Astronomical distances and velocities and special relativity

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ABSTRACT. We show that some primary special relativity effects, which are believed to be hardly detectable in everyday life, such as time dilation, relativistic Doppler effect, and length contraction, should tangibly and spectacularly show up here on the Earth. They should occur in ordinary observations of known astronomical phenomena, also when these phenomena involve astronomical systems that move with very low velocities relative to us but are very distant. We shall do that by providing a reanalysis of the so-called Andromeda paradox and by revisiting the standard explanation of the muon lifetime dilation given when this phenomenon is observed from muon's perspective. Ultimately, we shall show that if Lorentz transformations (and basically, special relativity) are meant to entail real physical consequences, then the observable Universe should appear very differently from what we see every clear night.

1   Introduction

It is well known that most of the primary special relativity effects, such as time dilation, relativistic Doppler effect, and length contraction, become macroscopically observable only when the velocity v of the physical system, relative to the observer, approaches the speed of light c. There is one notable exception, though. According to Purcell's explanation of magnetic forces, the magnetic force acting upon a single charge moving parallel to a neutral current-carrying wire (Lorentz force) is, in fact, a macroscopic manifestation of the relativistic length contraction of the distances between the moving conduction electrons in the wire, even

though the velocities involved are always $v \ll c$. The contraction, which is observable only in the reference frame of the moving single charge, allegedly causes an unbalance in the charge density of the wire that results in the attraction (or repulsion) of the moving single charge. However, the present author has already shown [1, 2] that this mechanical/dynamical approach to the explanation of magnetic forces is problematic and we do not deal with it here.

To the author's knowledge, it is less widely known that special relativity effects[1] should macroscopically show up also in other physical systems moving with very low relative velocities ($v \ll c$), provided that they are placed at huge distances d from the observer (with $d/c^2 \gtrsim 1\,\mathrm{s}^2/\mathrm{m}$). Astronomical objects, with their huge distances and fairly high velocities relative to the Earth, are thus good candidates to actually observe special relativity effects.

In the following two sections we describe two examples of relativistic effects which should allegedly show up in plain observations of astronomical objects (very distant and/or very fast) made here on the Earth: the first example is related to the so-called 'Andromeda paradox', while in the second one we compare the relativistic explanation of the muon retarded decay, given when the phenomenon is analyzed from the muon reference frame, to what we should see from the Earth when we observe relatively fast astronomical objects.

2   The Andromeda paradox

The Andromeda paradox, also known with the name of Rietdijk–Putnam–Penrose argument [3, 4, 5, 6], gives a colorful demonstration that if special relativity is true, then observers moving at different relative velocities (any velocity, also non-relativistic) have different sets of events that are *present* for them. In particular, if two people walk past each other in the street and one of the people was walking towards the Andromeda galaxy, then the events in this galaxy that are simultaneous with the present time of this observer might be hours or even days advanced of the events on Andromeda simultaneous with the person walking in the other direction.

This argument has been introduced in the past to support the philosophical stance known as 'four-dimensionalism' (or 'block Universe'

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[1]To make the picture clearer from here on out, with 'special relativity effects' we actually mean effects which are the mathematical consequence of the application of the Lorentz transformations.

view), namely that an object's persistence through time is like its extension through space (for an entertaining and accessible presentation of the philosophical and physical theories of Time see, for instance, [7]).

2.1 Simple derivation of the paradox

The Andromeda paradox can be explained by recurring to the *planes of simultaneity* in the space-time diagram (Minkowski diagram). Here, instead, we make use of the plain Lorentz transformations

$$
\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \\
y' &= y, \\
z' &= z, \\
t' &= \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}},
\end{aligned}
$$

where the non-primed coordinates (x, y, z, t) refer to the reference frame assumed to be at rest and v is the velocity of the primed frame relative to the non-primed one along the x-axis.

Consider an observer A here on the Earth who moves towards the Andromeda galaxy (relative distance d, the direction Earth-Andromeda being along the x-axis) at a relative velocity v with $v \ll c$. For the sake of derivation, we shall equivalently consider the Andromeda galaxy as approaching the observer, and thus the velocity to be inserted in the Lorentz transformations is $-v$. Observer B is also on the Earth. He is initially close to the place where A starts walking, but he is at rest. It is further assumed that the relative velocity between the Earth and the Andromeda galaxy is negligible, and thus the relative velocity of observer B relative to Andromeda is taken as zero. According to the Lorentz transformations, if t_A and t_B are the present instants of time of observer A and observer B respectively (with $t_A \simeq t_B$, since $v \ll c$ and the observers' clocks can be considered as continuously synchronized), then the instant of time on Andromeda simultaneous with t_A is

$$
t'_A = \frac{t_A + \frac{vd}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}, \tag{1}
$$

while the instant of time on Andromeda simultaneous with t_B ($\simeq t_A$) can be taken as simply $t'_B = t_B$.

Since the distance d between the Andromeda galaxy and the Earth is huge, we have that $\frac{vd}{c^2}$ can be much greater than unity, even with $v \ll c$, and then eq. (1) can be approximated to

$$t'_A \simeq t_A + \frac{vd}{c^2}. \qquad (2)$$

This has the paradoxical consequence that although observer A and observer B always experience the same 'present instant' of time ($t_A \simeq t_B$), the events on Andromeda simultaneous with observer A are events subsequent (instant of time $t'_A \simeq t_A + \frac{vd}{c^2}$) to the events on the same galaxy that are simultaneous with observer B (instant of time $t'_B = t_B \simeq t_A$). For instance, it might well happen that, in the *plane of simultaneity* of observer A, a supernova has just exploded in some part of the Andromeda galaxy while, in the *plane of simultaneity* of observer B, the same event has not yet happened.

2.2   Going further

In the literature, the extent of the paradox's consequences has been partially downplayed by noticing that the observers cannot actually see what is happening in Andromeda since it is light-years away, and then the paradox is only that they have different ideas of what is happening "now" in Andromeda.

We believe that there is more to it. There is something that can be in principle physically measured. Suppose that, for the sake of argument, both observes can live for millions of years and both decide, starting at time $t_A \simeq t_B$, to wait an interval of time equal to d/c and see what happens. This interval is the time needed by a light signal emitted in the Andromeda galaxy to reach the Earth. Please note that observer A does not keep moving for the whole interval of time d/c: observer A is near observer B, moves a bit, and then comes back near to B almost immediately.

Now, the problem is: what will observer A and observer B see after the interval of time d/c has passed? Will they see the same events or not?

What observer A sees after the interval d/c are the events that were simultaneous with the instant of time t_A of observer A exactly d/c years

ago and we have just seen that these events are surely different from the events that were simultaneous with the instant of time t_B ($\simeq t_A$) of observer B exactly d/c years ago. All this means that after the same interval of time d/c has passed, observer A and observer B, *who are at rest and close to one another already for a time nearly equal to d/c*, will *actually* see different events while observing the very same galaxy at the very same time here on the Earth (e.g. observer A detects the explosion of a supernova and observer B does not).

Let us linger over this with the following more direct representation. Observer Bob is at rest on the Earth, sitting on a bench and staring at the Andromeda galaxy (which is d away from the Earth). Observer Alice passes by with a velocity $v \ll c$. After few meters traveled (or, equivalently, after few seconds), Bob shouts "Now!" at Alice. Both Alice and Bob start their stopwatches. Then, Alice suddenly stops walking away, makes a U-turn, and goes sitting close to Bob. They both shut their eyes and wait an interval of time equal to d/c before opening their eyes again. Since $v \ll c$, their proper times are the same, their stopwatches are synchronized, their distance from Andromeda is the same (d) and the time they have to wait before opening their eyes is also the same (d/c). What will they see when they open their eyes? Bob will surely see events in Andromeda that were simultaneous with Bob's present when he yelled "Now!", and Alice will surely see events in Andromeda that were simultaneous with her present when Bob yelled "Now!". But, according to the Lorentz transformation of the time coordinate, the Andromeda events that were simultaneous with Alice's present when Bob yelled "Now" are $\simeq vd/c^2$ subsequent in time to those simultaneous with Bob's present when he yelled "Now!".

This is a bizarre situation: people staying in the same place at the same time and staring at the same source in the sky see different events. But, this is a strict logical consequence of the accepted laws of physics[2]. Moreover, the very same logic could be applied to the past, namely to events that happened millions of years ago. Today, we should see a rainbow of different and simultaneous events while observing the Andromeda galaxy. Millions of years ago, we were not yet born and it would be difficult to define the velocity v of the observers then (and even just define the 'observers'), but we are sure that the reader has got the point.

By the way, eq. (2) should have an observable consequence today: it

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[2]Lorentz transformations intended as physical laws.

is possible to demonstrate that even the periodic movement of the Earth around the Sun should induce a sort of visible (wild and haphazard) 'Doppler oscillations' of the radiation coming from very distant astronomical sources. Note that the frequency shift we are referring to here is not the standard Doppler shift due to the (usually high) relative velocity between the source and the observer. It is an exclusively relativistic effect. For the sake of derivation, let us focus on the velocity variation of the Earth with respect to the Andromeda galaxy during the Earth revolution around the Sun. The galaxy is assumed to be at rest with respect to the Sun. Let us further consider only a small trait of the Earth orbit, where the change of velocity (acceleration a) can be taken as constant (and obviously $a\Delta t \ll c$, for every Δt considered). The acceleration is assumed to be directed along the line of sight. Suppose that at initial Earth time instant t_{E_1} the velocity of the Earth relative to Andromeda is equal to zero and then the simultaneous instant of time in Andromeda is $t_{A_1} = t_{E_1}$. At Earth time instant t_{E_2}, the relative velocity of the Earth has increased to $a(t_{E_2} - t_{E_1}) = a\Delta t_E$ and thus, by making use of the differential form of eq. (2), we have the following result for the interval of time elapsed in Andromeda corresponding to the interval of time Δt_E elapsed on the Earth

$$dt_A \simeq dt_E + \frac{d}{c^2}dv \quad \rightarrow \quad \Delta t_A \simeq \Delta t_E + \frac{d}{c^2}a\Delta t_E = \Delta t_E \left(1 + \frac{ad}{c^2}\right), \text{ (3)}$$

where d is, as before, the distance between the Earth and the Andromeda galaxy.

Now, if there is a star in Andromeda that emits radiation at a frequency ν_0 for an interval of time Δt_A, then it will emit a number of periods equal to $\nu_0 \Delta t_A$. This number of periods will also be observed here on the Earth (after the traveling time d/c) but as emitted in the shorter time interval Δt_E and thus the frequency of the radiation seen here on the Earth will be higher, equal to

$$\nu_E = \nu_0 \left(1 + \frac{ad}{c^2}\right). \tag{4}$$

Since the motion of the Earth is not uniform around the Sun and since there are other dynamical mechanisms that contribute to the relative motion of the Earth with respect to distant galaxies (e.g. motion of the

Sun around the center of the Galaxy, relative motion of galaxies, not to mention the proper motion of the stars that emit radiation from inside the Andromeda galaxy), we should observe the light of distant galaxies weirdly and haphazardly Doppler shifted. The effects we would observe today would be due to radiation emitted a very long time ago ($\sim d/c$, where d is the astronomical distance of the source from the Earth), but this delay does not cancel out the phenomenon, we simply do not see it live.

3   Muon decay and length contraction

In the '40s, studies conducted on muons generated by cosmic rays in the upper atmosphere suggested that what was thought to be an anomalous absorption of these particles by the atmosphere itself was in fact due to their spontaneous decay and that the decay-rate depended upon muons' momentum [8, 9]. The decay-rate dependence on momentum has been interpreted in the framework of special relativity as one of the neatest experimental verification of the time dilation of a 'moving clock'. Although muons mean lifetime τ_0 is of only $\sim 2.2\,\mu s$ and thus not enough to guarantee their arrival at the lower atmosphere, their high abundance at this atmospheric depth is explained by the fact that their lifetime measured in the reference frame of the Earth is relativistically dilated to $\tau = \tau_0/\sqrt{1 - v^2/c^2}$ (due to their high relative velocity, $v \simeq 0.99\,c$); this is just the amount needed to explain their anomalous lower atmosphere abundance.

But, how is the same phenomenon explained when it is seen in the reference frame of the traveling muon? In the muon's rest frame, the particle decays, on average, after a time τ_0, and from its perspective the rate of clocks on the Earth is slowed down. Therefore, from its perspective, an observer on the Earth should measure a *decrease* and not an *increase* of its lifetime and thus a decrease and not an increase in the number of muons in the lower atmosphere. However, all relativists explain the phenomenon simply by invoking the length contraction of the atmosphere: for a muon, the atmosphere is thinner and the particle has the time to penetrate it deeper.

At first sight, this explanation appears quite neat and it is considered as a solid proof of the internal coherence and strength of special relativity. Under close inspection, however, it is a bit problematic and, apparently, it has never been recognized as such before.
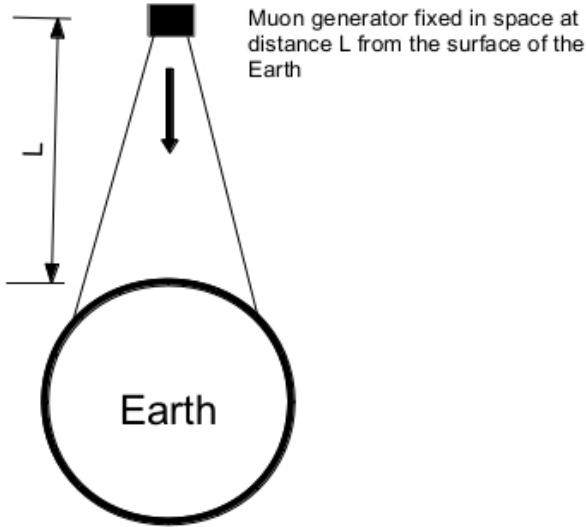
Figure 1: Setup described in the text.

For the sake of simplicity, consider the setup shown in Figure 1. With regard to the key aspects of the process, it is completely equivalent to the process observed in nature. The proper mean lifetime of a muon is τ_0. This means that if we travel with the muon we will see it decay after an interval of time τ_0. Observers on the Earth, instead, see the muon decay after a dilated interval of time $\tau = \tau_0/\sqrt{1 - v^2/c^2}$, since $v \simeq c$. During this time, for the observers on the Earth, the muon travels a distance $L = v \cdot \tau = v \cdot \tau_0/\sqrt{1 - v^2/c^2}$. For the sake of argument, the muon generator has been placed exactly at distance L from the surface of the Earth and thus muons can reach the surface just before decay.

In the reference frame of the muon, however, the particle sees the Earth approaching at speed v and thus, from its point of view, during that interval of time the distance covered by the Earth before muon decay is $v \cdot \tau_0 < L$. Namely, the muon disintegrates before touching the surface of the Earth. This result simply comes from elementary kinematics. Here, we only appeal to the principle of relativity by which
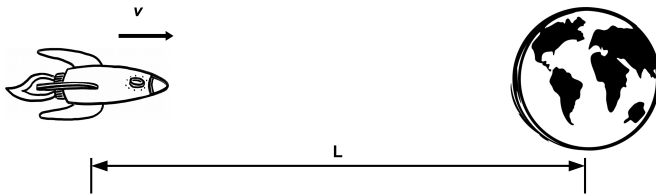
Figure 2: Muons as interstellar travellers.

the laws of physics (e.g. kinematics) are the same in every inertial frame.

The only possibility to reconcile these two different views is the widely known and accepted explanation (e.g. see [11]) that in the muon reference frame the distance that separates the muon (generator) from the surface of the Earth is Lorentz contracted,

$$L' = L \cdot \sqrt{1 - v^2/c^2} = v \cdot \tau_0/\sqrt{1 - v^2/c^2} \cdot \sqrt{1 - v^2/c^2} = v \cdot \tau_0. \quad (5)$$

This means that, from muon's perspective, the Earth's surface appears to be (and actually is) closer than L. This effect is also considered when an interstellar journey of a spaceship traveling at speeds close to that of light is analyzed from the perspective of the astronaut. ¿From the perspective of the Earth, time on spaceship dilates and the astronaut can cover a huge distance in a relatively short period of his own time. From the perspective of the astronaut, though, his time rate does not change and the only possibility to match the observations made by the observers on the Earth is that the distance to travel actually shortens for the astronaut. Consider the situation depicted in Figure 2. A spaceship is located at a distance L from the Earth and heads towards our planet at constant velocity $v \lesssim c$. The distance L is intended as measured from the Earth. Suppose that the time L/v needed by the spaceship to reach us is greater than 100 years. According to special relativity, if v is suitably high, the observers on the Earth will measure a time dilation within the spaceship that makes it possible for the astronaut to reach the Earth in a shorter period of his own time, say 8 years. Now, in the reference frame of the astronaut, the same outcome can only be explained with length contraction. In order for the astronaut to reach the Earth in 8

years of his own time, the distance L should be suitably shorter from his perspective. The other possibility, namely that the Earth appears faster to the astronaut, cannot be accepted owing to the principle of relativity. The very same principle of relativity, however, discloses a problem with the length contraction explanation. According to this principle, there is no reason to believe that the spaceship moves and the Earth is a rest. It may well be the other way around. In that case, it should be the distance seen by the Earth to be contracted. At any rate, the distance measured from the Earth should be equal to that measured by the astronaut in the reference frame of the spaceship because nobody can say who is moving and who is at rest.

Now, in the previous paragraph change the words 'astronaut' or 'spaceship' with 'muon' and it should be evident why length contraction cannot be an acceptable explanation for the muon problem when it is analyzed in the muon rest frame.

To recapitulate, two observations follow in order. First, if the principle of relativity holds, one may equivalently assume that the Earth is actually moving towards the muon and thus the distance L that separates us from the place where the particle originated (generator) is already 'shrunk'. Or, better, for the principle of relativity if we measure a distance equal to L, then also the muon must see the Earth distant L from itself. Owing to the principle of relativity, the two distances (contracted or not) must be equal. *Length contraction, like time dilation, is symmetrical[3] when the relative velocity is uniform, as is in this case.* Thus, the standard explanation of the muon lifetime dilation from muon's perspective becomes inconsistent, to say the least.

Secondly, what about observations of astronomical objects (matter) moving towards our position at relativistic speeds? Consider, for instance, relativistic jets of particles moving towards our position from Active Galactic Nuclei (AGNs)[4]. According to the principle of relativity and the relativity of uniform motion, we may equivalently consider the Solar System (or our galaxy) as moving towards the jet particles at relativistic speeds and thus, according to special relativity, these jets should appear to us a lot closer than the AGNs that have generated them. If the length contraction explanation of the muon decay phenomenon has a real physical meaning (it is physically real), then we should observe a

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[3]Unless we want to resort to Lorentz ether theory.

[4]Consider, for instance, the pulsar IGR J11014-6103: the estimated speed of its jet is $0.8c$.

weird distribution of matter in deep space, due to the existence of objects with different (and relativistic) relative velocity with respect to our reference frame.

4 Conclusion

We have shown that some primary special relativity effects, which are believed to be hardly detectable in everyday life, like time dilation, relativistic Doppler effect, and length contraction, should tangibly and spectacularly show up here on the Earth: they should occur in ordinary observations of known astronomical phenomena, also when the observations involve astronomical systems that move with very low relative velocities ($v \ll c$) but are placed at huge distances d from us (with $d/c^2 \gtrsim 1\,\mathrm{s}^2/\mathrm{m}$). In that regard, we have offered two examples: the first involves the so-called Andromeda paradox, and the second, inter alia, calls into question the standard special relativity explanation of the muon lifetime dilation when the phenomenon is analyzed from muon's perspective. These two examples ultimately imply that if special relativity consequences (basically the consequences deriving from the application of the Lorentz transformations) are real physical consequences, then the observable Universe should appear very differently from what we actually see every clear night. Unfortunately, none of the effects described in this paper, and that necessarily and strictly follows from special relativity, seem to have been ever observed. Thus, there are concrete elements to believe that something is actually not as it should be in the physical interpretation of Lorentz transformations and the allegedly real physical consequences of special relativity. A discussion on this last aspect from different standpoints can be found in [1]. We want to end this paper with two quotes from the renowned physicist Mendel Sachs that appear to be particularly pertinent here:

> "I believe that Einstein's identification of the Lorentz transformation with a *physical cause-effect* relation, and the subsequent conclusion about asymmetric ageing, *was a flaw*, not in the theory of relativity itself, [...], but rather *a flaw in the reasoning that Einstein used in this particular study*–leading him to an inconsistency with the meaning of space and time, according to his own theory. [10]" [emphasis added]

> "The crux of my argument was that the essence of Einstein's theory implies that the space-time transformations between

relatively moving frames of reference must be interpreted strictly kinematically, rather than dynamically. Thus, according to this theory, the transformations are not more than the necessary scale changes that must be applied to the *measures* of space and time, when comparing the expressions of the laws of nature in relatively moving frames of reference, so as to satisfy the principle of relativity–that is, to ensure that their expressions in the different reference frames are in one-to-one correspondence. [12]" [emphasis in the original text]

Acknowledgments

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