Louis de Broglie Realistic Research Program and the experimental detection of Quantum Waves

J.R. Croca[1,2], P. Castro[2], M. Gatta[2,3] and R. N. Moreira[2]

University of Lisbon, Faculty of Sciences, Department of Physics[1]
Center for Philosophy of Sciences of the University of Lisbon (CFCUL)[2]
CINAV / Escola Naval (Portuguese Naval Academy)[3]

Abstract: We give a brief historical review of Louis de Broglie realistic research program as developed in Lisbon's research group about nonlinear quantum physics. We propose and discuss "yes-no" type experiments to physically detect quantum waves, independently of the associated corpuscles. Thus, deciding if they are real physical perturbations or mere probability waves. We finally present a possible technological application for the detection of such waves in quantum communication.

Keywords: Louis de Broglie realistic research program, quantum waves, empty waves quantum yes-no experiments to test the physical reality of de Broglie waves, nonlinear quantum physics.

1 Introduction

To some physicists the debate on the ontic nature of the quantum waves, whether they are mere probability waves or, on the contrary, real physical entities, is devoided of any sense. They argue that they don't care because it is not a relevant to the actual quantum calculations.

Still, if we look briefly at the history of science, we see that at the end of the ninetieth century there were also an enormous controversy on the ontic nature of the atom. Mach, Ostwald, Avenarius and many other thinkers, the so-called positivists and neopositivists, claimed that the atoms were mere conceptual constructs, thus devoided of any ontic physical reality. Opposing this view, there were, among many others, Boltzmann, Maxwell and Einstein sustaining that the atoms were much more than mere conceptual constructs and consequently that they had physical reality. Following these beliefs, in 1905, Einstein[1] published a work in which he explained the Brownian motion in terms of the physical reality of atoms.

In 1909, Jean Perrin[2] did very interesting experiments on the Brownian motion that contributed to the clarification of the ontic nature of the atoms, thus contributing in a practical manner to the discussion whether they were mere theoretical constructs or, on the contrary, parts of physical reality.

Now, more than a century after these events, we may ask ourselves whether it was useful, for the development of science, to clarify the ontic nature of the atoms. Without the belief in the physical reality of the atoms our science, namely, molecular chemistry and solid-state physics, would not be possible.

Precisely on the same foot stands the controversy on the ontic nature of the quantum waves. If indeed quantum waves are real physical waves, as some experiments seem to indicate[3,4], then a whole new universe of theoretical and technological possibilities will open.

In order to solve the riddle posed by the dualism wave-corpuscle in a causal realistic way, de Broglie[5] assumed that a real physical quantum particle is a complex entity, composed of an extended part, a wave, with a relatively minute energy, plus a complex and highly localized energetic part, the corpuscle, as represented in the next picture, Fig.1

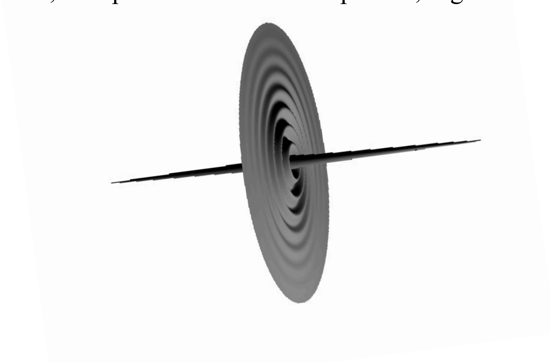

Fig.1 - De Broglie complex realistic quantum particle

The extended part of the particle is also called quantum wave, empty wave, de Broglie wave, pilot wave, subquantum wave, zero-point field wave or by us, theta wave, θ [6]. We also named the corpuscle acron, ξ, from the Greek word acropolis, the high pick, since one could then generalize and extend the pilot wave model also to macroscopic cases, as in the case of the Titius-Bode Law[7] or of the beautiful experiments performed by Yves Couder[8] and John Bush[9] with walking droplets.

We may recall the great contribution of de Broglie for a better understanding of the quantum realm. Against all traditional classical physics, he dared to propose at the time, the existence of a nonlinear process in which a minor action would, in adequate conditions, give rise to an enormous reaction. He expressed this nonlinear process as the Guiding Principle, also known as the pilot wave principle[10]. We have called it the Principle of eurhythmy (the best rhythm), emphasising the idea that the corpuscle (or acron) will tend to move along trajectories that maximize the particle's structural integrity. This basic principle tells us that the theta wave, the subquantum wave, in average, guides the acron in a nonlinear way to the regions where the density of the global wave, in which it is immerged, is relatively greater.

2 Experiments designed to test the ontic nature of the quantum waves

As was already mentioned, experimental evidence of the subquantum waves, beyond its mere operative aspect, would change our worldview in much the same manner as real physical atoms did. To try and reach such evidence, our aim is to devise concrete realizable yes-no type experiments that could decide on the ontic nature of the quantum waves. It is convenient to clarify the basics of the problem. The experiments must allow us to decide between the two opposite assumptions:

1 - The Copenhagen approach
Quantum waves are mere probability waves, devoided of any physical ontic content.

2 - De Broglie realistic causal approach
Quantum waves, subquantum waves, empty waves, de Broglie waves, pilot waves or theta waves, are real physical waves.

Without denying the existence of identical efforts elsewhere in the world, we present in a very succinct way our own history concerning the detection of quantum waves. In 1972, P. Neves[11], then a young student of João Andrade e Silva, who was a direct collaborator of de Broglie some years early, indicated in a scholar work a possible experiment to detect empty waves. Sometime later, in 1980, Andrade e Silva and his wife, Maria Andrade e Silva[8] further advanced the idea, writing "*Une expérience possible concernant la nature du dualisme onde-corpuscule*" [12]. In 1983, Andrade e Silva together with F. Selleri and J.P. Vigier, discussed still in a conceptual idealistic way "*Some possible experiments on quantum waves*"[13], further developing Neves initial proposal. In 1985, the first concrete proposal of an experiment, although still lacking the technological means to do it, was proposed by J.R. Croca in the paper, *Can the existence of de Broglie's empty waves be proven experimentally?*[14]

The first feasible experiment, also indicating the available technology to employ, Q*uantum-Optical predictions for an experiment on de Broglie waves*[15], was proposed in 1990 by J. R. Croca, A. Garuccio, V. L. Lepore and R. N. Moreira. This experiment was done at the University of Rochester, in 1992, by X.Y. Zou, T. Grayson and L. Mandel[16].

To fully describe the experiment, we begin by the very simple setup in Fig.2.

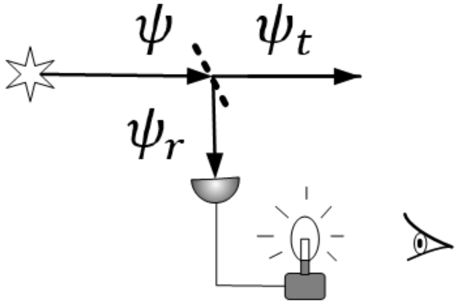

Fig.2 - A source emits quantum particles, one-by-one, towards a beamsplitter

In the setup above there is a quantum mono-particle source emitting photons one-by-one. Associated to each photon there is a quantum wave, described by the wave function, ψ, impinging on a beamsplitter. From the original wave results a transmitted wave, ψ_t, and a reflected wave, ψ_r.

Independently of which theoretical approach one uses to describe the phenomena, the photon will have an equal probability of being detected along both paths after the beamsplitter.

Consider the situation in which a detector, placed in the reflected path, clicks and an observer may, if he wants, see the lamp turn on. A question now arises: what happens in the transmitted path?

For this question there are two opposite answers:

a) According to the Copenhagen interpretation of quantum mechanics:

Before the measurement the initial wave function, ψ, containing all information about the quantum system, is the sum of the waves describing the two possible outcomes from the beamsplitter. The one associated to the transmitted wave path, and the other associated to the reflected wave path.

$$\psi = \frac{1}{\sqrt{2}}\psi_t + \frac{1}{\sqrt{2}}\psi_r. \tag{1}$$

After the measurement, since the waves are mere probability waves, there occurs the so-called collapse of the probabilities or reduction of the wave vector, that is,

$$\psi \longrightarrow \psi_r$$

and, consequently, the transmitted function nullifies itself

$$\psi_t = 0.$$

This implies that after measurement no physical action can be observed in the transmission path.

b) According to de Broglie realistic school.

Following this approach, since the waves are real physical waves, after the measurement, a perturbation of some sort must follow along the transmission path. The remaining real wave has been often called an empty wave. Using our own jargon, of a theta wave, θ, and its associated acron, ξ, (in the case, a photon) we have depicted the situation in Fig.3.

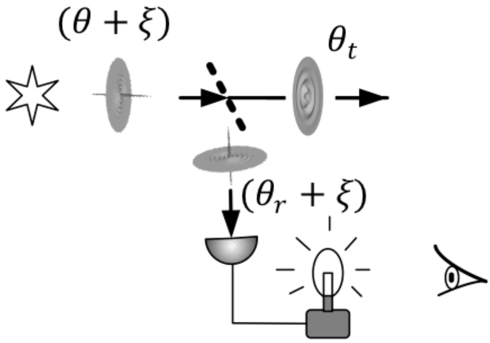

Fig.3 – Experimental situation according to de Broglie approach. After the measurement, an empty θ wave follows along the transmission path.

From this very simple situation, the setup may be improved to make what may be called a theta wave generator (TWG), as shown in the next drawing.

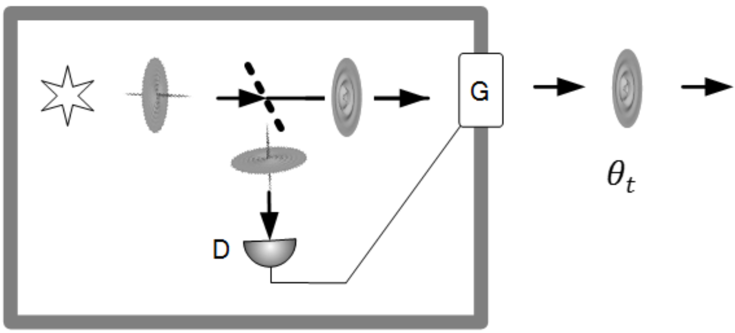

Fig.4 - TWG -Theta wave generator

In this situation the detector D will be connected to a gate G. Each time detector D registers the arrival of a photon, it sends a signal that opens gate G so that the theta wave may leave the apparatus. In all other cases the gate remains closed. In this way we can be sure that only empty waves (theta waves) are leaving the apparatus trough G.

Now the most important question. How can we be sure that something real leaves the device? To answer the question, we need to build a special device able to detect waves with a very, very minute amount of energy.

There are several possibilities for carrying out such a task[6].

Here we present a single possibility, that we came to acknowledge as readily feasible, as conceptually outlined in Fig.5.

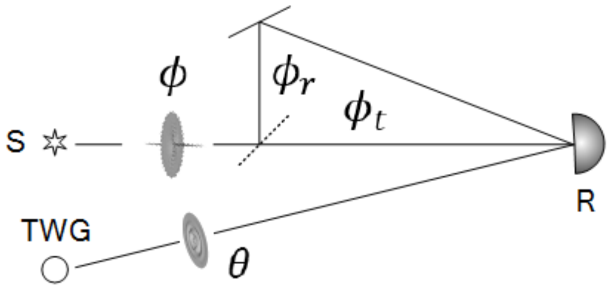

Fig.5 - The empty wave emitted by the TWG changes the interferometric pattern from the wave originating from source S at the detection region R.

In the picture a monoparticle source S emits photons one-by-one to the beamsplitter that produces two waves, ϕ_r and ϕ_t, one of which carrying the acron, and that are both led to superimpose, giving rise to an interferometric pattern at the detection region R. Here, the empty wave coming from the source S, either with or without corpuscle, was represented by the symbol ϕ. Only in the case when we are sure that we have an empty wave, a theta wave, the symbol θ is used. The theta wave, produced by the theta wave generator, TWG, superimpose to the two waves, ϕ_r and ϕ_t at the detection region R, altering the observable pattern.

So, for this experimental setup, there are two opposite predictions:

a) According to the orthodox interpretation of quantum mechanics:

Empty waves do not exist and therefore we have only two waves interfering, thus giving for the predicted observed intensity at the detection region R:

$$I_O = |\psi|^2 = |\psi_r + \psi_t|^2 = |\psi_r|^2 + |\psi_t|^2 + \psi_r^*\psi_t + \psi_r\psi_t^*, \tag{2}$$

If the waves have equal amplitude, we then have

$$I_O \propto (1 + \cos\delta), \tag{3}$$

in which, δ, represents the phase difference between the waves, ψ_r and ψ_t.

b) According to the causal de Broglie realistic approach:

Empty waves do exist. Since the two sources of waves, from the beamsplitter, and from the TWG, are incoherent, the empty wave coming from the theta wave generator will introduce noise at the detection region R, thus blurring the interference pattern.

In such conditions, at the detection region we have to consider the two waves, ϕ_r and ϕ_t, one of which is carrying the corpuscle, both coming from the beamsplitter and, also, the empty wave coming from the theta wave generator, TWG. Hence, the resulting intensity, I_c, seen by the detector may be written:

$$I_C = |\phi_r + \phi_t + \theta|^2. \tag{4}$$

If the waves coming from the monophotonic source have equal intensity, are coherent between each other, and are incoherent relative to the empty wave, θ, coming from the TWG, after some calculation (see reference 13) one gets:

$$I_C \propto (1 + \tfrac{2}{3}\cos\delta_\phi), \tag{5}$$

Where δ_ϕ represents the phase difference between waves ϕ_r and ϕ_t.

Concluding, according to the Copenhagen interpretation, since empty waves do not exist, a clear interference pattern with visibility equal to one shall be observed at the region R. On the other hand, if one accepts de Broglie causal approach, the observed interference pattern will become blurred, with a visibility factor of 2/3.

2.1 Actual implementation of a quantum wave detection experiment

The experiment conceptually described above, can be done using standard technology of quantum optics as depicted in Fig.6.

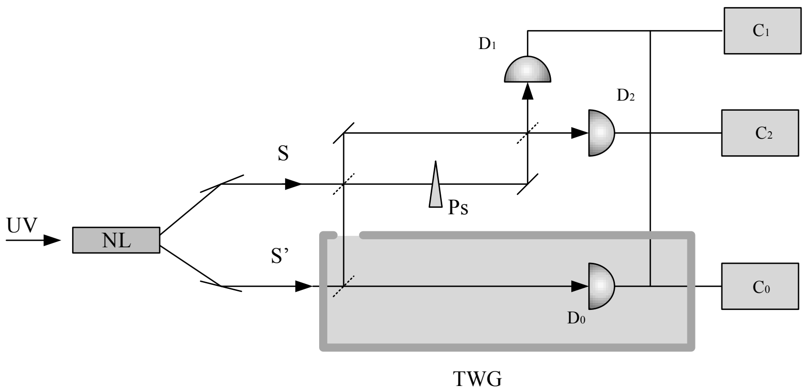

Fig.6 - Experimental setup for detection of theta waves

In the sketch we see a UV source acting on a nonlinear crystal, NL, producing in a parametric down conversion process two incoherent photons at the same time. Ps represents the phase shifting device. The signal photon coming from source S, that behaves like a single photon source, enters a Mach-Zehnder. The idler photon, from S', stands for the monophotonic source of the theta wave generator, TWG. Only when the detector D_0 is triggered by the arrival of the idler photon from S' will the gate in the TWG open, allowing for the empty wave to enter the Mach-Zehnder interferometer. Naturally, as indicated in the drawing, all detectors are in coincidence so that only when detector D_0 is activated will the counters be on, upon the eventual arrival of photons.

The expected results of this experiment are:

a) According to the Copenhagen interpretation of quantum mechanics:

Since the idler photon coming from S' was detected at D_0 there occurs the collapse of the wave function. In such conditions nothing coming from the source S' can enter the interferometer. In this situation, at the interferometer we will have only the photons coming from S. The predicted intensity seen at detectors D_1 and D_2 and registered at counters C_1 and C_2 will be respectively,

$$\begin{cases} I_{C1} = |\phi_{RR} + \phi_{TT}|^2 \\ I_{C2} = |\phi_{RT} + \phi_{TR}|^2 \end{cases} \tag{6}$$

giving,

$$\begin{cases} I_{C1} = \frac{1}{2} I_0 (1 - \cos\delta_\psi) \\ I_{C2} = \frac{1}{2} I_0 (1 + \cos\delta_\psi) \end{cases}, \tag{7}$$

in which, δ_ψ, represents the phase difference between the coherent waves originated by the source S.

Choosing for the relative phase shift difference, the null value, $\delta_\psi = 0$, one will finally get

$$\begin{cases} I_{C1} = 0 \\ I_{C2} = I_0 \end{cases} \tag{8}$$

b) According to the causal de Broglie realistic approach:

It is necessary to consider the waves coming from source S, ϕ_{ij}, plus the theta waves θ_{ij}, coming from the theta wave generator, TWG. Hence

$$\begin{cases} I_{C1} \propto |\phi_{RR} + \phi_{TT} + \theta_{TR} + \theta_{RT}|^2 \\ I_{C2} \propto |\phi_{RT} + \phi_{TR} + \theta_{RR} + \theta_{TT}|^2 \end{cases} \tag{9}$$

The sub-indices stand for the reflection and transmission at the beamsplitters. For instance, θ_{RT} indicates that the subquantum wave entering the interferometer is first reflected at the first beamsplitter and transmitted at the second.

Assuming that we are in a situation where all interfering subquantum waves have equal intensity

$$|\theta_{RT}|^2 = |\theta_{TR}|^2 = |\theta_{TT}|^2 = |\theta_{RR}|^2, \tag{10}$$

after some calculations[6] and recalling that the two sources have random relative phase, we have for the predicted final intensities at each output port C_1 and C_2:

$$\begin{cases} I_{C1} = \frac{1}{2} I_0 (1 - \cos\delta_\phi + \cos\delta_\theta) \\ I_{C2} = \frac{1}{2} I_0 (1 + \cos\delta_\phi - \cos\delta_\theta) \end{cases} \tag{11}$$

For equal optical pathlengths, we may write

$$\delta_\phi = \delta_\theta = 0 \tag{12}$$

which by substitution in the previous expression gives

$$\begin{cases} I_{C1} = \frac{1}{2} I_0 \\ I_{C2} = \frac{1}{2} I_0 \end{cases} \tag{13}$$

These results imply that for this choice of experimental parameters the predictions of the two approaches are quite different:

a) In the orthodox approach:
Subquantum waves do not exist, therefore only detector D_2 counts that is: $I_{C1} = 0,\ I_{C2} = I_0$

b) In the causal de Broglie approach:
Subquantum waves do have physical reality. In this case both detector count at the same rate $I_{C1} = I_{C2} = \frac{1}{2} I_0$.

Concluding, for these experimental conditions, the real observable physical action of the empty waves manifests itself through the change the overall probability scheme at the output ports of the interferometer :

De Broglie	Orthodox QM
$\begin{cases} I_{C1} = \frac{1}{2} I_0 \\ I_{C2} = \frac{1}{2} I_0 \end{cases}$	$\begin{cases} I_{O1} = 0 \\ I_{O2} = I_0 \end{cases}$

2.2 – The Mandel experiments at Rochester

A different opposite variant of the former experiment was performed by Mandel[16] and his collaborators, X.Y. Zou and T. Grayson, at the University of Rochester, USA, in 1992.

In this experiment the theta waves were led to produce interference while the full waves were to produce noise.

Accordingly, the causal prediction would be the interference pattern, giving:

$$I_C \propto (1 + \tfrac{1}{2}\cos\delta_\theta), \tag{14}$$

while the orthodox approach, with no interferences, would predict:

$$I_O \propto const. \tag{15}$$

That is, for this experimental setup the orthodox approach predicts no interference whereas de Broglie expects interferences with a visibility factor of 50%

The results of the experiment are shown in the next picture, Fig. 7.

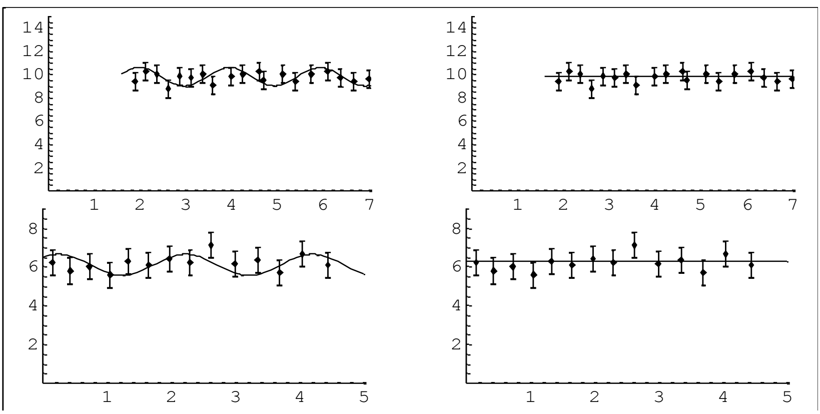

Fig. 7 - Results of the experiment performed at Rochester for the detection of theta waves.

The first line shows the published results, while the second line are the results privately communicated by Mandel to one of the authors.

The same data points are represented in two columns. The first column was fitted with an interference sinusoidal line by one of us, corresponding to a visibility factor of 10%. In the second column, the data is fitted with a straight constant line as suggested by Mandel and coworkers.

Mandel *et all* concluded in their paper[16] that the performed experiment disproved the existence of de Broglie waves. We think that this conclusion is not very sound since the same experimental data can also be fitted using a sinusoidal line representing a visibility of 10%. Even if, in theoretically ideal conditions, the expected visibility is 50%, in real concrete experiments this value is never attained. Mandel's results are undecidable at best. Furthermore, using empty waves to get interference may be technically more difficult than to use them as a noise source, as already described in this paper. The experiment needs to be redone, we propose, in better conditions, to clarify an important problem.

Another recent developed optical technique known under the name of ghost imaging[17] allows also for the concrete possibility of testing the ontic nature of de Broglie waves. Indeed, some experiments done by the German group of Menzel and collaborators[17, 18], elucidate, in our view, the question about the real physical nature of these subtle subquantum waves, with much lower levels of energy compared to the actual detectors' threshold.

Experimental detection of quantum empty waves needs to be redone, in better conditions, to clarify the matter.

3 Technological application of subquantum waves

In what follows we use the expression "subquantum wave" in place of "quantum wave", "de Broglie wave" and all the others. The main motivation is that, for one, such waves seem to possess much less energy than the particles, thus being unable to trigger actual common quadratic detectors. On the other hand, they seem to reside at a lesser scale than quantum particles. We thus expect such (empty) subquantum waves to penetrate mediums in a more efficient way than, for instance, common electromagnetic radiation. Most likely because EM waves carry photons in most circumstances.

One of the first and immediate consequences of the real physical nature of de Broglie waves is the possibility of developing concrete technological applications, namely subquantum wave telecommunication.

The next picture shows what we have called a possible subquantum wave emitter, Fig. 8.

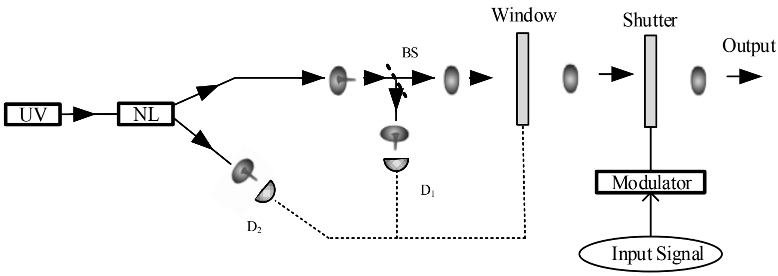

Fig.8 - Subquantum wave emitter

In this setup a nonlinear crystal is stimulated by a pump laser beam causing a parametric down conversion, with two photons being emitted at the same time. To make sure that we have a single photon, each impinging the beamsplitter BS at a time, the two detectors D_1 and D_2 are activated simultaneously. When the detector D_1 clicks it means that along that path, the reflection path, the reflected quantum wave went along with the photon. In this situation along the transmission path followed only the empty transmitted subquantum wave with a minute amount of energy.

A window connected electronically to detector, D_1, opens at the same time the former clicks, allowing the passage of the transmitted subquantum wave.

In front of this window is placed a Shutter operated by Modulator. This Modulator commands the opening of the Shutter allowing the subquantum waves to pass, according to the binary input string of information one wishes to transmit.

Due to the very feeble energy the subquantum waves the common detectors are unable to detect them. Thus, in order to detect subquantum waves a special kind of detector is needed. A possible detecting device is shown in Fig. 9.

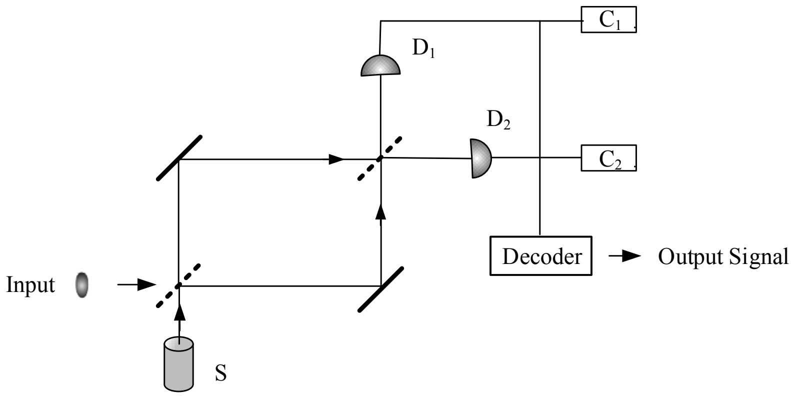

Fig. 9 - Subquantum wave detector

The device consists basically of a well-calibrated Mack-Zehnder interferometer, a very stable high intensity monophotonic source S plus the detectors D1 and D2 and a Decoder. The interferometer is adjusted in such a way that the waves are in phase at the output port that leads to detector D_1 and in phase opposition at the other output port. In such an arrangement only detector D_1 is activated and counts.

When the subquantum waves, coming from the emitter, enter the input port, they interfere additively with the other waves in the interferometer. Since there is no phase correlation between the incoming subquantum waves and the device internal generated waves, the former act like a kind of noise source, coming from the outside. So, depending on the strength and overlapping degree in the noise interference process, the influence of the incoming subquantum waves is felt by observing a decrease of the count rate at detector D_1 and, concomitantly, by an increase of the counting rate at detector D_2.

Calculations for the working of the subquantum wave detector are like the ones mentioned for the experiment proposed in 2.1.

When there are no incoming subquantum waves entering the Subquantum wave detector input, one has:

$$\begin{cases} I_1 = I_0 \\ I_2 = 0 \end{cases}$$

and the difference between the two output ports is, $\Delta = I_0$.

Each time a subquantum wave arrives, entering the input, one has:

$$\begin{cases} I_1 = \frac{1}{2} I_0 \\ I_2 = \frac{1}{2} I_0 \end{cases}$$

giving for the difference, $\Delta = 0$.

Summarizing:

When no subquantum wave has arrived from outside, only D_1 counts, and the Subquantum wave detector shows $\Delta = I_0$.

When there are subquantum waves arriving from the outside emitter, and assuming the ideal case of perfect overlapping and same intensity, both D1 and D2 count at the same rate. That is, the action of the entering subquantum wave changes the overall probabilities scheme in such a way that now both detectors register the same number of photons. Accordingly, the Subquantum wave detector now shows $\Delta = 0$.

These predictions, as stated, are valid in the ideal case where the two sources, the incoming subquantum wave and the internal produced waves, interfere as if they were all emitted at the same time. Furthermore, in the above, the waves have the same amplitude.

Still, this does not correspond to the real situation in which the two wave sources, external and internal to the Subquantum wave detector, emit independently. Even if the source S is very steady, the rate of the waves arriving from the subquantum emitter will depend on the modulation related with the information to be transmitted.

In these conditions, sometimes the waves arrive all at D1 and D2 precisely at the same time, which corresponds to a complete overlapping, while other times they don't even partially mix. If it happens that the external and internal waves do not overlap at the interference region, then there will be no output signal about the presence of a subquantum wave.

Between these two extreme situations there are, of course, all the intermediate cases of partial superposition as shown in the next picture, Fig.10.

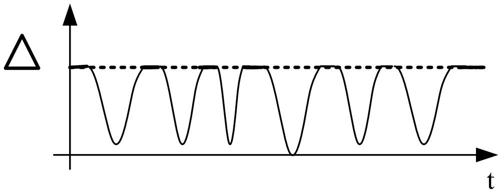

Fig.10 - Observed difference in the counting rate between C1 and C2 for the Subquantum wave detector

The straight line stands for no subquantum wave arrival. Each time a subquantum wave arrives and superimposes partially with the waves from the stable high intensity photonic source S a dip appears. The strength of the dip depends on the overlapping degree, as already mentioned. Naturally, the next step is to transform these dip bit rate into meaningful information by the Decoder. This can be done with the available standard electronics and computers, complying with error correcting coding if necessary.

4 Conclusion

The ontological nature of quantum waves is a problem that begs for clarification. In a certain sense it parallels the days when, just before Quantum Mechanics was formulated, the very existence of atoms was doubted. Furthermore, the existence of quantum waves suggests the possibility for greater intelligibility about quantum phenomena, a property than no theory should easily dismiss.

Presently, we have the technological means to investigate and detect quantum waves. We believe that such a discovery would allow for a new understanding of the world and, eventually, for a technological transformation to occur. Specifically in communications, but possibly also in quantum computation, since the decoherence problem attributed to the interaction between particles, could possibly be diminished using empty waves.

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