Divergence free self-interaction of particles with spin in quantum field theory

BISWARANJAN DIKSHIT

Laser and Plasma Technology Division
Bhabha Atomic Research Centre, Mumbai-400085, INDIA
and
Homi Bhabha National Institute, Mumbai-400094, INDIA

Email: bdikshit73@yahoo.co.in

ABSTRACT. In quantum field theory (QFT), when single loop Feynman diagrams representing photon or electron self-energy are evaluated, the probability amplitude diverges (becomes infinite). So, to make the QFT convergent, conventionally an infinite counter term is postulated which cancels the infinity generated by the diagram. This is known as renormalization procedure. However, proponents of renormalization method felt that this ad-hoc procedure of subtraction of infinity from infinity to reach at a finite value is not satisfactory and there is no physical basis for bringing in the counter term. So, it is desirable to establish a method in QFT which does not generate any infinite term, but which predicts the correct results. In this paper, we demonstrate such a technique for a particle with spin by taking self-interaction diagram of a photon (or vacuum polarization). In our method, no infinite term arises and so, renormalization is not necessary. Still, dependence of self-interaction amplitude on physical variables comes out to be same as that of conventional methods.

RÉSUMÉ. Dans la théorie des champs quantiques (QFT), lorsque des diagrammes de Feynman à une seule boucle représentant l'auto-énergie d'un photon ou d'un électron sont évalués, l'amplitude de probabilité diverge (devient infinie). Donc, pour rendre le QFT convergent, on postule classiquement un terme compteur infini qui annule l'infini généré par le diagramme. Ceci est connu comme procédure de renormalisation. Toutefois, les partisans de la méthode de renormalisation ont estimé que cette procédure ad hoc de soustraction de l'infini à l'infini pour atteindre une valeur finie n'est pas satisfaisante et qu'il n'existe aucune base physique permettant d'introduire le terme complémentaire. Il est donc souhaitable d’établir une méthode dans QFT qui ne génère pas de terme infini, mais qui prédit les résultats corrects. Dans cet article, nous démontrons une telle technique pour une particule avec spin en prenant un diagramme d'auto-interaction d'un photon (ou polarisation sous vide). Dans notre méthode, il n’ya pas de terme infini et la renormalisation n’est donc pas nécessaire. Néanmoins, la dépendance de l'amplitude d'auto-interaction à des variables physiques s'avère être la même que celle des méthodes conventionnelles.

1 Introduction

Feynman diagram is a very much convenient tool for calculating interaction probability amplitudes in quantum field theory (QFT). However, when self-interaction amplitudes for photon or electron represented as loops in Feynman diagrams are calculated, amplitude becomes infinite or divergent. Hence, the method of renormalization was invented to neutralize the infinity arising in calculations and to make experimentally testable predictions [1-6]. Renormalization involves two steps, regularization and renormalization. Regularization, in which the infinite term in mathematical expression of probability amplitude is isolated, can by of two types viz. cut off regularization or dimensional regularization. In cut off regularization [2], upper limit of four momentum of virtual particle is taken to be a large number such as Λ instead of ∞ so that algebraic operations involving it are possible. However, at the end when Λ is raised to ∞, the term containing Λ in amplitude becomes infinite. Similarly, in dimensional regularization, all calculations are carried out in $(4-\varepsilon)$ space-time dimensions and finally ε is taken to zero for predictions in 4-dimensions. At this point, as the self-interaction amplitude generally contains a term containing $1/\varepsilon$, the amplitude becomes infinite as $\varepsilon \to 0$. So, to rescue us from this situation, renormalization procedure is generally used. In this procedure, an additional infinite counter term is hypothesized whose amplitude exactly cancels the divergent part in regularized expression so that net amplitude remains finite. In other words, we simply subtract infinity from infinity to avoid the unwanted divergence.

Of course, there is no physical justification for this infinite counter term. The suggestion that infinite counter term might be coming from general relativistic effects [7-8] of point mass is not acceptable as we know particles are actually fields (or waves) spread out in space and they only act at a point during interaction. So, their small masses will not be sufficient to warp space-time to such an extent generating infinite gravitational energy. Lack of knowledge of high energy behavior of particles cannot be an excuse for ignoring the inconsistency especially when QFT we are developing is targeted for relativistic particles. Recently, Altaisky [9] suggested taking into account the resolution of the measuring instrument to make the QFT results finite. But, QFT must be made mathematically consistent irrespective of our measuring capability. Claims of reproducing QFT results using classical field theory without field quantization [10-11] has also been questioned by Bialynicki-Birula [12] and we cannot forgo field quantization as it is today a firmly and experimentally established fact.

Thus, the ad-hoc procedure of renormalization, although helps us in calculation of physical behavior, is not intellectually satisfying. So, Dirac had stated [13],

> “Most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory and we do not have to worry about it anymore.' I must say that I am very dissatisfied with the situation, because this so-called 'good theory' does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it is small – not neglecting it just because it is infinitely great and you do not want it!”

Similarly, Feynman, after development of quantum electrodynamics wrote [14] in 1985,

> “The shell game that we play ... is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate”.

In this paper, we describe a new approach in QFT calculation for photon self-energy (or vacuum polarization) which does not produce any infinite term in self-interaction probability amplitude, but reproduces the same finite experimentally verifiable result as conventional methods. Hence, we don’t need renormalization by infinite counter term. Dependence of the measurable parameter (charge) on the input variable (four momentum) in our approach is found to be same as in the conventional expression.

Structure of the paper is as follows. At first, in section-2, we will describe how the infinite term in probability amplitude appears in conventional method of calculation for single loop Feynman diagram and how it is removed by subtraction of infinite counter term which is commonly known as renormalization. In section-3, we demonstrate our technique that produces the probability expression without any infinite term so that renormalization by infinite counter term is not required.

2 Conventional approach for QED photon self-interaction

Consider a single loop self-interaction Feynman diagram of a photon as given in Fig.1 which is also known vacuum polarization. In this diagram, incoming photon A with four momentum q splits into two virtual particles

B and C (electron and positron), and finally photon A appears again by recombination. Four momentum of virtual particle B is (k+q) and four momentum of C is k.

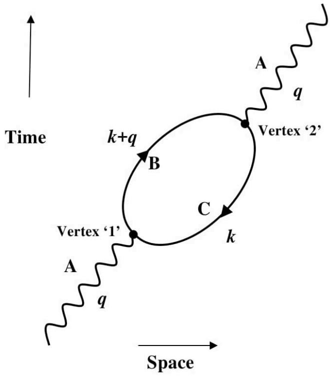

Figure 1. Single loop Feynman diagram for self-interaction
(Photon self-energy or vacuum polarization)

Working in natural units ($c = 1$ and $\hbar = 1$), probability amplitude of the diagram is given by (see Peskin and Shroeder [1]),

$$i\Pi^{\mu\nu}(q) = -4e^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^\mu (k+q)^\nu + k^\nu (k+q)^\mu - g^{\mu\nu}[k.(k+q) - m^2]}{(k^2 - m^2)[(k+q)^2 - m^2]} \quad (1)$$

Where e and m are charge and mass of electron.
After introducing Feynman parameter and performing Wick rotation in d-dimensional space-time, above expression becomes [1],

$$i\Pi^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\, i\Pi(q^2)$$

Where $$\Pi(q^2) = \frac{-8e^2}{(4\pi)^{d/2}} \int_0^1 dx\, x(1-x) \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \quad (2)$$

and $$\Delta = m^2 - x(1-x)q^2 \quad (3)$$

Putting d=4-ε, Eq. (2) becomes,

$$\Pi(q^2) = \frac{-2\alpha}{\pi} \int_0^1 dx\, x(1-x) \left(\frac{2}{\varepsilon} - \log\Delta - \gamma \right) \quad (4)$$

Where, $\alpha = e^2/4\pi$ (Fine structure constant)
γ=0.577 (Euler-Mascheroni constant)

For the physical world of four dimensions, when we put ε=0 in Eq. (4), $\Pi(q^2)$ becomes infinite! This is the infamous catastrophe in quantum field theory since the probability amplitude diverges. However, conventionally Eq. (4) is still used for prediction of physical events by exploiting a process known as renormalization in which diverging part is cancelled by adding an infinite counter term (of course without sufficient physical reason for origin of this term). The difference $[\Pi(q^2)-\Pi(0)]$ is unaffected since both the diverging term and counter term are constants and they are present in each of $\Pi(q^2)$ and $\Pi(0)$. It is the difference $[\Pi(q^2)-\Pi(0)]$ which decides the experimentally observed q^2 dependence of fine structure constant which is given by [1] (upto order α),

$$\alpha(q^2)=\frac{\alpha(0)}{1-[\Pi(q^2)-\Pi(0)]} \tag{5}$$

and

$$\Pi(q^2)-\Pi(0)=\frac{-2\alpha}{\pi}\int_0^1 dx\, x(1-x)\log\left(\frac{m^2}{m^2-x(1-x)q^2}\right) \tag{6}$$

However, getting an undesired infinite term in an expression for physical variable and then getting it artificially cancelled by assuming another infinite term is intellectually unsatisfying. In addition, since $\Pi(0)$ represents (e-e_0) where e_0 is bare charge (see [1]), the bare charge also becomes infinite which is certainly not a comfortable situation.

So, in the next section, we will derive a divergence-free expression for $\Pi(q^2)$ whose dependence on physical parameters (Δ or q and m) will still be same as in Eq. (4). We will also exactly reproduce the Eq. (6) in our new technique which is already experimentally tested.

3 New approach for divergence free QED photon self-interaction

Let us first justify the two assumptions that will be used in our formulation.

a) The problem of divergence in QFT appeared in a time when there was a hope of discovering hidden variables in quantum mechanics (EPR paradox [15]) and concept of "superposition or undefined state" was not fully accepted by the scientific community. But, presently quantum entanglement has experimentally established that spin of entangled pair of particles can remain in an undefined state until it is measured along a certain direction (i.e. until a direction is defined by an external interacting body). So, we will take the clue from quantum entanglement to assume that momentum of virtual particles perpendicular to two-dimensional plane containing momentum and displacement of parent particle from which they are created are undefined. In other words, virtual particles cannot discover a new direction in space which their parent didn't describe. For example, in Fig. 1, the parent particle A describes only two directions in space, one by its own momentum vector and other by its displacement vector from vertex 1 to 2. So, using the above hypothesis, if the two-dimensional plane containing 3-momentum and displacement of the parent particle is named as X-Y plane, then the momentum of two virtual particles will always lie on this X-Y plane. Hence, the undefined momentum q_3 should not be considered in integration in Eq. (1) to calculate the probability amplitude. So, we put *d*=3 in Eq. (2) and we get,

$$\Pi(q^2) = \frac{-8e^2}{(4\pi)^{3/2}} \int_0^1 dx\, x(1-x) \frac{\Gamma(2-3/2)}{\Delta^{2-3/2}}$$

Or $$\Pi(q^2) = \frac{-e^2}{\pi} \int_0^1 dx\, x(1-x) \frac{1}{\Delta^{1/2}} \quad (7)$$

Let us define a new variable,

$$S = \sqrt{m^2 - x(1-x)q^2} \quad (8)$$

So, using Eq. (3), we get, $\Delta^{1/2} = \pm S$.

To account for both the possible signs +*S* and -*S*, we have to multiply the amplitude by a factor of '2'. So, Eq. (7) becomes,

$$\Pi(q^2) = \frac{-2e^2}{\pi} \int_0^1 dx\, x(1-x) \frac{1}{S} \quad (9)$$

b) It is well known that virtual particles are off-shell, meaning they need not satisfy Einstein's relation ($E^2 \neq p^2c^2 + m^2c^4$). In addition to this, in our approach, we will assume that mass *m* of *virtual particle* has also a small amount of flexibility. Since *S* is a function of *m* (see Eq. (8)), *S* in Eq. (9) can also vary. Since *S* has the dimensions of momentum and it is now a variable depending upon mass of virtual particle, Eq. (9) should be integrated to include all possibilities in the total amplitude. Integration can be carried out from *S* to S_0 which is an arbitrary *finite* constant. Note that as we assume very small variation in *m*, S_0 is nearly equal to *S* and is never infinite. As usual, we have to divide the expression by 2π as we are going to integrate with respect to another momentum-like variable *S*. So, Eq. (9) becomes,

$$\Pi(q^2) = \frac{-e^2}{\pi^2}\int_0^1 dx\, x(1-x)\int_S^{S_0} \frac{dS}{S}$$

Or $$\Pi(q^2) = \frac{-e^2}{\pi^2}\int_0^1 dx\, x(1-x)(\log S_0 - \log S) \tag{10}$$

Using $\alpha = e^2/4\pi$ and as $\Delta = S^2$, Eq. (10) can be written as,

$$\Pi(q^2) = \frac{-2\alpha}{\pi}\int_0^1 dx\, x(1-x)(\log S_0^2 - \log \Delta) \tag{11}$$

Thus, the Δ dependent term in Eq. (11) is exactly same as the term in Eq. (4) where as the infinite term is absent in our expression given by Eq. (11). In addition, value of $\Pi(0)$ calculated from Eq. (11) is also finite given by,

$$\Pi(0) = \frac{-2\alpha}{\pi}\int_0^1 dx\, x(1-x)(\log S_0^2 - \log m^2) \tag{12}$$

As $\Pi(0) \neq \infty$, problem of infinite bare charge in avoided in our approach.

Now, using our Eq.(11) and (12), the value of $[\Pi(q^2) - \Pi(0)]$ can be calculated which decides the experimentally observed q^2 dependence of fine structure constant and it is given by,

$$\Pi(q^2) - \Pi(0) = \frac{-2\alpha}{\pi}\int_0^1 dx\, x(1-x)\log\left(\frac{m^2}{m^2 - x(1-x)q^2}\right) \tag{13}$$

Thus, the Eq. (13) of our approach is exactly same as conventionally derived expression given by Eq. (6). But, in our technique, we have avoided the appearance of infinite or diverging term in the intermediate steps.

4 Conclusion

In this paper, we have developed a technique to calculate the probability amplitude for self-interaction of a particle with spin in which four-momentum dependent term exactly matches with that of conventional method whereas no infinite term appears in our expression. For this demonstration, we have considered a single loop Feynman diagram representing photon self-energy or vacuum polarization. As no infinite term appears in expression for probability amplitude, we don't have the problem of infinite bare charge and we also don't need the infinite counter term to cancel the unwanted diverging term. We have thus escaped from the process of conventional renormalization method subtracting infinity from infinity which has been doubted as an illegitimate mathematical procedure even by proponents of QFT such as Dirac and Feynman. We could achieve this by assuming that momentum of virtual particles along a direction perpendicular to two-dimensional plane described by their parent particle remains undefined. This hypothesis was motivated from the observation that spin of a particle along a direction remains undefined until it is measured. We hope, in future, a similar procedure can be developed to make predictions in the field of quantum gravity which is at present non-renormalizable.

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