Quantum Mechanics of Inelastic Collision Processes with De Broglie-Barut Non-Spreading Waves

LAURENT BINDEL

800 rés. « Le parc des Eaux Vives », Palaiseau 91120, France

ABSTRACT. After having shown the interest of the De Broglie-Barut non-spreading wave for the treatment of the Rutherford scattering, which is elastic, the object of this communication is to show that the Barut wave and the associated mathematics allow as easily the treatment of the problem of inelastic scattering by an atom which was resolved in 1926 by Born in a famous paper which lead him to be awarded by the Nobel price. As a byproduct the general differential cross-section of any non-relativistic inelastic process is calculated and showed to be a slight generalization of the expression given by Dirac in his well-known treatise. This communication then constitutes a new chapter of the extension of Barut's program aiming at establishing a new Quantum Theory of single events.

RESUME. Après avoir montré dans une précédente communication l'intérêt de l'onde non-dispersive de De Broglie-Barut pour le traitement de la diffusion de Rutherford, laquelle est élastique, l'objet de cette communication est de montrer que cette ondelette et la mathématique associée permettent tout aussi aisément de traiter le problème de la diffusion inélastique par un atome résolu en 1926 par Born dans un célèbre papier qui lui a valu d'obtenir un prix Nobel. Comme sous-produit, la section efficace différentielle la plus générale pour un processus inélastique dans l'approximation non-relativiste est calculée et montrée être une généralisation légère de l'expression donnée par Dirac dans son fameux traité. Cette communication constitue donc un nouveau chapitre de la reprise et de l'extension du programme de Barut visant à établir une nouvelle Théorie Quantique de la particule individuelle.

1 INTRODUCTION

Does Quantum Mechanics really describe single events or is it just a purely statistical theory? This question went through the years until nowadays without receiving any response than the confrontation of two points of view, synthetized as the Einstein-Bohr debate. With time, this question and its ramifications have been dismissed as pertaining to the philosophical domain, and from now on the majority of professional physicists consider that it doesn't come up anymore. Indeed, the formalisms of this important branch of Physics allow to calculate anything one's need and the comparison of the statistical laws obtained, in general probabilities and cross-sections, with the experimental results is quantitatively from good to excellent. However these successes are obtained by mean of some technical drawbacks like the adiabatic switching of the potential energy and the "$[\delta(\ldots)]^2$ problem" when squaring the scattering amplitude to obtain a cross-section. These drawbacks are not details and are connected with the use of plane waves that fill all space to represent in/out particles.

Following A. O. Barut [1], the point of view here developed is that the question is always a current affair and that Quantum Mechanics with De Broglie-Barut non-spreading waves *in lieu* of plane waves could really describe single events. The conventional Quantum Mechanics is manipulating infinite norm waves forming a purely statistical theory. And this was precisely the intention to impose it to be able to describe single events which leaded to the main interpretative problems:

- the collapses of the wave function induced by the observation as a necessity to explain a single event (for example the arrival of an electron on a luminescent screen); this is the classical interpretation of London & Bauer (1939) that was later acknowledged by Wigner;
- the necessity of the Bohr complementarity principle for explaining without contradiction the connection between the spatio-temporal description and the state of a system, or more technically the connection between the conjugate spatio-temporal and momentum-energy variables: the one disappear when the other is observed.

However, a Quantum Mechanics of single events has to emerge in order to explain the experimental facts, notably the interference or the diffraction at very low incoming flux, without resorting to the mentioned interpretative considerations which are indeed out of Science since non-testable.

By the way, it is interesting to know that, in a small note to the French Academy of Science dated 1925 [2], De Broglie found a progressive wave analogous to the Barut one's but for the scalar Maxwell wave equation.

Strangely, this finding seems to have been forgotten with time and to the author knowledge the resurgence of the concept date to A.O. Barut in the beginning of the nineties (see [1] and the references therein). This is the reason why we can call the wave of this paper the De Broglie-Barut non-spreading wave.

The object of this communication is to show that the joint use of De Broglie-Barut non-spreading wave and the integral form of Schrödinger equation based upon propagators allows to give sense to single events without sacrificing the statistical content emerging from the use of Born postulate. In particular in this communication, we demonstrate in the non-relativistic case of spinless massive particles that:

- a De Broglie-Barut non-spreading wave colliding an atom is after interaction a linear superposition of De Broglie-Barut non-spreading waves of all accessible states of the atom as stated by Born [3];
- the cross-section of this inelastic process calculated by Dirac [5] is recovered along with an explicit summation over all accessible states.

2 SOLUTIONS OF THE SCHRÖDINGER EQUATION

2.1 The Barut wave as a new solution of the free Schrödinger equation

At the beginning of the nineties Asim Barut drew attention to the fact that the free Schrödinger equation possesses a non-dispersive solution [1]. In [6], it is demonstrated how to obtain such a wave and some of its main properties are given:

- propagation without deformation in free space;
- integral representation;
- normalisation;

The non-relativistic De Broglie-Barut non-spreading wave is obtained as the following product:

$$\psi(\vec{r},t) = F(\vec{r}-\vec{v}t)\; e^{\frac{i}{\hbar}(\vec{p}.\vec{r}-E.t)} \tag{1}$$

where F is a solution of both a structure equation $\frac{\partial F}{\partial t}+\vec{v}.\vec{\nabla}F=0$ and a Helmholtz equation $\Delta F+2(\frac{mc}{\hbar})^2 F=0$, $\vec{p}$ is the momentum ($\vec{v}=\vec{p}/m$ is the particle velocity) and E is the total energy. The phase exponential term is the famous De Broglie wave. In the original De Broglie work it is a relativistic phase with $E=\sqrt{(\vec{p}c)^2+(mc^2)^2}$ but in this article we will use the semi-relativistic version $E=mc^2+\frac{\vec{p}^2}{2m}$. The De Broglie-Barut non-spreading wave is the product of a localized progressive wave, i.e. a corpuscular trait, and a de Broglie phase wave, i.e. an undulatory or wave-like trait. The progressive nature of the amplitude F-term is fundamental for obtaining the propagation without deformation property in free space.

The De Broglie-Barut non-spreading wave is of a quite different nature from the super-luminal X-shaped non-spreading solutions of the Schrödinger equation [7] in that the progressive term ($\vec{r}-\vec{v}t$) is located in the amplitude F-term and not in the phase exponential. The De Broglie-Barut non-spreading wave phase is, as in conventional Quantum Mechanics, the De Broglie one's where the only super-luminal 'object' is the phase velocity ($v_\varphi=|\vec{p}|/E$) whose product with the particle velocity (group velocity) equals the square of the light velocity ($v_\varphi.v=c^2$).

In [6], the diffraction by a circular aperture and the elastic scattering by a Coulomb potential (Rutherford scattering) are worked out in detail to show the power of De Broglie-Barut non-spreading wave in giving clear calculations without ambiguities and consistent with the modern experiments where single "quantum dots" (one particle at a time) accumulates to form a statistical diffraction pattern.

The point of view developed here about the De Broglie-Barut non-spreading wave concept is:

- spatiotemporal because it develops in space-time (but without filling all of it uniformly),
- causal because effects are time-ordered from causes,
- non-deterministic because it doesn't make hidden variables intervene (this point was however not shared by A. O. Barut).

2.2 The free propagator of the free Schrödinger equation

As is well known the integral form of the free non-relativistic Schrödinger equation

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi(\vec{r},t) \tag{2}$$

is

$$\psi(\vec{r},t)=\left(\frac{m}{2i\pi\hbar(t-t')}\right)^{3/2}\iiint d^3\vec{r}'\, e^{-\frac{m}{2i\hbar(t-t')}(\vec{r}-\vec{r}')^2}\psi(\vec{r}',t') \tag{3}$$

This defines a propagator:

$$K^{(0)}(\vec{r}-\vec{r}',t-t')=\left(\frac{m}{2i\pi\hbar(t-t')}\right)^{3/2} e^{-\frac{m}{2i\hbar}\frac{(\vec{r}-\vec{r}')^2}{t-t'}} \tag{4}$$

which can be written as:

$$K^{(0)}(\vec{r}-\vec{r}',t-t')=\frac{1}{(2\pi\hbar)^3}\iiint d^3\vec{p}\; e^{\frac{i}{\hbar}\left(\vec{p}.(\vec{r}-\vec{r}')-\frac{\vec{p}^2}{2m}(t-t')\right)} \tag{5}$$

showing that it is a generalized δ-like function:

$$K^{(0)^*}(\vec{r}-\vec{r}',t-t')=K^{(0)}(\vec{r}'-\vec{r},t'-t),$$
$$K^{(0)}(\vec{r}-\vec{r}',0)=\delta^{(3)}(\vec{r}-\vec{r}'),$$
$$\iiint K^{(0)}(\vec{r}-\vec{r}',t-t')\, d^3\vec{r}=1.$$

This $K^{(0)}$ is a propagator without being a Green kernel of the Schrödinger equation since it is a solution of the free Schrödinger equation:

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\Delta_{\vec{r}}\right)K^{(0)}(\vec{r}-\vec{r}',t-t')=0 \tag{6}$$

Using equation (5), it is easy to demonstrate the self-reproducing property for which, as Barut said, “time adjust itself” during the spatial integration:

$$K^{(0)}(\vec{r}_2-\vec{r}_1,t_2-t_1)=\iiint_{]-\infty;+\infty[^3} d^3\vec{x}\; K^{(0)}(\vec{r}_2-\vec{x},t_2-t)\,K^{(0)}(\vec{x}-\vec{r}_1,t-t_1) \tag{7a}$$

The propagation without deformation of a De Broglie-Barut non-spreading wave is demonstrated in [6] and is expressed by the equation:

$$\psi(\vec{r}_2,t_2)=\iiint_{]-\infty;+\infty[^3} d^3\vec{r}_1\; K^{(0)}(\vec{r}_2-\vec{r}_1,t_2-t_1)\;\psi(\vec{r}_1,t_1) \tag{7b}$$

This is so because the De Broglie-Barut non-spreading wave and the free propagator are both solutions of the free Schrödinger equation (2).

2.3 Interpretation of the Barut wave

The De Broglie-Barut non-spreading wave is the product of a progressive Debye wave of velocity v as in $F(\vec{r}-\vec{v}t)$ and a de Broglie wave with a phase velocity $v_\varphi = E/p$ as in $e^{\frac{i}{\hbar}(\vec{p}.\vec{r}-E.t)}$. De Broglie showed that the group velocity of the phase wave is $v_g = dE/dp = v$ that is the velocity of the mobile, here represented by the Debye wave. Since in the relativistic case $v v_\varphi = c^2$ and $v < c$ this means that the phase velocity is supraliminal as is well known. The progressive Debye wave is localized in space around the moving point defined by $\vec{X}(\vec{r},t) = \vec{r} - \vec{v}t$ with a characteristic length of the order of the Compton wavelength ($\lambda_c = h/mc$) for the particle of mass m as it follows from the integral representation established in [6]:

$$\mathbf{F}(\vec{r},t) = \left(4\pi\lambda_0^3\right)^{-\frac{1}{2}} \iint_{S_1} d^2\vec{n}\; e^{2i\pi\frac{\vec{n}}{\lambda_0}.(\vec{r}-\vec{v}t)}$$

where $\lambda_0 = \lambda_c / \sqrt{2}$ is the said characteristic length.
The wave-particle is thus filling all space non-uniformly enabling it to interact with surrounding structures like atoms or screens.

2.4 Normalisation of the Barut wave

The normalisation property $\iiint \left|F(\vec{r},t)\right|^2 d^3\vec{r} = 1$ makes $\left|F(\vec{r},t)\right|^2$ a per volume density that allows to avoid all the difficulties connected with infinite norm wave functions:

- Born [4] recognizing the problem for plane waves ($\psi = c\, e^{\pm ikx}$) considered the mean of the normalization condition: $\lim_{a\to\infty} \frac{1}{2a} \int_{-a}^{+a} \left|\psi(k,x)\right|^2 dx = \lim_{a\to\infty} \frac{c^2}{2a} \int_{-a}^{+a} e^{-ikx} e^{ikx} dx = 1$, and concluded that c=1 in order to have “normalized” eigenfunctions.

- Dirac [5], also recognizing the problem, offered a lengthy discussion of it and finally considered the wave function normalized to a cell and the space being made of an infinite number of cells. This is the modern “box” approach where the wave functions are defined by: $\psi = \frac{1}{V^{3/2}} e^{-i\vec{k}\vec{r}}$ and the normalization condition applied to the cell of volume V: $\iiint\limits_{V} |\psi(\vec{r})|^2 d^3\vec{r} = \frac{1}{V^3} \iiint\limits_{V} d^3\vec{r} = 1$

In both cases there was a deviation from the textbooks definition $\iiint\limits_{all\ space} |\psi(\vec{r})|^2 d^3\vec{r} = 1$. In fact, this normalization condition comes from the solution of static (periodic) problems and is perfectly suited to interpret the formalism in a probabilistic way as discovered by Born.

But when applied to collision (aperiodic) problems the difficulty appeared and is still present nowadays, for example in the treatment of Quantum Electrodynamics questions.

This normalization disease generates a side effect connected with the adiabatic hypothesis employed in collision problems: the necessity to switch on and off the interaction potential energy in order to manipulate asymptotically free plane waves with definite energy-momentum. As has been shown in [6] and will be emphasized in the present paper, this second technical difficulty does not exist when using De Broglie-Barut non-spreading wave *in lieu* of plane ones.

Thus the De Broglie-Barut non-spreading wave cures two connected difficulties rooted to the application of Quantum Mechanics to collision problems.

3 INELASTIC SCATTERING BY AN ATOM

In his famous papers written in 1926 [3], Born noticed that the emerging Quantum Mechanics so far was convincingly explaining stationary problems (periodic systems was the term at that time) but has also to explain collision one's (aperiodic systems was the term at that time). He embarked himself on heavy calculations treating the collision problem by the use of asymptotic expansions of the stationary Helmholtz equation representing the time-independent Schrödinger equation. He then concluded about the interpretation one have to have about the question of indeterminism. In this section I would like to redo Born calculation's with De Broglie-Barut non-spreading waves and the propagator formalism instead of asymptotic (plane and spherical) stationary waves obeying Helmholtz equation.

3.1 Statement of the problem

The problem to be solved is

$$i\hbar\frac{\partial\Psi_{AP}}{\partial t}=\left(H_A(\vec r_A)+H_P(\vec r_P)+U_{AP}(\vec r_A,\vec r_P)\right)\Psi_{AP} \tag{8}$$

where $H_A(\vec r_A)$ is the Hamiltonian of the atom (coordinate $\vec r_A$), $H_P(\vec r_P)=-\dfrac{\hbar^2}{2M}\Delta_{\vec r_P}$ .is the Hamiltonian of the particle of mass M incident on the atom and $U_{AP}(\vec r_A,\vec r_P)$ is the time-independent potential energy between the atom and the incident particle, and $\Psi_{AP}(\vec r_A,\vec r_P,t)$ is the wave function of the atom and the particle.

The initial condition on Ψ_{AP} is the free time evolution factored states written as $\Psi^{(0)}_{AP}(\vec r_A,\vec r_P,t)=\varphi_A(\vec r_A,t).\varphi_P(\vec r_P,t)$ with:

- $i\hbar\dfrac{\partial\varphi_A}{\partial t}=H_A(\vec r_A)\,\varphi_A(\vec r_A,t)$, so that $\varphi_A(\vec r_A,t)=e^{-\frac{i}{\hbar}W^{(n)}t}\varphi_A^{(n)}(\vec r_A)$ if the initial state is the indeterminate initial stationary state $\varphi_A^{(n)}(\vec r_A)$ for which $H_A(\vec r_A)\,\varphi_A^{(n)}(\vec r_A)=W^{(n)}\varphi_A^{(n)}(\vec r_A)$ (stationary Schrödinger problem).
- $i\hbar\dfrac{\partial\varphi_P}{\partial t}=H_P(\vec r_P)\,\varphi_P(\vec r_P,t)$, so that $\varphi_P^{(\vec p_i)}(\vec r_P,t)=e^{-\frac{i}{\hbar}(E_it-\vec p_i.\vec r_P)}F(\vec r_P-\vec v_it)$ is the initial De Broglie-Barut non-spreading wave for which $\vec p_i=M\,\vec v_i$ and $E_i=\dfrac{\vec p_i}{2M}+Mc^2$ is the total semi-relativistic energy.

So the atom and the particle are described by a time dependent Schrödinger equation instead of a static one as in Born papers [4]. The atom is supposed to be infinitely heavy so that no recoil will be considered.

3.2 Calculation of the total wave function at first order

Doing a unitary transformation on equation (8) in order to remove the Hamiltonian of the atom $\Phi_{AP}(\vec{r}_A,\vec{r}_P,t)=e^{\frac{i}{\hbar}H_A(\vec{r}_A)t}\Psi_{AP}(\vec{r}_A,\vec{r}_P,t)$ we are left with ($H_A(\vec{r}_A)$ and $H_P(\vec{r}_P)$ commute):

$$i\hbar\frac{\partial\Phi_{AP}}{\partial t}=\left(H_P(\vec{r}_P)+\hat{U}_{AP}(\vec{r}_A,\vec{r}_P,t)\right)\Phi_{AP} \tag{9a}$$

where

$$\hat{U}_{AP}(\vec{r}_A,\vec{r}_P,t)=e^{+\frac{i}{\hbar}H_A(\vec{r}_A)t}U_{AP}(\vec{r}_A,\vec{r}_P)\ e^{-\frac{i}{\hbar}H_A(\vec{r}_A)t} \tag{9b}$$

The equation (9a) is a Schrödinger equation of motion of a particle in the time-dependent potential energy $\hat{U}_{AP}(\vec{r}_A,\vec{r}_P,t)$. Then, according to §IV.A in [6], the integral form of (9a) is:

$$\Phi_{AP}(\vec{r}_A,\vec{r}_P,t)=\iiint d^3\vec{\lambda}\ K^{(\hat{U})}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)\ \Phi_{AP}(\vec{\lambda},t_0) \tag{10a}$$

with

$$K^{(\hat{U})}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)=K^{(0)}(\vec{r}_P,t;\vec{\lambda},t_0)e^{-\frac{i}{\hbar}\int_{t_0}^{t}\tilde{U}(\vec{r}_A;\vec{\lambda},\tau,t_0)\,d\tau} \tag{10b}$$

and

$$\tilde{U}_{AP}(\vec{r}_A,\vec{\lambda},\tau,t_0)=e^{+\frac{i}{\hbar}\hat{E}_c(\vec{\lambda})(\tau-t_0)}\hat{U}_{AP}(\vec{r}_A,\vec{\lambda},\tau)\ e^{-\frac{i}{\hbar}\hat{E}_c(\vec{\lambda})(\tau-t_0)} \tag{10c}$$

where $\hat{E}_c(\vec{\lambda})=-\frac{\hbar^2}{2M}\Delta_{\vec{\lambda}}$ is the kinetic energy operator.

Then, developing the exponential in $K^{(\hat{U})}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)$, provided that $\left|\int_{t_0}^{t}\tilde{U}(\vec{r}_A;\vec{\lambda},\tau,t_0)\,d\tau\right|<<\hbar$, it follows from §IV.B of [6] that

$$K^{(\hat{U})}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)=K^{(0)}(\vec{r}_P,t;\vec{\lambda},t_0)+K^{(1)}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)+... \tag{11a}$$

with

$$K^{(1)}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)=-\frac{i}{\hbar}\int_{t_0}^{t}d\tau\iiint d^3\vec{x}\ K^{(0)}(\vec{r}_P,t;\vec{x},\tau)\ \hat{U}(\vec{r}_A;\vec{x},\tau)K^{(0)}(\vec{x},\tau;\vec{\lambda},t_0) \tag{11b}$$

The wave function can also be developed as:

$$\Phi_{AP}(\vec{r}_A,\vec{r}_P,t)=\Phi_{AP}^{(0)}(\vec{r}_A,\vec{r}_P,t)+\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)+...$$

Then we get the infinite system of integral equations:

$$\Phi_{AP}^{(0)}(\vec{r}_A,\vec{r}_P,t)=\iiint d^3\vec{\lambda}\ K^{(0)}(\vec{r}_P,t;\vec{\lambda},t_0)\ \Phi_{AP}^{(0)}(\vec{r}_A,\vec{\lambda},t_0) \quad (12a)$$

$$\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)=\iiint d^3\vec{\lambda}\ K^{(1)}(\vec{r}_A;\vec{r}_P,t;\vec{\lambda},t_0)\ \Phi_{AP}^{(0)}(\vec{r}_A,\vec{\lambda},t_0) \quad (12b)$$

…

The equation (12a) is an identity which means that at zero order there is no interaction between the incident particle and the atom. The second one (12b) gives the effect on the total wave function of the interaction of the incident particle and the atom at the first order of perturbation.

Injecting equation (11b) into equation (12b) and using the transport property $\Phi_{AP}^{(0)}(\vec{r}_A,\vec{x},\tau)=\iiint d^3\vec{\lambda}\ K^{(0)}(\vec{x},\tau;\vec{\lambda},t_0)\ \Phi_{AP}^{(0)}(\vec{r}_A,\vec{\lambda},t_0)$ which is the same as equation (12a) with variables $(\vec{x},\tau)$ instead of $(\vec{r}_P,t)$, we are left with:

$$\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)=-\frac{i}{\hbar}\int_{t_0}^{t}d\tau\iiint d^3\vec{x}\ K^{(0)}(\vec{r}_P,t;\vec{x},\tau)\ \hat{U}(\vec{r}_A,\vec{x},\tau)\,\Phi_{AP}^{(0)}(\vec{r}_A,\vec{x},\tau)$$

Recalling that $\Phi_{AP}(\vec{r}_A,\vec{r}_P,t)=e^{\frac{i}{\hbar}H_A(\vec{r}_A)t}\Psi_{AP}(\vec{r}_A,\vec{r}_P,t)$, it follows from equation (9b) that $\hat{U}_{AP}(\vec{r}_A,\vec{x},\tau)\Phi_{AP}^{(0)}(\vec{r}_A,\vec{x},\tau)=e^{+\frac{i}{\hbar}H_A(\vec{r}_A)\tau}U_{AP}(\vec{r}_A,\vec{x})\ \Psi_{AP}^{(0)}(\vec{r}_A,\vec{x},\tau)$ with $\Psi_{AP}^{(0)}(\vec{r}_A,\vec{x},\tau)=e^{-\frac{i}{\hbar}W^{(n)}\tau}\varphi_A^{(n)}(\vec{r}_A)\ \varphi_P^{(\vec{p}_i)}(\vec{x},\tau)$ according to the discussion at the beginning of the section where the atom is initially (far away in the past) supposed to be in the (n) state.

Now, following Born [4] and due to the completeness of the discrete $\varphi^{(m)}(\vec{r}_A)$ atomic states, we can expand:

$$U_{AP}(\vec{r}_A,\vec{x})\varphi^{(n)}(\vec{r}_A)=\sum_m U_{nm}(\vec{x})\ \varphi^{(m)}(\vec{r}_A) \quad (13)$$

which means that $U_{nm}(\vec{x})=\iiint d^3\vec{r}_A\ \varphi^{(m)*}(\vec{r}_A)U_{AP}(\vec{r}_A,\vec{x})\varphi^{(n)}(\vec{r}_A)$ since the (discrete) atomic states are supposed to form a complete and orthogonal set.

This leads to:

$$\hat{U}_{AP}(\vec{r}_A,\vec{x},\tau)\Phi_{AP}^{(0)}(\vec{r}_A,\vec{x},\tau)=\sum_m e^{-\frac{i}{\hbar}(W_n-W_m)\tau}U_{nm}(\vec{x})\ \varphi^{(m)}(\vec{r}_A)\varphi_P^{(\vec{p}_i)}(\vec{x},\tau)$$

and so

$$\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)=-\frac{i}{\hbar}\int_{t_0}^{t}d\tau\iiint d^3\vec{x}\;K^{(0)}(\vec{r}_P,t;\vec{x},\tau)\sum_m e^{-\frac{i}{\hbar}(W_n-W_m)\tau}U_{nm}(\vec{x})\;\varphi^{(m)}(\vec{r}_A)\,\varphi_P^{(\vec{p}_i)}(\vec{x},\tau)$$

which is the same as:

$$\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)=-\frac{i}{\hbar}\sum_m \varphi^{(m)}(\vec{r}_A)\int_{t_0}^{t}d\tau\, e^{-\frac{i}{\hbar}(W_n-W_m)\tau}\iiint d^3\vec{x}\;K^{(0)}(\vec{r}_P,t;\vec{x},\tau)\;U_{nm}(\vec{x})\;\varphi_P^{(\vec{p}_i)}(\vec{x},\tau)$$

Then, analogously to Born ansatz (13) for atomic states, the continuous De Broglie-Barut non-spreading wave functions are developed as:

$$U_{nm}(\vec{x})\;\varphi_P^{(\vec{p}_i)}(\vec{x},\tau)=\iiint d^3\vec{p}\;\bigcup_{nm}(\vec{p},\vec{p}_i;\tau)\;\varphi_P^{(\vec{p})}(\vec{x},\tau) \qquad (14)$$

where $\varphi_P^{(\vec{p})}(\vec{x},\tau)=F(\vec{x}-\frac{\vec{p}}{M}\tau)\;e^{\frac{i}{\hbar}(\vec{p}.\vec{x}-E\tau)}$ is the De Broglie-Barut non-spreading wave of momentum $\vec{p}$ and energy E. It is to be noted that there is a residual time dependence in the super-matrix $\bigcup_{nm}(\vec{p},\vec{p}_i;\tau)$.

Then, with (14) $\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)$ becomes:

$$\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)=-\frac{i}{\hbar}\sum_m \varphi^{(m)}(\vec{r}_A)\int_{t_0}^{t}d\tau\, e^{-\frac{i}{\hbar}(W_n-W_m)\tau}\iiint d^3\vec{p}\;\bigcup_{nm}(\vec{p},\vec{p}_i;\tau)\;\varphi_P^{(\vec{p})}(\vec{r}_P,t)$$

where the free transport property of the De Broglie-Barut non-spreading wave (7b) is used to perform the spatial integral.

Now, the super-matrix $\bigcup_{nm}(\vec{p},\vec{p}_i;\tau)$ has to be worked out. To this end both sides of the equation (14) are multiplied by $\varphi_P^{(\vec{q})*}(\vec{x},\tau)$ and integrated on $\vec{x}$:

$$\iiint d^3\vec{x}\;\varphi_P^{(\vec{q})*}(\vec{x},\tau)\,U_{nm}(\vec{x})\,\varphi_P^{(\vec{p}_i)}(\vec{x},\tau)=\iiint d^3\vec{p}\;\bigcup_{nm}(\vec{p},\vec{p}_i;\tau)\iiint d^3\vec{x}\;\varphi_P^{(\vec{q})*}(\vec{x},\tau)\;\varphi_P^{(\vec{p})}(\vec{x},\tau)$$

Recalling that De Broglie-Barut non-spreading waves are localized and progressive, there must be a point in space, let's call it $\vec{x}_c$, at which the non-spreading wave transfers energy-momentum to the atom. If it were not the case then we should be prepared to think about a kind of trans-location: a particle disappear at a point and reappear at another one. The conservation of energy-momentum would then not be strictly conserved at a collision.

Then the Debye waves are located at their common intersection $\vec{x}_c$ and:

$$|F(\vec{x}_c)|^2\iiint d^3\vec{x}\;e^{-\frac{i}{\hbar}\left[(\vec{q}-\vec{p}_i).\vec{x}-(\frac{\vec{q}^2}{2M}-\frac{\vec{p}_i^2}{2M}).\tau\right]}U_{nm}(\vec{x})=|F(\vec{x}_c)|^2\iiint d^3\vec{p}\;\bigcup_{nm}(\vec{p},\vec{p}_i;\tau)\;(2\pi\hbar)^3\,e^{\frac{i}{\hbar}(\frac{\vec{q}^2}{2M}-\frac{\vec{p}^2}{2M}).\tau}\;\delta^{(3)}(\vec{q}-\vec{p})$$

so, the super-matrix appears as proportional to the Fourier transform of U_{nm}:

$$\bigcup_{nm}(\vec{q},\vec{p}_i;\tau)=\frac{1}{(2\pi\hbar)^3}e^{+\frac{i}{\hbar}(\frac{\vec{q}^2}{2M}-\frac{\vec{p}_i^2}{2M}).\tau}\iiint d^3\vec{x}\;e^{-\frac{i}{\hbar}(\vec{q}-\vec{p}_i).\vec{x}}\;U_{nm}(\vec{x})\;=\;e^{+\frac{i}{\hbar}(\frac{\vec{q}^2}{2M}-\frac{\vec{p}_i^2}{2M}).\tau}.\mathfrak{F}[U_{nm}](\vec{q}-\vec{p}_i)$$

Injecting this expression into $\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)$ it comes:

$$\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t) = -\frac{i}{\hbar}\sum_m \varphi^{(m)}(\vec{r}_A)\iiint d^3\vec{p}\;\; \mathfrak{F}[U_{nm}](\vec{p}-\vec{p}_i)\; \varphi_P^{(\vec{p})}(\vec{r}_P,t)\int_{t_0}^{t} d\tau\, e^{-\frac{i}{\hbar}(W_n - W_m + \frac{\vec{p}_i^2}{2M} - \frac{\vec{p}^2}{2M})\tau} \tag{15}$$

We shall deduce from this the asymptotic wave function and the differential cross-section of the inelastic process, both at order one of perturbation expansion.

3.3 The asymptotic wave function

Taking the limit of (15) at both sides of the time axis and noting $\Phi_\infty^{(1)}(\vec{r}_A,\vec{r}_P) = \lim_{t\to+\infty}\Phi_{AP}^{(1)}(\vec{r}_A,\vec{r}_P,t)$ and $\varphi^{(\vec{p})}(\vec{r}_P,+\infty) = \lim_{t\to+\infty}\varphi_P^{(\vec{p})}(\vec{r}_P,t)$, we have to evaluate:

$$\Phi_\infty^{(1)}(\vec{r}_A,\vec{r}_P) = -2i\pi\sum_m \varphi^{(m)}(\vec{r}_A)\iiint d^3\vec{p}\;\; \mathfrak{F}[U_{nm}](\vec{p}-\vec{p}_i)\; \varphi_P^{(\vec{p})}(\vec{r}_P,+\infty)\; \delta(W_n - W_m + \frac{\vec{p}_i^2}{2M} - \frac{\vec{p}^2}{2M})$$

The last step consists in the integration on the modulus p of $\vec{p} = p.\vec{\Omega}$ ($d^3\vec{p} = p^2 dp\, d^2\vec{\Omega}$) by using two well-known properties of the Dirac distribution:

$$\delta\left(W_n - W_m + \frac{\vec{p}_i^2}{2M} - \frac{\vec{p}^2}{2M}\right) = 2M\,\delta\left(p^2 - P_{nm}^2\right) = \frac{M}{P_{nm}}\delta\left(p - P_{nm}\right) \quad where\ P_{nm} = \sqrt{\vec{p}_i^2 + 2M(W_n - W_m)}$$

since p, as a modulus, cannot be negative. The conservation of energy imposes W_m to be such that $\vec{p}_i^2 + 2M(W_n - W_m) \geq 0$. This defines m* ($W_{m^*} \leq W_n + \vec{p}_i^2/2M$).

The asymptotic atom-particle wave function is then:

$$\Phi_\infty^{(1)}(\vec{r}_A,\vec{r}_P) = -2i\pi M \sum_{m<m^*} P_{nm}\iint_{4\pi} d^2\vec{\Omega}\; \mathfrak{F}[U_{nm}](P_{nm}\vec{\Omega} - \vec{p}_i)\; \varphi_P^{(P_{nm}\vec{\Omega})}(\vec{r}_P,+\infty)\,\varphi^{(m)}(\vec{r}_A) \tag{16}$$

The meaning of this result is the following: a collision between a particle represented by a De Broglie-Barut non-spreading wave and an atom in a stationary state leads to a final state which is a linear superposition of all the stationary states of the atom weighted by a De Broglie-Barut non-spreading wave with a momentum satisfying the energy conservation law, the proportionality coefficient being proportional to the Fourier transform of the matrix element of the potential energy between the initial and the final atomic state.

This result is obtained much more easily than in Born original paper [4]. The famous author made use of asymptotic analysis of the Helmholtz form of the time independent Schrödinger equation and discussed at length the problem of manipulating non-normalized wave function. Here the use of localized and normalized De Broglie-Barut non-spreading wave functions simplify the calculation and allows to avoid the technical difficulties associated with the use of plane or spherical waves that fill all space.

Our result compares well with that of Born which he wrote as [3]:

$$\psi^1_{n\tau}(x,y,z;q_k)=\sum_m \iint_{\alpha x+\beta y+\gamma z>0} d\omega\, \Phi_{nm}(\alpha,\beta,\gamma)\,\sin\left[k_{nm}\,(\alpha x+\beta y+\gamma z+\delta)\right]\varphi^0_m(q_k)$$

In Born's expression, xyz represents the colliding particle coordinates and q_k the atom internal coordinates. The sine term represents the colliding particle time-independent final wave function. The k_{nm} term is the wave number associated with the n→m transition and is essentially our $P_{nm}/\hbar$. The Φ_{nm} term represent what he called the yield function and interpreted in a footnote as being connected (after squaring) to the probability for the incident particle to be thrown in the direction α, β, γ with the phase change δ after the collision.

Due to the use of non-normalized plane waves in Born calculation the normalization constant ($-2i\pi M P_{nm}$) is not recovered.

3.4 *The differential cross-section*

Multiplying equation (15) by its adjoint, integrating on $\vec{r}_A$ and $\vec{r}_P$, and using the normalization property of:

- atomic states: $\iiint d^3\vec{r}_A\;\; \varphi^{(m)*}(\vec{r}_A)\,\varphi^{(m')}(\vec{r}_A) = \delta_{m\,m'}$
- De Broglie-Barut non-spreading waves:

$$\iiint d^3\vec{r}_P\;\; \varphi_P^{(\vec{p})*}(\vec{r}_P,t)\;\varphi_P^{(\vec{q})}(\vec{r}_P,t) = (2\pi\hbar)^3\left|F(\vec{r}_C)\right|^2\delta^{(3)}(\vec{p}-\vec{q})$$

where $\vec{r}_C$ is the common point of in and out trajectories, we are left with:

$$\iiint d^3\vec{r}_A\iiint d^3\vec{r}_P\left|\Phi^{(1)}_{AP}(\vec{r}_A,\vec{r}_P,t)\right|^2=\frac{(2\pi\hbar)^3\left|F(\vec{r}_C)\right|^2}{\hbar^2}\sum_m\iiint d^3\vec{p}\left|\mathfrak{F}[U_{nm}](\vec{p}-\vec{p}_i)\right|^2\left|\int_{t_0}^{t}d\tau\, e^{-\frac{i}{\hbar}(W_n-W_m+\frac{\vec{p}_i^2}{2M}-\frac{\vec{p}^2}{2M})\tau}\right|^2$$

This quantity is the scattering probability. Dividing it by $(t-t_0)$ we get the probability per unit time. Dividing it again by $\left|\mathbf{F}(\vec{r}_C)\right|^2\left|\vec{v}_i\right|$ i.e. the flux of incoming particles and taking the limits $t_0\to-\infty, t\to+\infty$, this defines the total cross-section σ of the process under consideration:

$$\sigma_{n,p_i} = \lim_{t\to+\infty,t_0\to-\infty} \frac{(2\pi)^3\hbar}{(t-t_0).|\vec{v}_i|} \sum_m \iiint d^3\vec{p}\left|\mathfrak{F}[U_{nm}](\vec{p}-\vec{p}_i)\right|^2 \left|\int_{t_0}^{t} d\tau\, e^{-\frac{i}{\hbar}(W_n-W_m+\frac{\vec{p}_i^2}{2M}-\frac{\vec{p}^2}{2M})\tau}\right|^2$$

According to [6] section 4.4,

$$\lim_{t_0\to-\infty,t\to+\infty} \frac{1}{t-t_0}\left|\int_{t_0}^{t} d\tau\, e^{-\frac{i}{\hbar}(W_n-W_m+\frac{p_i^2}{2M}-\frac{p^2}{2M})\tau}\right|^2 = 2\pi\hbar\,\delta\left(W_n-W_m+\frac{p_i^2}{2M}-\frac{p^2}{2M}\right)$$

Then,

$$\sigma_{n,p_i} = \frac{(2\pi)^4\hbar^2}{|\vec{v}_i|}\sum_m \iiint d^3\vec{p}\left|\mathfrak{F}[U_{nm}](\vec{p}-\vec{p}_i)\right|^2 \delta(W_n-W_m+\frac{\vec{p}_i^2}{2M}-\frac{\vec{p}^2}{2M})$$

The last step consists in the integration of the modulus p of $\vec{p}=p.\vec{\Omega}$ ($d^3\vec{p}=p^2dp\,d^2\vec{\Omega}$) by using the same argument as in section 3.3:

$$\delta\left(W_n-W_m+\frac{\vec{p}_i^2}{2M}-\frac{\vec{p}^2}{2M}\right)=\frac{M}{P_{nm}}\delta\left(p-P_{nm}\right) \quad where\ P_{nm}=\sqrt{\vec{p}_i^2+2M(W_n-W_m)}$$

The conservation of energy imposes W_m to be such that $\vec{p}_i^2+2M(W_n-W_m)>0$. This defines m*.

Writing $\frac{1}{|\vec{v}_i|}=\frac{M}{p_i}$ the differential cross-section of the inelastic process under consideration is then given by the expression:

$$\frac{d\sigma_{n,p_i}}{d\Omega}=4\pi^2M^2h^2\sum_{m<m^*}\frac{P_{nm}}{p_i}\left|\mathfrak{F}[U_{nm}](P_{nm}\vec{\Omega}-\vec{p}_i)\right|^2 \qquad (17)$$

To the sum over final atomic states, here restrained to a finite set because of energy conservation, this equation is exactly the same as the one calculated by Dirac in [5] (section 49 Eq. (15) page 193). The equation (17) is a generalization of Dirac result in that the summation over all energetically accessible states is here explicitly obtained by our method.

4 CONCLUSIONS

In this communication we showed how the use of the De Broglie-Barut non-spreading wave and the propagator approach considerably simplify the calculation of some early results of Quantum Mechanics. Indeed, in the case of inelastic scattering, there is no need to work out the sophisticated asymptotic analysis of the Helmholtz form of the time-independent Schrödinger equation. Instead, a pure quantum calculation with time-dependent normalized

state wave functions is done and makes the energy conservation appears at the end when taking the time(s) limit(s).
Our calculation shows that a De Broglie-Barut non-spreading wave is transformed into a sum of De Broglie-Barut non-spreading waves during an inelastic collision process weighted by the Fourier transform of the potential energy operator taken between the initial and final atomic state. In the classical description of such process (following the Born and Dirac asymptotic approach), the asymptotic scattered wave is imposed to be a spherical outgoing one with an angular dependence found to be (after squaring) proportional to the scattering probability amplitude. This is precisely this spherical (but anisotropic) outgoing wave after collision that lead to the necessity of its collapse when one wish to interpret the arrival of a single dot on a screen located behind the collision region for very low incoming flux experiments. On another side, the De Broglie-Barut non-spreading wave is a localized object, so that once a particular outcome has been probabilistically selected, a single dot is to be observed on a detection screen located behind the collision region (see Eq. (16)). Then, the De Broglie-Barut non-spreading wave might be a candidate for explaining why at low intensity the measurements always reveal the corpuscular trait of elementary particles (i.e. a dot on a screen) and at the same time the undulatory trait of elementary particles (i.e. the distribution of dots on the screen). This, in a strict unitary way in accordance with Quantum Mechanics principles.
The De Broglie-Barut non-spreading wave offers a clear representation of a mobile (or wave-particle) and its interaction with surrounding structure like atoms. This non-spreading wave possess all the required qualities to pretend to be the representation of quantum objects. This is due to the fact that the wave-particle duality is engraved in the De Broglie-Barut non-spreading wave and not in the "apparatus" used to measure the result of an experiment. Indeed, being the product of two waves, a Debye one which represent the particle side and a De Broglie one which represents the wave side, the De Broglie-Barut non-spreading wave is a quantum object that is altogether suited for being the representation of a single quantum "particle".

Acknowlegment

The author wishes to thank his referee for constructive comments that encourage him to explore in more depth the Barut mathematical formulation of quantum mechanics.

References

[1] Asim O. BARUT, ″Diffraction and interference of single de Broglie wavelets – Deterministic wave mechanics″, in Courants, Amers, Ecueils en Microphysique, Fondation L. de Broglie 1993, p. 27.

[2] Louis De Broglie, ″Sur la fréquence propre de l'électron″, C.R. Académie des Sciences, 1925, p. 498.

[3] Max BORN, ″On the Quantum Mechanics of Collisions″, [Preliminary communication], originally published under the title ”Zur Quantenmechanik der Stossvorgänge”, Zeitschrift für Physik, 37, 863-67 (1926) translation into French in ″Sources et évolution de la physique quantique – Textes fondateurs″ by J. LEITE LOPES and B. ESCOUBES, Masson (1995), or in English by J.A. WHEELER and W.H. ZUREK, 1981.

[4] Max BORN, ″Quantum Mechanics of Collision Processes″, translated by D. BROOKS (www.physics.rutgers.edu/~dbrooks/Born/Born.pdf).

[5] Paul A. M. DIRAC, ″The Principles of Quantum Mechanics″, Oxford University Press, Fourth Edition, 1958.

[6] Laurent BINDEL, ″Théorie non-relativiste d'une particule individuelle″, Annales de la Fondation Louis de Broglie, volume 37 N°2, 2012, p. 143-171 (in French, the Englich translation is available on request).

[7] Michel RACHED and Erasmo RECAMI, "Soliton-like solutions to the ordinary Schroedinger equation", arXiv:1008.3087v1 [quant-ph] 17 Aug 2010.

(Manuscrit reçu le 29 juin 2015)