner effect.

Equation (36) shows that the magnetic field has a poloidal direction around the inner superconducting spherical shell and does not penetrate it, except for the residual magnetic field $\vec{B}_{res} = m\vec{B}_{apl}/|m_{Ws}|$. As $m/|m_{Ws}| \ll 1$ we have $|\vec{B}_{res}|/|\vec{B}_{apl}| \ll 1$. That is, the magnitude of the residual magnetic field inside the superconducting spherical shell is much smaller than the magnitude of the applied magnetic field. This property represents the essence of the Meissner effect.

4.4 A Spherical Shell Inside a Cylindrical Shell

We now perform a similar calculation considering a superconducting spherical shell of radius R_1 inside a resistive cylindrical shell of radius $R_2 > R_1$, figure 6. They are centered on the origin of the coordinate system, with the axis of the cylindrical shell along the z axis. They have positive and negative charge densities represented by, respectively: $\sigma_1 > 0$, $-\sigma_1$, $\sigma_2 > 0$ and $-\sigma_2$. We assume that the positive charges of both shells will remain at rest relative to the inertial frame S. The negative charges of both shells are considered initially at rest. As before, we assume that during the time interval $0 < t < t_f$ an external source accelerates azimuthally the negative charges of the outer shell with a variable and given angular velocity $\vec{\Omega}_{2-}(t) = \Omega_{2-}(t)\hat{z}$ up to a final and constant angular velocity $\Omega_{2-f}\hat{z}$. Our goal is to calculate the induced motion of the conduction electrons of the superconducting inner shell. We will also obtain the magnetic field produced by this system. As before, we are only interested in the force and motion along the azimuthal φ direction. We will once more neglect the coulombian and velocity terms of Weber's force, so that the only remaining component will be that given by equation (2).

The force acting on any conduction electron of the inner shell will be due to four sets of charges, namely, the positive and negative charges of the outer shell, together with the positive and remaining negative charges of the inner shell. The force exerted by the first two sets of

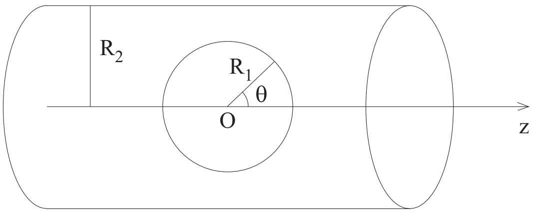

Figure 6: A superconducting spherical shell inside a resistive cylindrical shell.

charges is given by equation (3) with the appropriate values of σ, R and Ω. The force exerted by the last two sets of charges is given by equation (16) with the appropriate values of σ, R and Ω. We now add the forces due to these four sets of charges utilizing charge neutrality, $\sigma_{2-} = -\sigma_2$ and $\sigma_{1-} = -\sigma_1$, together with the assumption that the positive charges of both shells remain at rest during the time interval $0 < t < t_f$, so that $\Omega_{2+}(t) = \Omega_{1+}(t) = 0$. We consider a test electron of the inner shell located at $(R_1,\ \theta_1,\ \varphi_1)$ moving with angular velocity ω_{1-}. Its azimuthal equation of motion is then given by:

$$-\frac{\mu_o}{2}q_1\rho_1\left[\sigma_2R_2(0-\dot{\omega}_{1-})-\sigma_2R_2(\dot{\Omega}_{2-}-\dot{\omega}_{1-})\right]\hat{\varphi}_1$$

$$-\frac{\mu_o q_1\sigma_1R_1^2\sin\theta_1}{3}\left(0-\dot{\omega}_{1-}\right)\hat{\varphi}_1+\vec{0}=mR_1\sin\theta_1\dot{\omega}_{1-}\hat{\varphi}_1\ . \qquad (39)$$

With $\rho_1 = R_1\sin\theta_1$ and $q_1 = -e < 0$ we obtain:

$$\dot{\omega}_{1-}=-\frac{3}{2}\frac{\sigma_2R_2}{\sigma_1R_1}\frac{|m_{Ws}|}{m+|m_{Ws}|}\dot{\Omega}_{2-}\approx-\frac{3}{2}\frac{\sigma_2R_2}{\sigma_1R_1}\left(1-\frac{m}{|m_{Ws}|}\right)\dot{\Omega}_{2-}\ , \qquad (40)$$

where $m/|m_{Ws}| \ll 1$. Here $|m_{Ws}| \equiv \mu_o e\sigma_1R_1/3 > 0$ is the magnitude of the weberian electromagnetic mass for this spherical geometry. This equation shows that $\dot{\omega}_{1-}$ will have the same value for all charges, no matter the polar angle θ where the test electron is located. Therefore, all negative charges of the inner superconducting shell will be accelerated azimuthally as a rigid body.

Integrating equation (40) from $t = 0$ to $t = t_f$ yields:

$$\omega_{1-f} = -\frac{3}{2}\frac{\sigma_2 R_2}{\sigma_1 R_1}\frac{|m_{Ws}|}{m+|m_{Ws}|}\Omega_{2-f} \approx -\frac{3}{2}\frac{\sigma_2 R_2}{\sigma_1 R_1}\left(1-\frac{m}{|m_{Ws}|}\right)\Omega_{2-f} \ . \tag{41}$$

The magnetic field can now be obtained combining the field due to the motion of the conduction electrons around the cylindrical outer shell given by equation (9), with the magnetic field due to the motion of the conduction electrons around the inner spherical shell given by equation (21). Inside the inner superconducting spherical shell the magnetic field is given by:

$$\vec{B}_{res}(r < R_1) = -\mu_o R_2 \sigma_2 \Omega_{2-f}\hat{z} - \frac{2}{3}\mu_o R_1 \sigma_1 \omega_{1-f}\hat{z}$$

$$= -3\frac{m}{e}\frac{\sigma_2 R_2}{\sigma_1 R_1}\Omega_{2-f}\hat{z} = \frac{m}{|m_{Ws}|}\vec{B}_{apl} \ , \tag{42}$$

where $\vec{B}_{apl} \equiv -\mu_o R_2 \sigma_2 \Omega_{2-}\hat{z}$ is the applied magnetic field. The net magnetic field in the region $r < R_1$ would have the value of the applied magnetic field $\vec{B}_{apl}$ if the superconducting inner spherical shell were not present. The presence of the inner spherical shell modifies the magnetic field inside it, so that the net value is now given by $\vec{B}_{res}$. As $m \ll |m_{Ws}|$ we have $|\vec{B}_{res}| \ll |\vec{B}_{apl}|$. Therefore the magnitude of the remaining residual magnetic field inside the superconducting spherical shell is much smaller than the magnitude of the applied magnetic field. This same property happened in the case of two cylindrical shells.

The magnetic field in the region between the inner spherical shell and the outer cylindrical shell can also be obtained by equations (9) and (21). In this region equation (41) can be approximated to $\omega_{1-f} \approx -3\sigma_2 R_2 \Omega_{2-f}/(2\sigma_1 R_1)$ because $m/|m_{Ws}| \ll 1$. Utilizing that $\hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$, the magnetic field at the surface of the inner superconducting shell will be given by:

$$\vec{B}(R_1) = -\frac{1}{2}\left[\mu_o R_2 \sigma_2 \Omega_{2-f}\hat{z} + \lim_{r\to R_1}\frac{\mu_o R_1^4 \sigma_1 \omega_{1-f}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{3r^3}\right]$$

$$= \frac{3\mu_o R_2 \sigma_2 \Omega_{2-f}\sin\theta}{4}\hat{\theta} - \frac{1}{4}\mu_o R_2 \sigma_2 (2\cos\theta\hat{r} + \sin\theta\hat{\theta})\frac{m}{|m_{Ws}|}\Omega_{2-f}$$

$$\approx \frac{3\mu_o R_2 \sigma_2 \Omega_{2-f} \sin\theta}{4} \hat{\theta} \ . \tag{43}$$

In the region between the inner spherical shell and the outer cylindrical shell the magnetic field will be given by:

$$\vec{B} = -\mu_o R_2 \sigma_2 \Omega_{2-f} \hat{z} - \frac{\mu_o R_1^4 \sigma_1 \omega_{1-f} (2\cos\theta \hat{r} + \sin\theta\hat{\theta})}{3r^3}$$

$$\approx -\mu_o R_2 \sigma_2 \Omega_{2-f} (\cos\theta \hat{r} - \sin\theta\hat{\theta}) + \frac{\mu_o R_1^3 R_2 \sigma_2 \Omega_{2-f} (2\cos\theta\hat{r} + \sin\theta\hat{\theta})}{2r^3} \ . \tag{44}$$

Once more the present situation will be equivalent to that of figure 4 with one difference. In the situation of two ideal cylindrical shells of infinite lengths carrying azimuthal currents, the magnetic field between the shells will always be the same, no matter the value of the current in the inner shell. In the more realistic situation of a superconducting spherical shell inside a resistive cylindrical shell, on the other hand, the magnetic field between them will be the uniform magnetic field produced by the outer cylindrical shell, combined with the dipolar magnetic field produced by the inner superconducting spherical shell. They have the same orders of magnitude, as can be seen from equation (44).

Equation (44) indicates that the magnitude of the magnetic field at $\theta = \pi/2 \ rad$ just outside the superconducting spherical shell is larger than the magnitude of the applied magnetic field $\vec{B}_{apl} \equiv -\mu_o R_2 \sigma_2 \Omega_{2-} \hat{z}$. This behavior is typical in the Meissner effect.

Equations (42) and (43) show that the magnetic field has a poloidal direction on the inner superconducting spherical shell and does not penetrate it, except for the residual magnetic field $\vec{B}_{res} = m\vec{B}_{apl}/|m_{Ws}|$. As $m/|m_{Ws}| \ll 1$ we have $|\vec{B}_{res}| \ll |\vec{B}_{apl}|$. This property represents the essence of the Meissner effect. It has been deduced in this paper as a consequence of Weber's electrodynamics combined with Newton's second law of motion.

In this Section we considered only case (I) where a magnetic field does not penetrate into a superconductor. This is not too surprising as a temporary skin effect is already observed and explained in normal conductors. A real challenge is posed when the magnetic field is expelled once a current-free conductor is cooled down to the superconducting state. This case (II) will not considered in the present work.

5 London Penetration Depth

5.1 Superconducting Cylinder

We now consider a superconducting cylinder of infinite length, inner radius R_i and outer radius R_o centered along the z axis. We assume that this hollow superconductor is inside a normal resistive cylindrical shell of radius $R_2 > R_o$. Their axes are located along the z axis. Consider a test electron $q_1 = -e < 0$ of the superconductor at a distance ρ from the z axis, with $R_i \leq \rho \leq R_o$. We assume that at the initial time $t = 0$ all charges are at rest. During the time interval $0 < t < t_f$ an external source generates a given variable azimuthal angular velocity $\Omega_{2-}(t)$ for the electrons located at the external resistive cylindrical shell R_2, figure 7. Our goal is to obtain the angular velocity $\omega(\rho,\ t)$ induced in the test electron as a function of its distance ρ and time t. This problem will be solved with Weber's electrodynamics utilizing equation (3) coupled with Newton's second law of motion.

Let R' be a variable distance of integration in the superconducting cylindrical material, with $R_i \leq R' \leq R_o$. There are six groups of charges exerting forces on a specific conduction electron of the superconductor located at a distance ρ from the z axis, namely, (a) the positive and negative charges of the superconductor located in the region $R_i < R' < \rho$; (b) the positive and negative charges of the superconductor located in the region $\rho < R' < R_o$; together with (c) the positive and negative charges of the resistive cylindrical shell located at R_2. As usual, we will assume charge neutrality of the superconductor cylinder, together with charge neutrality of the external cylindrical shell. Moreover, we will assume that the positive charges of the superconductor and external shell will always remain at rest. The external shell has positive and negative surface charge densities given by $\sigma_{2+} \equiv \sigma_2 > 0$ and $\sigma_{2-} = -\sigma_2$, respectively. The positive and negative volume charge densities of the superconductor will be written as $ne > 0$ and $-ne$, respectively, where $n > 0$ represents the number density of the conduction free electrons, while $e = 1.6 \times 10^{19}\ C > 0$ is the magnitude of their charge. We now apply equation (3) to these six groups of charges when acting on the conduction electron located at a distance ρ from the z axis.

We first need to obtain the azimuthal force exerted by the charges of

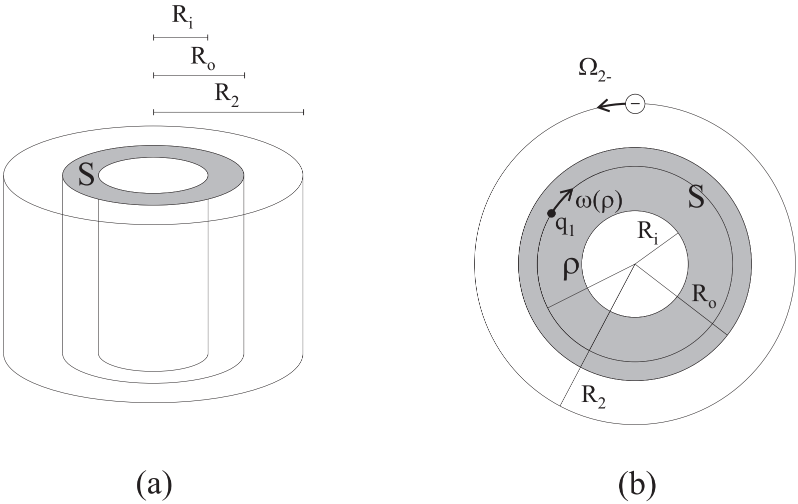

Figure 7: (a) Superconducting cylinder S of inner radius R_i and outer radius R_o inside a resistive cylindrical shell of radius R_2. (b) Cross-section of the system with a conduction electron $q_1 = -e < 0$ of the superconductor at a distance ρ from the z axis.

the superconducting cylinder located between R_i and ρ acting on a test electron located at a distance ρ from the z axis. To this end, we first consider a cylindrical shell of radius $R_2 = R'$ and thickness dR' such that $R_i \leq R' \leq \rho$. The volume charge densities of this cylindrical shell will be represented by $\pm ne$, where the upper (lower) sign represents the positive (negative) charges of the cylinder. The angular velocities of the positive and negative charges of this cylindrical shell will be represented by 0 and $\omega(R')$, respectively. We then utilize the third line of equation (3) and integrate it from $R' = R_i$ up to $R' = \rho$. Therefore, the sum of the azimuthal force exerted by the positive and negative charges of the cylinder acting on the conduction electron with charge $q_1 = -e < 0$ located at a distance ρ from the z axis will be given by:

$$\vec{F} = \frac{\mu_o e^2 n \hat{\varphi}}{2\rho} \int_{R'=R_i}^{\rho} R'^3 [0 - \dot{\omega}(\rho)] dR' - \frac{\mu_o e^2 n \hat{\varphi}}{2\rho} \int_{R'=R_i}^{\rho} R'^3 [\dot{\omega}(R') - \dot{\omega}(\rho)] dR'$$

$$= -\frac{\mu_o e^2 n\hat{\varphi}}{2\rho} \int_{R'=R_i}^{\rho} R'^3 \dot{\omega}(R') dR' \ . \tag{45}$$

Analogously the force exerted by the positive and negative charges of the cylinder located between ρ and R_o acting on a test electron located at ρ can be obtained integrating the first line of equation (3), yielding:

$$\vec{F} = \frac{\mu_o e^2 n\rho\hat{\varphi}}{2} \int_{R'=\rho}^{R_o} R'[0 - \dot{\omega}(\rho)] dR' - \frac{\mu_o e^2 n\rho\hat{\varphi}}{2} \int_{R'=\rho}^{R_o} R'[\dot{\omega}(R') - \dot{\omega}(\rho)] dR'$$

$$= -\frac{\mu_o e^2 n\rho\hat{\varphi}}{2} \int_{R'=\rho}^{R_o} R' \dot{\omega}(R') dR' \ . \tag{46}$$

Consider now the external and resistive cylindrical shell of radius R_2 with a positive surface charge density $\sigma_{2+} \equiv \sigma_2 > 0$ always at rest, $\Omega_{2+}(t) = 0$, and a negative surface charge density $\sigma_{2-} = -\sigma_2$ moving with angular velocity $\Omega_{2-}(t)$ around the z axis. The force exerted by these two shells and acting on a test electron located at a distance ρ from the z axis and moving around it with angular velocity ω can also be obtained by the first line of equation (3), yielding:

$$\vec{F} = \frac{\mu_o e\sigma_2\rho\hat{\varphi}}{2} R_2[0 - \dot{\omega}(\rho)] - \frac{\mu_o e\sigma_2\rho\hat{\varphi}}{2} R_2[\dot{\Omega}_{2-} - \dot{\omega}(\rho)] = -\frac{\mu_o e\sigma_2\rho\hat{\varphi}}{2} R_2 \dot{\Omega}_{2-} \ . \tag{47}$$

According to equation (4) the azimuthal component of Newton's second law of motion applied to the test electron located at a distance ρ from the z axis can be written as:

$$\vec{F} = m\vec{a} = m\rho\dot{\omega}(\rho)\hat{\varphi} \ , \tag{48}$$

where $\vec{F}$ represents the total force acting on this test electron.

The sum of equations (45), (46) and (47) yields the net force $\vec{F}$ acting on the conduction electron located at ρ. Applying this net force to the azimuthal component of Newton's second law of motion yields:

$$-\frac{\mu_o e^2 n\hat{\varphi}}{2\rho} \int_{R'=R_i}^{\rho} R'^3 \dot{\omega}(R') dR' - \frac{\mu_o e^2 n\rho\hat{\varphi}}{2} \int_{R'=R_i}^{\rho} R' \dot{\omega}(R') dR'$$

$$-\frac{\mu_o e\sigma_2\rho\hat{\varphi}}{2}R_2\dot{\Omega}_{2-} = m\rho\dot{\omega}(\rho)\hat{\varphi} \ . \tag{49}$$

This is an integral equation with the unknown $\dot{\omega}(\rho)$. We can obtain the differential equation satisfied by $\dot{\omega}(\rho)$ utilizing that [25, p. 44]:

$$\frac{\partial}{\partial\rho}\int_{R'=f(\rho)}^{R'=g(\rho)} F(\rho,R')dR' = \int_{R'=f(\rho)}^{R'=g(\rho)} \frac{\partial F(\rho,R')}{\partial\rho}dR'$$

$$+\left\{\frac{\partial g(\rho)}{\partial\rho}F[\rho,g(\rho)] - \frac{\partial f(\rho)}{\partial\rho}F[\rho,f(\rho)]\right\} \ . \tag{50}$$

This equation is valid for arbitrary functions $f(\rho)$, $g(\rho)$ and $F(\rho,R')$.

We define a positive magnitude λ_L by

$$\lambda_L \equiv \sqrt{\frac{m}{\mu_o ne^2}} \ . \tag{51}$$

This positive magnitude is the so-called London penetration depth.

Deriving twice equation (49) as a function of ρ, utilizing equations (50) and (51) and rearranging the terms, we obtain the following second order differential equation:

$$\lambda_L^2\rho\frac{d^2\dot{\omega}}{d\rho^2} + 3\lambda_L^2\frac{d\dot{\omega}}{d\rho} - \rho\dot{\omega} = 0 \ . \tag{52}$$

We now define the dimensionless magnitude $u \equiv \rho/\lambda_L$ and utilize a change of variable by defining the magnitude $v(u)$ as $v(u) \equiv u\dot{\omega}$. With these definitions equation (52) can then be written as:

$$u^2v'' + uv' - (u^2+1)v = 0 \ , \tag{53}$$

where $v' \equiv dv/du$ and $v'' \equiv d^2v/du^2$. This is the modified Bessel function of order 1. Its general solution is given by

$$v(u) = c_1 I_1(u) + c_2 K_1(u) \ , \tag{54}$$

where c_1 and c_2 are arbitrary constants, while $I_1(u)$ is the modified Bessel function of order 1 and $K_1(u)$ is the Hankel function of order 1.

According to equation (49) we will need to integrate these modified Bessel and Hankel functions from $u = R_i/\lambda_L$ up to $u = R_o/\lambda_L$. Typical values of λ_L range from 50 to 500 nm, while R_i and R_o have macroscopic values, so that $R_o/\lambda_L > R_i/\lambda_L \gg 1$. We can then approximate $I_1(u)$ and $K_1(u)$ by their asymptotic behaviors

$$I_\nu(u \gg 1) = \frac{e^u}{\sqrt{2\pi u}} \ , \tag{55}$$

and

$$K_\nu(u \gg 1) = \sqrt{\frac{\pi}{2u}} e^{-u} \ . \tag{56}$$

These asymptotic behaviors are valid for arbitrary values of the modified Bessel and Hankel functions of the order ν.

In these limits we then have:

$$\dot{\omega} = \frac{v(u)}{u} = \frac{c_1 I_1(u) + c_2 K_1(u)}{u} = \frac{c_1}{\sqrt{2\pi}} \frac{e^u}{u^{3/2}} + \sqrt{\frac{\pi}{2}} c_2 \frac{e^{-u}}{u^{3/2}} \approx c_3 \frac{e^u}{u^{3/2}} \ . \tag{57}$$

We defined here the magnitude c_3 by $c_3 \equiv c_1/\sqrt{2\pi}$ and we utilized once more that we are interested only in values of $u \equiv \rho/\lambda_L$ when $u \gg 1$.

We can now apply equation (57) into equation (49). After performing the two integrations and utilizing once more $R_o/\lambda_L > R_i/\lambda_L \gg 1$ we obtain

$$c_3 = -\frac{2|m_{Wc2}|}{m} \sqrt{\frac{R_o}{\lambda_L}} e^{-R_o/\lambda_L} \dot{\Omega}_{2-} \ . \tag{58}$$

Here $|m_{Wc2}| \equiv \mu_o e \sigma_2 R_2/2 > 0$ is the magnitude of the weberian electromagnetic mass for this cylindrical geometry.

By equations (57) and (58) we obtain that $\dot{\omega}$ given by $\dot{\omega} = c_3 e^u/u^{3/2}$ will be proportional to $\dot{\Omega}_{2-}$, no matter how Ω_{2-} changes with time. Integrating $\dot{\omega}$ in time, we obtain the angular velocity of the test electron, $\omega(t)$, as being proportional to the angular velocity $\Omega_{2-}(t)$ of the electrons located at the external resistive cylindrical shell R_2. Both angular velocities, $\omega(t)$ and Ω_{2-}, begin from rest, that is, $\omega(0) = \Omega_{2-}(0) = 0$. When Ω_{2-} reaches its final and constant value Ω_{2-f} at $t = t_f$, then ω

will also reach its final and constant value $\omega_f(\rho)$. The integrated result of equations (57) and (58) yields this final angular velocity of any conduction electron located at a distance ρ from the z axis as given by:

$$\omega_f(\rho) = \frac{e^u}{u^{3/2}} \int_{t=0}^{t_f} c_3 dt = -\frac{2|m_{Wc2}|}{m} \sqrt{\frac{R_o}{\lambda_L}} e^{-R_o/\lambda_L} \frac{e^{\rho/\lambda_L}}{(\rho/\lambda_L)^{3/2}} \Omega_{2-f} \ . \tag{59}$$

The uniform applied magnetic field $\vec{B}_{apl}$ generated by the current flowing in the external shell R_2 is given by:

$$\vec{B}_{apl} = -\mu_o R_2 \sigma_2 \Omega_{2-f} \hat{z} = -\mu_o \sigma_2 V_{2-d} \hat{z} \ , \tag{60}$$

where V_{2-d} is the drifting velocity of the conduction electrons at the resistive shell R_2 given by $V_{2-d} \equiv R_2 \Omega_{2-f}$.

Equations (59) and (60) yield the value of the final angular velocity $\omega_f(R_o)$ of the conduction electrons located at the outer surface R_o of the superconducting cylinder as given by:

$$\omega_f(R_o) \equiv -\frac{2|m_{Wc2}|}{m} \frac{\lambda_L}{R_o} \Omega_{2-f} = -\frac{\mu_o e \sigma_2 R_2}{m} \frac{\lambda_L}{R_o} \Omega_{2-f} = -\frac{e\lambda_L |\vec{B}_{apl}|}{m R_o} \ . \tag{61}$$

Equations (51), (60) and (61) yield the tangential velocity $v_t(R_o)$ of the free electrons at the outer surface of the superconducting cylinder as given by:

$$v_t(R_o) \equiv \omega_f R_o = -\frac{e\lambda_L |\vec{B}_{apl}|}{m} = -\sqrt{\frac{\sigma_2^2 \mu_o}{mn}} V_{2-d} \ . \tag{62}$$

Equation (59) shows that we could then obtain how the angular velocities $\omega_f(\rho)$ of the conduction electrons of the superconductor vary as a function of their distance ρ to the z axis. We now calculate the value of this angular velocity very close to the outer surface of the superconductor, that is, at a distance $\rho = R_o - d$, with $0 \leq d \ll R_o$. Equation (59) yields:

$$\omega_f(R_o - d) = \omega_f(R_o) e^{-d/\lambda_L} \ . \tag{63}$$

This equation indicates that the induced current in the superconductor exists essentially only at its surface, decreasing exponentially as we go inside the body.

The magnetic field inside the superconductor can be obtained by the sum of two fields, namely, (a) the uniform applied magnetic field, $\vec{B}_{apl}$, generated by the external resistive shell; and (b) the magnetic field due to the induced currents in the superconductor. The magnetic field at a distance ρ from the z axis due to the induced currents can be obtained utilizing the first line of equation (9). To this end we consider a cylindrical shell of radius R' such that $\rho < R' < R_o$, thickness dR', with positive charges at rest and negative charges rotating around the z axis with an angular velocity $\omega_f(R')$. We then replace the surface charge density σ of this shell as appearing in equation (9) by $\pm nedR'$, where the upper (lower) sign represents the positive (negative) charges of this shell. We then add the applied magnetic field due to the resistive cylindrical shell of radius R_2 with the magnetic fields due to these positive and negative cylindrical shells of radius R' and thickness dR'. After integration in R' we obtain that the net magnetic field at a distance ρ from the z axis will be then given by:

$$\vec{B}(\rho \leq R_o) = \vec{B}_{apl} - \mu_o ne\hat{z} \int_{R'=\rho}^{R_o} R'\omega_f(R')dR' \ . \tag{64}$$

Utilizing equation (59) we can integrate equation (64). For $\rho \gg \lambda_L$ this integration yields:

$$\vec{B}(\rho \leq R_o) = \vec{B}_{apl} \left[1 - e^{-R_o/\lambda_L} \left(e^{R_o/\lambda_L} - e^{\rho/\lambda_L} \sqrt{\frac{R_o}{\rho}} \right) \right] \ . \tag{65}$$

Finally, the magnetic field very close to the outer surface of the superconductor, at a distance $\rho = R_o - d$, with $0 \leq d \ll R_o$, is given by:

$$\vec{B}(R_o - d) = \vec{B}_{apl} e^{-d/\lambda_L} = -\mu_o R_2 \sigma_2 \Omega_{2-f} \hat{z} e^{-d/\lambda_L} \ . \tag{66}$$

This equation shows that the magnetic field penetrates into a superconductor typically only up to the London penetration depth λ_L. When $d = \lambda_L$ the value of the magnetic field will be decreased to $1/e$ of the value of this field in the external surface R_o of the material, being negligible beyond this point.

5.2 Superconducting Sphere

We now consider a more realistic model of a superconducting sphere inside a resistive cylindrical shell. They have a common center and the cylindrical shell is aligned along the z axis. The superconductor has an inner radius R_i and outer radius R_o, while the cylindrical shell has a radius $R_2 > R_o > R_i$. As usual we will utilize charge neutrality and assume that the positive charges of the superconductor and cylindrical shell remain at rest during all the time. Consider a test electron $q_1 = -e < 0$ of the superconductor located at the polar angle θ from the z axis and at a distance r from the origin, with $R_i \leq r \leq R_o$, figure 8. We assume that at the initial time $t = 0$ all charges are at rest. During the time interval $0 < t < t_f$ an external source generates a given variable azimuthal angular velocity $\Omega_{2-}(t)$ for the electrons located at the external resistive cylindrical shell R_2 until they reach a final given angular velocity Ω_{2-f}. Our goal is to obtain the angular velocity $\omega(r, t)$ induced in the test electron moving around the z axis during the time interval $0 < t < t_f$. This problem will be solved with Weber's electrodynamics utilizing equation (16) coupled with Newton's second law of motion.

Let R' be a variable distance of integration in the superconducting spherical material, with $R_i \leq R' \leq R_o$. Once more there are six groups of charges exerting forces on a specific electron of the superconductor located at a polar angle θ and at a distance r from the origin, namely, (a) the positive and negative charges of the superconductor located in the region $R_i < R' < r$; (b) the positive and negative charges of the superconductor located in the region $r < R' < R_o$; together with (c) the positive and negative charges of the resistive cylindrical shell R_2. The external shell has positive and negative surface charge densities given by $\sigma_{2+} \equiv \sigma_2 > 0$ and $\sigma_{2-} = -\sigma_2$, respectively. The positive and negative volume charge densities of the superconductor will be written as $ne > 0$ and $-ne$, respectively, where $n > 0$ represents the number density of the conduction free electrons and $e = 1.6 \times 10^{-19}$ $C > 0$ is the magnitude of their charge. We now apply equation (8) to these six groups of charges when acting on the conduction electron located at a distance r from the origin.

The azimuthal component of Newton's second law of motion applied to a conduction electron of the superconductor located at the polar angle

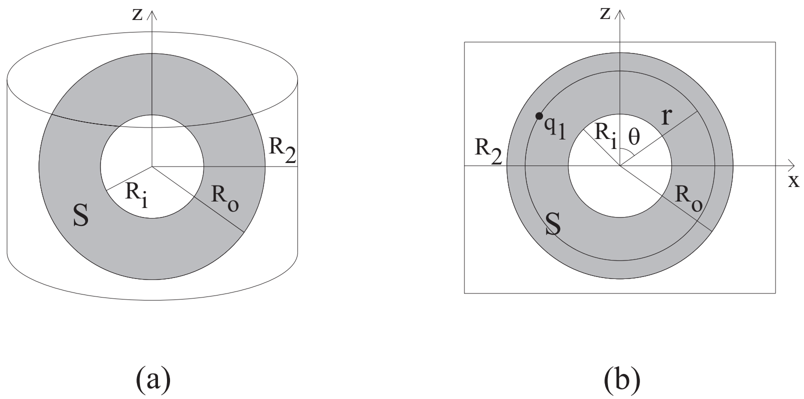

Figure 8: (a) Superconducting sphere S of inner radius R_i and outer radius R_o inside a resistive cylindrical shell of radius R_2 aligned with the z axis. (b) Cross-section of the system in the xz plane with a conduction electron $q_1 = -e < 0$ of the superconductor at a polar angle θ from the z axis and at a distance r from the origin.

θ from the z axis and at a distance r from the origin is given by equation (17), namely:

$$\vec{F} = mr \sin\theta \dot{\omega}(r)\hat{\varphi} \ , \tag{67}$$

where $\vec{F}$ represents the total force acting on it along the azimuthal φ direction.

Following the procedure of Subsection 5.1 we obtain that the azimuthal component of Newton's second law of motion of this test electron is given by:

$$\vec{F} = -\frac{\mu_o e^2 n \sin\theta\hat{\varphi}}{3r^2}\int_{R'=R_i}^{r} R'^4 \dot{\omega}(R')dR' - \frac{\mu_o e^2 n r \sin\theta\hat{\varphi}}{3}\int_{R'=r}^{R_o} R'\dot{\omega}(R')dR'$$

$$-\frac{\mu_o e \sigma_2 r \sin\theta\hat{\varphi}}{2} R_2 \dot{\Omega}_{2-} = mr\sin\theta\dot{\omega}(r)\hat{\varphi} \ . \tag{68}$$

There is a common factor $\sin\theta$ in all terms of this equation which can be canceled out. Therefore all conduction electrons of any spherical shell of radius r inside the superconductor will rotate together around the z axis as a rigid body.

The integral equation (68) has the unknown $\dot{\omega}(r)$. Deriving twice this equation as a function of r, utilizing equation (50) and rearranging the terms yields the following second order differential equation:

$$\lambda_L^2 \rho \frac{d^2\dot{\omega}}{dr^2} + 4\lambda_L^2 \frac{d\dot{\omega}}{dr} - \rho\dot{\omega} = 0 \ , \tag{69}$$

where $\lambda_L \equiv \sqrt{m/(\mu_o n e^2)}$.

We now define the dimensionless magnitude $u_2 \equiv r/\lambda_L$ and utilize the change of variables $v_2(s_2) \equiv s_2^{3/2}\dot{\omega}$. With these definitions equation (69) can then be written as:

$$u_2^2 v_2'' + u_2 v_2' - \left[u_2^2 + \left(\frac{3}{2}\right)^2\right] v_2 = 0 \ , \tag{70}$$

where $v_2' \equiv dv_2/du_2$ and $v_2'' \equiv d^2v_2/du_2^2$. This is the modified Bessel function of order 3/2. Its general solution is given by

$$v_2(u_2) = c_1 I_{3/2}(u_2) + c_2 I_{-3/2}(u_2) \ , \tag{71}$$

where c_1 and c_2 are arbitrary constants, while $I_{3/2}(u_2)$ and $I_{-3/2}(u_2)$ are the modified Bessel functions of orders 3/2 and $-3/2$, respectively.

According to equation (68) we will need to integrate these modified Bessel functions from $u_2 = R_i/\lambda_L$ up to $u_2 = R_o/\lambda_L$. As before, we also have $R_o/\lambda_L > R_i/\lambda_L \gg 1$. We can then approximate $I_{\pm 3/2}(u_2)$ by their asymptotic behaviors given by equation (55).

Therefore

$$\dot{\omega} = \frac{c_1 I_1(u_2) + c_2 K_1(u_2)}{u_2^{3/2}} \approx c_4 \frac{e^{u_2}}{u_2^{3/2}} \ , \tag{72}$$

where we have defined the magnitude c_4 by $c_4 \equiv (c_1 + c_2)/\sqrt{2\pi}$.

We can now apply equation (72) into equation (68). After performing the two integrations and utilizing once more $R_o/\lambda_L > R_i/\lambda_L \gg 1$ we obtain

$$c_4 = -\frac{3|m_{Wc2}|}{m}\sqrt{\frac{R_o}{\lambda_L}} e^{-R_o/\lambda_L} \dot{\Omega}_{2-} \ . \tag{73}$$

Here $|m_{Wc}| \equiv \mu_o e \sigma_2 R_2/2 > 0$ is the magnitude of the weberian electromagnetic mass of the electrons of the cylindrical shell R_2.

Therefore $\dot{\omega} = c_4 e^{u_2}/u_2^{3/2}$ will be proportional to $\dot{\Omega}_{2-}$, no matter how Ω_{2-} changes with time. Integrating in time $\dot{\omega}$ yields $\omega(t)$ proportional to $\Omega_{2-}(t)$. Both angular velocities begin from rest. After Ω_{2-} reaches its final value Ω_{2-f}, then the final value $\omega_f(r)$ of the angular velocity of any conduction electron located at a distance r from the origin will be then given by:

$$\omega_f(r) = \frac{e^{u_2}}{u_2^{3/2}} \int_{t=0}^{t_f} c_4 dt = -\frac{3|m_{Wc2}|}{m} \sqrt{\frac{R_o}{\lambda_L}} e^{-R_o/\lambda_L} \frac{e^{r/\lambda_L}}{(r/\lambda_L)^{3/2}} \Omega_{2-f} \ . \tag{74}$$

The value of the angular velocity of the conduction electrons at the surface of the superconducting spherical shell, $\omega_f(R_o)$, is then given by:

$$\omega_f(R_o) \equiv -\frac{3|m_{Wc2}|}{m} \frac{\lambda_L}{R_o} \Omega_{2-f} = -\frac{3e\mu_o\sigma_2 R_2}{2m} \frac{\lambda_L}{R_o} \Omega_{2-f} = -\frac{3e\lambda_L |\vec{B}_{apl}|}{2mR_o} \ , \tag{75}$$

where $\vec{B}_{apl} = -\mu_o R_2 \sigma_2 \Omega_{2-f} \hat{z}$ is the uniform applied magnetic field generated by the current flowing in the external resistive shell R_2.

Equation (74) shows that we could then obtain how the angular velocities $\omega_f(r)$ of the conduction electrons of the superconductor vary as a function of their distance r to the origin. We now calculate the value of this angular velocity very close to the outer surface of the superconductor, that is, at a distance $r = R_o - d$, with $0 \leq d \ll R_o$. Equation (74) yields:

$$\omega_f(R_o - d) = \omega_f(R_o) e^{-d/\lambda_L} \ . \tag{76}$$

This equation indicates that the induced current in the superconductor exists essentially only at its surface, decreasing exponentially as we go inside the body.

The magnetic field inside the superconductor can be obtained combining the uniform applied field $\vec{B}_{apl}$ generated by the external shell with the field due to the induced currents. The magnetic field at a distance r from the origin due to the induced currents can be obtained utilizing equation (21). To this end we consider a spherical shell of radius R' with

$R_i < R' < R_o$, thickness dR', with positive charges at rest and negative charges rotating around the z axis with an angular velocity $\omega_f(R')$. We then replace in equation (21) the surface charge density σ of this shell by $\pm nedR'$, where the upper (lower) sign represents the positive (negative) charges of this shell. We then add the applied magnetic field due to the resistive cylindrical shell of radius R_2 with the magnetic fields due to these positive and negative spherical shells of radius R' and thickness dR'. After integration in R' the net magnetic field at a distance r from the origin will be then given by:

$$\vec{B}(R_i \leq r \leq R_o) = \vec{B}_{apl} - \frac{\mu_o ne}{3r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta}) \int_{R'=R_i}^{r} R'^4 \omega_f(R') dR'$$

$$- \frac{2}{3}\mu_o ne\hat{z} \int_{R'=r}^{R_o} R' \omega_f(R') dR' \ . \quad (77)$$

Utilizing equation (74) we can integrate equation (77). For $\lambda_L \ll R_i < R_o < R_2$ this integration yields:

$$\vec{B}(\lambda_L \ll R_i \leq r \leq R_o) = \vec{B}_{apl}$$

$$+ \frac{ne^2\mu_o^2\sigma_2 R_2}{2m}\frac{\lambda_L^5}{r^3}\sqrt{\frac{R_o}{\lambda_L}}e^{-R_o/\lambda_L}\Omega_{2-f}(2\cos\theta\hat{r} + \sin\theta\hat{\theta})\left[e^{r/\lambda_L}(r/\lambda_L)^{5/2}\right.$$

$$\left. - \ e^{R_i/\lambda_L}(R_i/\lambda_L)^{5/2}\right]$$

$$+\frac{ne^2\mu_o^2\sigma_2 R_2}{m}\lambda_L^2\sqrt{\frac{R_o}{\lambda_L}}e^{-R_o/\lambda_L}\Omega_{2-f}\hat{z}\left[\frac{e^{R_o/\lambda_L}}{(R_o/\lambda_L)^{1/2}} - \frac{e^{r/\lambda_L}}{(r/\lambda_L)^{1/2}}\right] \ . \quad (78)$$

Therefore, utilizing $\hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$, the magnetic field at the outer surface R_o of the sphere is given by:

$$\vec{B}(R_o) = \frac{3}{2}\mu_o R_2\sigma_2\Omega_{2-f}\sin\theta\hat{\theta} = \frac{3}{2}|\vec{B}_{apl}|\sin\theta\hat{\theta} \ . \quad (79)$$

That is, it is a poloidal magnetic field.

Finally, the magnetic field very close to the outer surface of the superconductor, at a distance $r = R_o - d$, with $0 \leq d \ll R_o$, is given by:

$$\begin{aligned} \vec{B}(R_o - d) = \vec{B}(R_o)e^{-d/\lambda_L} &= \frac{3}{2}\mu_o R_2 \sigma_2 \Omega_{2-f} \sin\theta\hat{\theta}e^{-d/\lambda_L} \\ &= \frac{3}{2}|\vec{B}_{apl}| \sin\theta\hat{\theta}e^{-d/\lambda_L} \ . \end{aligned} \tag{80}$$

This equation shows that the magnetic field penetrates into a superconductor typically up to the London penetration depth λ_L. When $d = \lambda_L$ the value of the magnetic field will be decreased to $1/e$ of the value of this field in the external surface of the material, being negligible beyond this point.

6 General Discussion

If the effects of superconductivity have a classical explanation, it should also be applicable to cases at very low resistivity as Edwards noted already [8, 9]. On the other hand, one finds magnetic fields frozen into collision-less plasmas, but not expelled like it is observed in cases of a fully developed Meissner effect. An explanation for the "critical field" where superconductivity cannot be maintained any longer is beyond the scope of this work.

All calculations presented in this paper were based essentially on Weber's electrodynamics coupled with Newton's second law of motion applied to a conduction electron of the superconductor. In particular, we utilized the component of Weber's force which depends on the accelerations $\vec{a}_2$ and $\vec{a}_1$ of the source and test charges, q_2 and q_1, respectively, as given by equation (2).

We first deduced the London moment showing that when a superconductor rotates relative to an inertial coordinate system with an angular velocity $\vec{\Omega}$, an internal magnetic field is produced given by $\vec{B} = 2m\vec{\Omega}/e$. This effect was deduced utilizing the force exerted by the positive lattice of the superconductor acting on its conduction electrons. This force depends on the time variation of the angular velocity of the conductor. It acts while the material increases its angular velocity. Weber's force has an azimuthal force component proportional to $d\vec{\Omega}/dt$.

In this work we also showed that when a magnetic field is produced around a superconductor, permanent currents will be induced on its surface. The magnetic field produced inside the superconductor by these surface currents will cancel almost exactly the applied magnetic field, except for an extremely small residual value. This behavior represents the essence of the Meissner effect. This fact was deduced utilizing the component of Weber's force which depends on the time variation of the angular velocity of the conduction electrons located at the outer resistive shell and producing the applied magnetic field. That is, we utilized the component of Weber's force which is proportional to $d\vec{\Omega}_{2-}/dt$.

We also deduced the London penetration depth. We showed that the current induced in the superconductor exists essentially only at its surface, decreasing exponentially as we penetrate the superconducting material. The net magnetic field at the outer surface of the superconductor is tangential to its surface.

In the original works of Becker, London *et al.* the masses of the free electrons appearing in the London moment and also in the Meissner effect were introduced *ad hoc* in purely electrodynamic equations. In particular, these masses were introduced in equations describing the magnetic field of the superconducting material. In our treatment, on the other hand, the mass of the test particle was introduced where it really belongs, namely, in Newton's second law of motion. This is the main advantage of the present treatment of these topics.

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