Rebuttal to W. Engelhardt Paper on the Relativistic Explanation of the Sagnac Experiment

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ABSTRACT. In a recent paper published in the Annales de la Fondation Louis de Broglie. W. W. Engelhardt has produced an incorrect explanation of the Sagnac experiment leading him to the false conclusion that light speed does not travel at "c" in vacuum in a small domain of a rotating platform. These two conclusions are contradicted by the correct application of the theory of relativity, as we will show in this short note and, equally important, by a multitude of experiments. In the following, we will give the correct relativistic explanation pointing out where Engelhardt has made his errors in his paper.

RESUME. Dans un article récent publié dans les Annales de la Fondation Louis de Broglie, W. W. Engelhardt a fourni une explication erronée de l'expérience de Sagnac le menant à la conclusion fausse que la vitesse de la lumière ne se déplace pas à "c" dans le vide dans un petit domaine d'une plate-forme tournante. Ces deux conclusions sont contredites par l'application correcte de la théorie de la relativité, comme nous allons le montrer dans cette courte note et, tout aussi important, par une multitude d'expériences. Dans la suite, nous allons donner l'explication relativiste correcte indiquant où Engelhardt a fait ses erreurs dans son papier.

Keywords: Sagnac experiment, light speed frame invariance
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1 INTRODUCTION

One cannot disprove a theory via attempting to apply it incorrectly. This is exactly what W.W. Engelhardt did in a recent paper [1]. In the current rebuttal we will give a succinct explanation of the Sagnac experiment applying the theory in cause (special relativity). We pinpoint the exact mistakes in the Engelhardt paper and we conclude by citing just a sample (of the many) papers that contradict his claims. A tenet of physics is that a theory can be refuted via experiment, there are many experiments that refute the theory espoused in [1].

2 THE SPECIAL RELATIVITY THEORY OF THE SAGNAC EXPERIMENT

We start the explanation by using a practical case, the case of a well known device called fiber optic gyroscope (FOG). An FOG senses changes in orientation, thus performing the function of a mechanical gyroscope. However its principle of operation is instead based on the interference of light which has passed through a coil of optical fiber. Two beams from a laser are injected into the same fiber but in opposite directions. Due to the Sagnac effect, the beam travelling against the rotation experiences a slightly shorter path delay than the other beam. The resulting differential phase shift is measured through interferometry, thus translating one component of the angular velocity into a shift of the interference pattern which is measured.

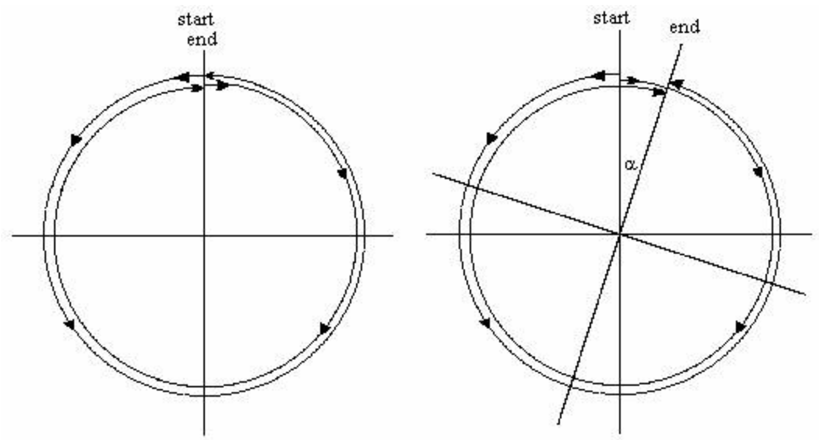

Figure 1. Explanation of the Sagnac experiment

In the following, all calculations are done from the perspective of the inertial frame having the origin coincident with the center of rotation. The right hand side of Fig. 1 illustrates what happens if the loop itself is rotating. The symbol denotes the angular displacement of the loop during the time required for the pulses to travel once around the loop. For any positive value of , the pulse traveling in the same direction as the rotation of the loop must travel a slightly greater distance than the pulse traveling in the opposite direction. As a result, the counter-rotating pulse arrives at the "end" point slightly earlier than the co-rotating pulse. Quantitatively, if we let denote the angular speed of the loop, then the circumferential tangent speed of the end point is . The respective angles traveled by the two light fronts are, in vacuum:

$$\phi_+ = 2\pi + \alpha_+ = \frac{ct_+}{R} \tag{2.1}$$

for the co-rotating front

$$\phi_- = 2\pi - \alpha_- = \frac{ct_-}{R} \tag{2.2}$$

for the counter-rotating front,
where c is the light speed **in vacuum** (as measured in the inertial frame) and:

$$\alpha_+ = \omega t_+ \tag{2.3}$$

for the co-rotating front

$$\alpha_- = \omega t_- \tag{2.4}$$

for the counter-rotating front.

Substituting (2.3) in (2.1) and (2.4) in (2.2) we get:

$$t_+ = \frac{2\pi R}{c - \omega R} \tag{2.5}$$

for the co-rotating front

$$t_- = \frac{2\pi R}{c + \omega R} \tag{2.6}$$

for the counter-rotating front. From (2.5) and (2.6) it follows that:

$$\Delta T_{total} = t_+ - t_- = \frac{4\pi R^2\omega}{c^2 - R^2\omega^2} = \frac{4A\omega}{c^2 - R^2\omega^2} \tag{2.7}$$

where A is the area of the interferometer loop. The above is the exact formula. For $R\omega << c$ we recover the formula used in practice for detecting angular speed via the Sagnac experiment [5,6]:

$$\Delta T_{total} = \frac{4A\omega}{c^2} \tag{2.8}$$

The formula shows that the phase difference between the two counter-propagating light signals is, at low angular speeds, proportional to the angular speed and to the area enclosed by the interferometer loop. The first to perform a ring interferometer experiment aimed at observing the correlation of angular velocity and phase-shift was G. Sagnac [2] in 1913 with the purpose of detecting "the effect of the relative motion of the ether". In 1926 a very ambitious ring interferometry experiment was set up by A. Michelson and H.Gale [3]. The aim was to find out whether the rotation of the Earth has an effect on the propagation of light in the vicinity of the Earth. The Michelson-Gale experiment used a very large ring interferometer, with a perimeter of 1.9 kilometer, so it was large enough to detect the angular velocity of the Earth. The outcome of the experiment was that the angular velocity of the Earth as measured by astronomical methods was confirmed to within measuring accuracy.

The situation is a little more complicated **in the case of using a fiber optic of refraction index** n **:**

$$c_+ = \frac{\frac{c}{n} + \omega R}{1 + \frac{\omega R}{nc}} \qquad c_- = \frac{\frac{c}{n} - \omega R}{1 - \frac{\omega R}{nc}} \tag{2.9}$$

In the above, $\frac{c}{n}$ is the speed of light with respect to the fiber optic and $c_{\pm}$ is the resultant light speed in the inertial frame for the two light wavefronts ("plus" for co-rotating with the fiber and "minus" for counter-rotating). For $n=1$ we recover the vacuum case explained above. Substituting (2.9) into (2.5)-(2.6):

$$t_{+} = \frac{2\pi R}{c_{+} - \omega R} = 2\pi R \frac{1 + \frac{\omega R}{nc}}{\frac{c}{n} - \frac{(\omega R)^2}{nc}} \tag{2.10}$$

for the co-rotating front

$$t_{-} = \frac{2\pi R}{c_{-} + \omega R} = 2\pi R \frac{1 - \frac{\omega R}{nc}}{\frac{c}{n} - \frac{(\omega R)^2}{nc}} \tag{2.11}$$

for the counter-rotating front, resulting into a total time:

$$\Delta T = t_{+} - t_{-} = \frac{4\pi R^2 \omega}{c^2 - R^2\omega^2} \tag{2.12}$$

At this point, Engelhardt incorrectly concludes [1] that $t_{+} - t_{-} = 0$. Engelhardt writes [1]:

"*The obvious consequence of formula (10) above is that coherent beams leaving the beam splitter at the same time in opposite directions will return at the same time as they both travel at the same speed c according to (12). The relativistically correct result is, therefore, not Post's formula (23 P), but simply* $\Delta t' = t'^{+} - t'^{-} = L/c - L/c = 0$ *(13)*"

The error can be traced to two things:

The distances traveled by the two light beams are not the same (L) and the speeds, in the lab frame, are not the same either. The correct expressions for the transition times are given by (2.10) and (2.11) above. Even if $n = 1$ $\Delta t' \neq 0$ contrary to Englehardt's claim.

A confusion between the inertial frame (where the velocity composition formulas are applied and where the phase detector is located) and the rotating frames attached to the fiber optic

In conclusion, one cannot "disprove" special relativity by an incorrect application of the said theory.

Interestingly enough, the outcome of the experiment does not depend on the refraction index of the fiber optic. The SR prediction from expression (2.12) fully coincides with the experimental results [5]. One of the important advantages of FOGs, besides the absence of any moving parts is the fact that the optic cables can be wrapped around k times resulting into an "amplification" of the net effect:

$$\Delta T = k \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \tag{2.13}$$

The resulting phase difference is [4,5]:

$$\Delta S = c\Delta T = kc \frac{4\pi R^2 \omega}{c^2 - R^2 \omega^2} \approx \frac{4\pi R^2 k\omega}{c} \tag{2.14}$$

that is, the effect is the first order in $\frac{\omega}{c}$, "amplified" by the length of the fiber, $L = 2\pi Rk$ and by the radius of the gyroscope, R. Formula (2.14) (non-null phase difference) is confirmed by a large number of experiments, contradicting Engelhardt's conclusions. The error is even more egregious given the large number of experiments that verify the correctness of the application of the theory of special relativity to the Sagnac experiment. Experiments [5-16] are just a small sample of the most recent such confirmations.

3 CONCLUSION

We have rebutted the incorrect conclusions of the Engelhardt paper by showing the correct application of the theory of relativity, confirmed by a multitude of experiments. We have also pointed out the two major errors in the reasoning behind the Engelhardt paper.

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