Rotation of a Superconductor Due to Electromagnetic Induction Using Weber's Electrodynamics

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ABSTRACT. We consider a resistive solenoid around a type-I superconductor which is free to turn around its axis. When a variable current flows through the solenoid, a current is induced in the superconductor. The free electrons of the superconductor begin to rotate in one direction, while the positive lattice of the superconductor rotates in the opposite direction. Moreover, there will be no net angular momentum given to the superconducting material as a whole (composed of the positive lattice plus the free electrons), as this net or total angular momentum will be always zero. The same conclusion takes place by replacing the superconductor with a resistive material.

Key Words: Superconductivity, Weber's electrodynamics, induced current.

PACS: 41.20.-q (applied classical electromagnetism), 74.20.-z (theories and models of superconducting state).

1 Weber's Force

In this work we utilize Weber's electrodynamics to study the rotation of a type-I superconductor due to electromagnetic induction. This paper continues our study of superconductivity utilizing Weber's electrodynamics.[1]

We will consider here the translational motion of the conduction electrons of the superconductor induced by an external variable current in a surrounding solenoid and the opposite translational motion of the positive lattice of the superconductor also induced by the same external source of variable current. The effect discussed here is different from the mechanical effect acting on a long cylindrical bar of iron accompanying the act of magnetization predicted by O. W. Richardson in 1908 and investigated experimentally by Einstein, de Haas, Stewart and others.[2] Richardson's effect happens through the orientation of the magnetic moment of iron's atoms due to the application of an external magnetic field, while the effect discussed here is related to the macroscopic translational motion of the conduction electrons through the positive lattice of the superconductor material.

Consider an inertial frame of reference S with origin O and a point particle 1 electrified with charge q_1 located at point P_1. Let $\vec{r}_1 = x_1\hat{x} + y_1\hat{y} + z_1\hat{z}$ be its position vector relative to the origin O of S, while $\vec{r}_2 = x_2\hat{x}+y_2\hat{y}+z_2\hat{z}$ is the position vector of another point particle 2 electrified with charge q_2 and located at point P_2. The velocities and accelerations of these charges in S are given by, respectively: $\vec{v}_1 = d\vec{r}_1/dt$, $\vec{v}_2 = d\vec{r}_2/dt$, $\vec{a}_1 = d\vec{v}_1/dt = d^2\vec{r}_1/dt^2$ and $\vec{a}_2 = d\vec{v}_2/dt = d^2\vec{r}_2/dt^2$.

The position vector pointing from q_2 to q_1 will be defined by $\vec{r}_{12} \equiv \vec{r}_1 - \vec{r}_2$. We also define in this reference frame the relative velocity vector $\vec{v}_{12}$ and the relative acceleration vector $\vec{a}_{12}$ by the following expressions: $\vec{v}_{12} \equiv \vec{v}_1 - \vec{v}_2$ and $\vec{a}_{12} \equiv \vec{a}_1 - \vec{a}_2$. The charges are separated by a distance $r_{12} \equiv \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$. The unit vector $\hat{r}_{12}$ pointing from 2 to 1 can be written as $\hat{r}_{12} \equiv (\vec{r}_1 - \vec{r}_2)/r$.

In the International System of Units and in vector notation Weber's force $\vec{F}_{21}$ exerted by particle 2 on particle 1 located at point P_1 is given by:[3]

[1] [1] and [2].
[2] [3], [4], [5], [6, p. 244], [7] and [8].
[3] [9], [10], [11], [12], [13], [14], [15] and [16].

$$\vec{F}_{21} = -\vec{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{12}}{r_{12}^2}\left(1 - \frac{\dot{r}_{12}^2}{2c^2} + \frac{r_{12}\ddot{r}_{12}}{c^2}\right)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{12}}{r_{12}^2}\left[1 + \frac{1}{c^2}\left(\vec{v}_{12}\cdot\vec{v}_{12} - \frac{3}{2}\left(\hat{r}_{12}\cdot\vec{v}_{12}\right)^2 + \vec{r}_{12}\cdot\vec{a}_{12}\right)\right] . \quad (1)$$

Here $\vec{F}_{12}$ is the force exerted by q_1 on q_2 located at point P_2. Moreover, $\dot{r}_{12} \equiv dr_{12}/dt$ is the relative *radial* velocity between them, while $\ddot{r}_{12} \equiv d\dot{r}_{12}/dt = d^2 r_{12}/dt^2$ is the relative *radial* acceleration between the charges. In vector notation these magnitudes can be written as:

$$\dot{r}_{12} = \frac{dr_{12}}{dt} = \hat{r}_{12}\cdot\vec{v}_{12} \ , \quad (2)$$

and

$$\ddot{r}_{12} = \frac{d\dot{r}_{12}}{dt} = \frac{d^2 r_{12}}{dt^2} = \frac{\vec{v}_{12}\cdot\vec{v}_{12} - (\hat{r}_{12}\cdot\vec{v}_{12})^2 + \vec{r}_{12}\cdot\vec{a}_{12}}{r_{12}} . \quad (3)$$

The constant $c \equiv 1/\sqrt{\mu_o\varepsilon_o} = 2.998 \times 10^8\ m/s$ in equation (1) is the ratio of electromagnetic and electrostatic units of charge. Its experimental value was first determined by W. Weber and R. Kohlrausch. Its value is the same as light velocity in vacuum. In the expression $c \equiv 1/\sqrt{\mu_o\varepsilon_o}$ the magnitude $\mu_o = 4\pi \times 10^{-7}\ H/m$ is called the permeability of free space, while $\varepsilon_o = 8.85 \times 10^{-12}\ F/m$ is the permittivity of vacuum. We are using here an Eulerian description of the electric matter that will be put in motion.

In this work we will be dealing with neutral materials, so that the electrostatic or coulombian component of equation (1), $q_1 q_2 \hat{r}_{12}/(4\pi\varepsilon_o r_{12}^2)$, will not need to be considered in the calculations. The acquired velocities of the test particles considered here will be much smaller than light velocity, so that $v_1 \ll c$ and $v_2 \ll c$, where $v_1 \equiv |\vec{v}_1|$ and $v_2 \equiv |\vec{v}_2|$. Therefore, the velocity components of Weber's force (1) will be neglected in the following calculations. The only remaining term of Weber's force which will need to be considered here is the last component depending on the accelerations $\vec{a}_1$ and $\vec{a}_2$, namely:

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_o}\frac{\hat{r}_{12}}{r_{12}^2}\frac{\vec{r}_{12}\cdot\vec{a}_{12}}{c^2} = \frac{\mu_o q_1 q_2}{4\pi}\frac{\hat{r}_{12}}{r_{12}}\left(\hat{r}_{12}\cdot\vec{a}_{12}\right) . \quad (4)$$

2 Rotation of a Superconductor Due to Electromagnetic Induction

We first consider a resistive cylindrical shell of radius R_2 and infinite length with its axis along the z axis. It is around a superconducting cylindrical shell of radius $R_1 < R_2$ coaxial with the resistive shell. We suppose the internal cylinder to be composed of a single monoatomic layer of superconducting material.

The cylindrical shells 1 and 2 are assumed to be electrically neutral, being composed of positive lattices with surface charge densities $\sigma_{2+} > 0$ and $\sigma_{1+} > 0$, respectively, together with free electrons having surface charge densities $\sigma_{2-} = -\sigma_{2+}$ and $\sigma_{1-} = -\sigma_{1+}$. The positive lattice of the superconductor with surface charge density σ_{1+} will be identified with the macroscopic superconducting sample. Therefore, the rotation of the lattice means a rotation of the macroscopic sample being considered here.

We assume that the positive cylindrical shells and the conduction electrons are initially at rest relative to the inertial frame of reference S. We also assume that the positive lattice of the resistive shell will always remain at rest, with zero angular velocity: $\vec{\omega}_{2+} = \vec{0}$. In the time interval from $t = 0$ to the final value $t = t_f$ a given external applied current begins to flow azimuthally in the outer resistive cylinder, so that all its electrons move with a given angular velocity $\vec{\omega}_{2-}(t) = \omega_{2-}(t)\hat{z}$, reaching a final angular velocity $\vec{\omega}_{2-}(t_f) = \omega_{2-f}\hat{z}$. Our goal is to calculate the induced angular velocities of the positive lattice of the superconductor, $\vec{\omega}_{1+}(t) = \omega_{1+}(t)\hat{z}$, and the induced angular velocity of its free electrons, $\vec{\omega}_{1-}(t) = \omega_{1-}(t)\hat{z}$.

We first consider a generic charged cylindrical shell of radius R_2 and surface charge density σ_2 rotating with angular velocity $\omega_2(t)$ around the z axis. We calculate Weber's force exerted by this shell acting on a test charge q_1 located at a distance R_1 from the axis of the shell, figure 1.

We utilize cylindrical coordinates $(\rho,\ \varphi,\ z)$. Consider an element of source charge dq_2 of the cylindrical shell having a surface charge density σ_2 and area da_2. In cylindrical coordinates $da_2 = R_2 d\varphi_2 dz_2$, where φ is the azimuthal angle. Therefore, $dq_2 = \sigma_2 da_2 = \sigma_2 R_2 d\varphi_2 dz_2$. When the cylindrical shell is rotating with angular velocity $\omega_2(t) = d\varphi_2(t)/dt$ around the z axis, the position vector, velocity and acceleration of this element of charge relative to the inertial frame of reference S are given by, respectively: $\vec{r}_2 = R_2\hat{\rho}_2 + z_2\hat{z}$, $\vec{v}_2 = R_2\omega_2\hat{\varphi}_2$ and $\vec{a}_2 = -R_2\omega_2^2\hat{\rho}_2 +$

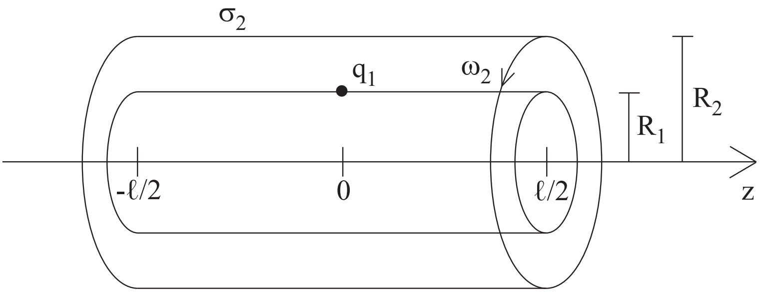

Figure 1: Cylindrical shell of length ℓ, radius R_2, surface charge density σ_2 rotating relative to the inertial frame of reference S with an angular velocity $\omega_2(t)$ around the z axis and acting on a test charge q_1 at a distance R_1 from the z axis.

$R_2\dot{\omega}_2\hat{\varphi}_2$, where $\hat{\rho}_2$, $\hat{\varphi}_2$ and $\hat{z}$ are the unit vectors of cylindrical coordinates at the location of dq_2, while $\dot{\omega}_2 \equiv d\omega_2/dt$. The test charge q_1 is located at the internal cylindrical shell of radius $\rho_1 = R_1$. It may also have centripetal and tangential components of its acceleration. We assume that contact forces maintain this test charge at a constant distance R_1 from the axis of the cylinder, so that $\dot{\rho}_1 = 0$ and $\ddot{\rho}_1 = 0$. We consider the test charge located at $z_1 = 0$. We assume that it will move along the tangential direction φ with an angular velocity $\omega_1(t) = d\varphi_1(t)/dt$. Its position vector, velocity and acceleration relative to the inertial frame of reference S are then given by, respectively: $\vec{r}_1 = \rho_1\hat{\rho}_1$, $\vec{v}_1 = \rho_1\omega_1\hat{\varphi}_1$ and $\vec{a}_1 = -\rho_1\omega_1^2\hat{\rho}_1 + \rho_1\dot{\omega}_1\hat{\varphi}_1$, where $\dot{\omega}_1 \equiv d\omega_1/dt$.

Equation (4) yields the force exerted by the source charge dq_2 acting on the conduction electron q_1. Integrating equation (4) over the surface of the cylindrical shell of radius R_2 yields the following net force acting on the test electron along the tangential or azimuthal direction $\hat{\varphi}_1$:[4]

$$\vec{F} = \int_{z_2=-\infty}^{\infty}\int_{\varphi_2=0}^{2\pi} \frac{\mu_o q_1 dq_2}{4\pi}\frac{\hat{r}_{12}}{r_{12}}\left(\hat{r}_{12}\cdot\vec{a}_{12}\right)$$

$$= \begin{cases} -\mu_o q_1\sigma_2 R_2(\dot{\omega}_2 - \dot{\omega}_1)\rho_1\hat{\varphi}_1/2 \ , & \text{if } \rho_1 < R_2 \ , \\ -\mu_o q_1\sigma_2 R_2(\dot{\omega}_2 - \dot{\omega}_1)R_2\hat{\varphi}_1/2 \ , & \text{if } \rho_1 = R_2 \ , \\ -\mu_o q_1\sigma_2 R_2(\dot{\omega}_2 - \dot{\omega}_1)R_2^2\hat{\varphi}_1/(2\rho_1) \ , & \text{if } \rho_1 > R_2 \ . \end{cases} \tag{5}$$

[4] [17] and [1].

We now apply Newton's second law of motion to the test charge q_1 of inertial mass m_1 located at $\rho_1 = R_1$ and moving only along the azimuthal direction, namely:

$$\vec{F} = m_1 \vec{a}_1 = m_1 R_1 \dot{\omega}_1 \hat{\varphi}_1 \ . \tag{6}$$

Here $\vec{F}$ represents the total force acting on the test charge. We are considering only the azimuthal component of this force along the φ direction, as contact forces prevent the test particle from moving along the radial ρ direction.

We assume two neutral cylindrical shells of infinite lengths and radii R_1 and $R_2 > R_1$ concentric along the z axis. The outer shell is a normal conductor with positive surface charge density $\sigma_{2+} > 0$ and negative surface charge density $\sigma_{2-} = -\sigma_{2+}$, while the inner shell is superconducting with positive surface charge density $\sigma_{1+} > 0$ and negative surface charge density $\sigma_{1-} = -\sigma_{1+}$. All these charges are supposed initially at rest relative to the inertial frame of reference S. Our goal is to calculate the motion induced in the positive and negative charges of the inner shell when an external azimuthal current is applied to the outer shell. We will assume that the positive lattice of the external resistive shell remains stationary during the whole process. The conduction electrons of the outer shell will be accelerated by an external source along the azimuthal direction during the time interval $0 < t < t_f$, moving around the z axis with a variable and given angular velocity $\vec{\omega}_{2-}(t) = \omega_{2-}(t)\hat{z}$. They begin at rest and at the end of this time interval they will be moving with the final and constant angular velocity $\omega_{2-f}\hat{z}$.

The test charge q_1 at the superconducting shell is located at $(\rho_1, \ \varphi_1, \ z_1) = (R_1, \ \varphi_1, \ 0)$. It can rotate freely around the z axis in the time interval $0 < t < t_f$ with a variable angular velocity $d\varphi_1/dt \equiv \omega_1(t)$ which needs to be calculated. There are four sets of source charges which exert a force on any test charge of the inner shell in the time interval $0 < t < t_f$, namely: (a) the stationary positive lattice of the outer shell with surface charge density σ_{2+} and zero angular velocity $\omega_{2+}(t) = 0$; (b) the set of negative conduction electrons of the outer shell with surface charge density $\sigma_{2-} = -\sigma_{2+}$ moving with an angular velocity $\vec{\omega}_{2-}(t) = \omega_{2-}(t)\hat{z}$; (c) the positive lattice of the inner shell with surface charge density σ_{1+} and angular velocity $\omega_{1+}(t)$; and (d) the set of negative conduction electrons of the inner shell with surface charge density $\sigma_{1-} = -\sigma_{1+}$ and angular velocity $\omega_{1-}(t)$. These four forces acting on a test particle with mass $m_1 > 0$ and charge q_1 will be represented by, respectively, $\vec{F}_{2+,\ q_1}$, $\vec{F}_{2-,\ q_1}$, $\vec{F}_{1+,\ q_1}$ and $\vec{F}_{1-,\ q_1}$.

The total force $\vec{F}$ in equation (6) will be given by equation (5) exerted by the four sets of source charges presented in the previous paragraph, namely, (a), (b), (c) and (d), acting on the the test charge.

We first consider the test charge to be a positive ion of the superconducting lattice with mass M and charge $q_1 = Ne = N \times 1.6 \times 10^{-19}\ C > 0$, where $N > 0$ represents the number of free electrons per atom and $e = +1.6 \times 10^{-19}\ C > 0$ represents the charge of the proton. The angular velocity of this positive charge q_1 will be represented by ω_{1+}. As it will be moving together with all other positive ions of the inner shell, we have $\dot{r} = 0$ and $\ddot{r} = 0$ for any pair of positive ions of the inner shell, so that $\vec{F}_{1+,\ q_1} = \vec{0}$. Combining equations (5) and (6) for these four sets of source charges acting on this positive ion yields:

$$\vec{F}_{2+,\ Ne} + \vec{F}_{2-,\ Ne} + \vec{F}_{1+,\ Ne} + \vec{F}_{1-,\ Ne}$$

$$= \frac{\mu_o NeR_1R_2\sigma_{2+}\dot{\omega}_{1+}}{2}\hat{\varphi}_1 + \frac{\mu_o NeR_1R_2\sigma_{2+}\left(\dot{\omega}_{2-} - \dot{\omega}_{1+}\right)}{2}\hat{\varphi}_1$$

$$+ \vec{0} + \frac{\mu_o NeR_1^2\sigma_{1+}\left(\dot{\omega}_{1-} - \dot{\omega}_{1+}\right)}{2}\hat{\varphi}_1$$

$$= MR_1\dot{\omega}_{1+}\hat{\varphi}_1 \ . \tag{7}$$

Let $|m_{W1}| \equiv \mu_o eR_1\sigma_{1+}/2 > 0$ and $|m_{W2}| \equiv \mu_o eR_2\sigma_{2+}/2 > 0$ be the magnitudes of the weberian electromagnetic masses for this cylindrical geometry.[5] With these definitions of $|m_{W1}|$ and $|m_{W2}|$ equation (7) can be written as:

$$N|m_{W2}|\dot{\omega}_{2-} + N|m_{W1}|\left(\dot{\omega}_{1-} - \dot{\omega}_{1+}\right) = M\dot{\omega}_{1+} \ . \tag{8}$$

We now consider the test charge to be a free electron of the superconducting lattice with mass $m = 9.1 \times 10^{-31}\ kg$ and charge $q_1 = -e = -1.6 \times 10^{-19}\ C < 0$. Its angular velocity will be represented by ω_{1-}. As it will be moving together with all other free electrons of the inner shell,

[5] [17], [18] and [1].

we have $\dot{r} = 0$ and $\ddot{r} = 0$ for any pair of electrons of the inner shell, so that $\vec{F}_{1-,\ q_1} = \vec{0}$. Combining equations (5) and (6) for the four previous sets of source charges (a), (b), (c) and (d) acting on this free electron yields:

$$\vec{F}_{2+,\ -e} + \vec{F}_{2-,\ -e} + \vec{F}_{1+,\ -e} + \vec{F}_{1-,\ -e}$$

$$= -\frac{\mu_o e R_1 R_2 \sigma_{2+} \dot{\omega}_{1-}}{2} \hat{\varphi}_1 - \frac{\mu_o e R_1 R_2 \sigma_{2+} \left(\dot{\omega}_{2-} - \dot{\omega}_{1-} \right)}{2} \hat{\varphi}_1$$

$$+ \frac{\mu_o e R_1^2 \sigma_{1+} \left(\dot{\omega}_{1+} - \dot{\omega}_{1-} \right)}{2} \hat{\varphi}_1 + \vec{0}$$

$$= m R_1 \dot{\omega}_{1-} \hat{\varphi}_1 \ . \tag{9}$$

With the previous definitions of $|m_{W1}|$ and $|m_{W2}|$ this equation can be written as:

$$-|m_{W2}|\dot{\omega}_{2-} + |m_{W1}| \left(\dot{\omega}_{1+} - \dot{\omega}_{1-} \right) = m \dot{\omega}_{1-} \ . \tag{10}$$

Equations (8) and (10) yield:

$$\dot{\omega}_{1-} = -\frac{M|m_{W2}|}{mM + mN|m_{W1}| + M|m_{W1}|} \dot{\omega}_{2-} \ , \tag{11}$$

and

$$\dot{\omega}_{1+} = \frac{Nm|m_{W2}|}{mM + mN|m_{W1}| + M|m_{W1}|} \dot{\omega}_{2-} = -\frac{Nm}{M} \dot{\omega}_{1-} \ . \tag{12}$$

We can also integrate these two equations in time utilizing the initial conditions $\omega_{1+}(0) = \omega_{1-}(0) = \omega_{2+}(0) = \omega_{2-}(0) = 0$. After integration we obtain our final results, namely:

$$\omega_{1-}(t) = -\frac{M|m_{W2}|}{mM + mN|m_{W1}| + M|m_{W1}|} \omega_{2-}(t) \ , \tag{13}$$

and

$$\omega_{1+}(t) = -\frac{Nm}{M}\omega_{1-}(t) \ . \tag{14}$$

Let us give some orders of magnitude. The free electron mass is given by $m = 9.1 \times 10^{-31}$ kg. Suppose $R_1 = 0.1$ m and a lead superconductor with four free electrons per atom ($N = 4$) and a mass of its positive ion given by $M = 3.44 \times 10^{-25}$ $kg \gg m$. Its number density is $n = 3.3 \times 10^{28}$ $atoms/m^3$, equivalent to 1.0×10^{19} $atoms/m^2$. The surface charge density of these positive lead ions (each ion charged with four protons) is then given by $\sigma_{1+} \approx 6.6$ C/m^2. With an outer copper conductor with $R_2 = 0.2$ m and $\sigma_{2+} \approx 3$ C/m^2 we have $|m_{W1}| \equiv \mu_o e R_1 \sigma_{1+}/2 \approx 6.6 \times 10^{-26}$ kg and $|m_{W2}| \equiv \mu_o e R_2 \sigma_{2+}/2 \approx 6.0 \times 10^{-26}$ kg, such that:

$$m \ll |m_{W2}| < |m_{W1}| < M \ . \tag{15}$$

Applying equation (15) with the appropriate orders of magnitude for the masses into equations (13) and (14) yields:

$$\omega_{1-}(t) \approx -\frac{\sigma_{2+}R_2}{\sigma_{1+}R_1}\omega_{2-}(t) \approx -0.91\omega_{2-} \ , \tag{16}$$

and

$$\omega_{1+}(t) = -\frac{Nm}{M}\omega_{1-}(t) \approx 1.06 \times 10^{-5}\omega_{1-} \ . \tag{17}$$

Therefore, $|\omega_{1+}| \ll |\omega_{1-}|$, while $|\omega_{1-}| \approx |\omega_{2-}|$. Although the angular velocities of the free electrons in shells 1 and 2 have the same order of magnitude, they rotate in opposite directions. That is, if the electrons in the external shell begin to rotate in the clockwise direction, the induced motion of the free electrons in the internal superconducting sample will take place in the counter-clockwise direction.

Multiplying equation (14) by R_1^2 shows that the sum of the angular momentum acquired by the each positive ion of the superconducting lattice with the angular momentum acquired by its N free electrons goes to zero, namely:

$$MR_1^2\omega_{1+}(t) + NmR_1^2\omega_{1-}(t) = 0 \ . \tag{18}$$

Equations (13), (14) and (18) show that when the electrons are forced by external means to rotate in the resistive conductor, the free electrons of the internal superconductor will be induced to rotate in the opposite direction, while the positive lattice of the superconductor will rotate in the same direction as the electrons of the external resistive conductor, Figure 2.

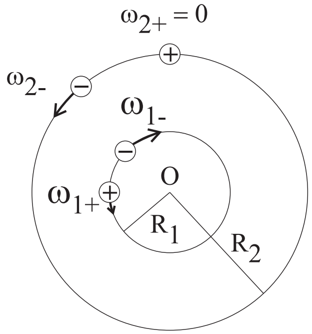

Figure 2: When the electrons of the resistive cylindrical shell 2 are rotated by external means around the z axis with an angular velocity ω_{2-}, they induce the free electrons of the superconducting shell 1 to rotate in the opposite direction with an angular velocity ω_{1-}. They also induce the positive ions of the superconductor to rotate in the same direction as the electrons of shell 2 with a much smaller angular velocity $|\omega_{1+}| \ll |\omega_{2-}|$.

Moreover, there will be no net angular momentum given to the superconducting material as a whole, considered as being composed of the positive lattice plus the free electrons of the superconductor. This net or total angular momentum will be always zero according to equation (18).

3 Rotation of a Resistive Conductor Due to Electromagnetic Induction

It is now simple to include resistivity in the inner conductor. As is well known, the resistive force responsible for Ohm's law can be expressed microscopically as a frictional force proportional to the relative velocity between the conduction electrons and the lattice of the metal. This resistive force tries to decrease the relative motion between the lattice and the free electrons of the metal. We can then add to the left side of equation (9) the resistive force exerted by the lattice on any free electron as given by $\vec{F}^{res}_{latt,\ -e} = -b\,(\vec{v}_{1-} - \vec{v}_{1+})$, where $b = e^2 n/g > 0$ is the positive coefficient of the frictional force. Moreover, $n > 0$ represents the number density of the resistive conductor and g its conductivity. Analogously, by Newton's third law of motion, we must also add to the left side of equation (7) the resistive force of reaction exerted by the conduction electrons acting on the lattice, namely, $\vec{F}^{res}_{-e,\ latt} = -\vec{F}^{res}_{latt,\ -e} = b\,(\vec{v}_{1-} - \vec{v}_{1+})$. As these two forces comply with action and reaction, they will not generate any net angular momentum on the resistive conductor. Therefore equation (18) will remain valid for a resistive internal cylinder. That is, the external variable current can produce a rotation of the positive lattice of the resistive conductor in one direction (supposing that the lattice is free to rotate). However, the external current will also produce a rotation of the conduction electrons of this resistive conductor in the opposite direction. These opposite rotations will not generate any total angular momentum of the internal cylinder considered as a single body (that is, composed by the positive lattice together with the negative conduction electrons).

4 Discussion and Conclusion

Although it is beyond the scope of this paper, it would be interesting to perform the calculation of the induced torque, on each type of charge, using the standard Maxwellian electrodynamics. In this formulation the external variable current creates a magnetic field and an electric field which act on the inner non-resistive or resistive neutral system by mutual induction between cylinders.

In this work we showed with Weber's electrodynamics that a variable current flowing in an external cylinder can induce rotations in one direction of the positive lattice of an internal superconducting cylinder, while the induced rotation of the free electrons belonging to the sample will take place in the opposite direction. The material composed of positive lattice and corresponding free electrons will receive no net angular momentum from the external source. We also reached a similar conclusion by extending our treatment to a resistive inner sample.

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