Collective effects in droplets dynamics: the rosette model

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Bouncing oil droplets are complex systems. They present some analogy with quantum (de Broglie) particles, in the sense that their trajectories are guided by a self-generated "pilot" wave. Their dynamics is also characterized by a non-negligible memory time which often imposes, if we wish to theoretically grasp their behaviour, to resort to rather heavy and opaque numerical treatments. Moreover the interaction between two droplets is not simple: it exhibits a spatial alternance of repulsive and attractive behaviours. When many droplets are present, the collective dynamics is even more complex. It may thus happen to be helpful to adopt a simplified, effective description of their dynamics if we wish to be able to predict certain aspects of their collective behaviour. The model presented here (rosette model) encapsulates the main features of the two by two droplets interaction and makes it possible to predict some properties of the dynamics of an isolated droplet orbiting around a dense "core" where many droplets are present, in analogy with the dynamics of a star orbiting around the dense centre of a galaxy.

1 Introduction

In ref. [1] one can read:

...It is worth noting that, quite recently, de Broglie's point of view has been revived, be it indirectly, by experimental observations in hydrodynamics, which show that certain macroscopic objects, so-called walkers (bouncing oil droplets), exhibit many of the features of the de Broglie-Bohm (dB-B) dynamics [2, 3, 4, 5]. These unexpected developments not only show that de Broglie's ideas encompass a large class of systems, but they might in the future also allow us to build a bridge between quantum and classical mechanics, where ingredients such as nonlinearity, solitary

waves and wave monism play a prominent role... ...These (bouncing oil droplets or walkers) *take the form of oil droplets bouncing off the surface of a vibrating bath of oil, excited in a Faraday resonance regime (the walkers are prevented from coalescing into the bath, the vibration of which creates a thin film of air between its surface and the droplet, and therefore seem to levitate above it). Walkers exhibit rich and intriguing properties. Roughly summarized, they were shown experimentally to follow dB-B-like quantum trajectories. For instance, when the walker passes through one slit of a two-slit device, it undergoes the influence of its"pilot-wave" passing through the other slit, in such a way that, after averaging over many dB-B like trajectories, the interference pattern typical of a double-slit experiment is restored and this despite the fact that each walker passes through only one slit. The average trajectories of the drops exhibit several other quantum features such as orbit quantization, quantum tunneling, single-slit diffraction, the Zeeman effect and so on.*

At this level it is worth noting that the analogy with quantum (de Broglie) particles is de facto limited: two droplets entanglement does not exist in nature, as far as we know, and the dynamics of droplets, as it is modeled in a classical hydrodynamical approach, is characterized by a non-negligible memory time, which maybe explains why up to now it was impossible to derive an effective (Markovian) Schrodinger equation on the basis of the (non-Markovian) droplets phenomenology [6, 7]. For the same reason, the connection [2, 3, 8, 9] with dB-B dynamics [10, 11, 12] and de Broglie's double solution program [13, 14] has still today merely the status of a stimulating analogy. Actually, the dynamics of a single droplet exhibits memory effects [15, 16], self-interaction [4, 17], ergodicity [18], spinning [19] and wave-particle duality [5] altogether, which makes it particularly complex, even in the single particle (droplet) case.

The interaction between two droplets has also been shown to exhibit a rich and remarkable behaviour [20, 21]: depending on the distance between the two droplets the interaction will alternatively be attractive and repulsive.

In ref. [3], for instance, it is mentioned that: *...Depending on the value of d (which represents the impact parameter of the collision) the interaction is either repulsive or attractive. When repulsive, the drops follow two approximately hyperbolic trajectories. When attractive, there is usually a mutual capture of the two walkers into an orbital motion similar to that of twin stars ... .*

As has been emphasised in ref.[22] this observation is seemingly related to the properties of the Green function of the Helmholtz equation which is ubiquitous in the literature related to bouncing oil droplets [23]. In section 2, we apply to droplets a model [24] previously conceived by one of us (TD) in another context (galactic rotation curves), from now on called the rosette model, because it predicts that trajectories have the shape of a rosette. In this model, the Green function of the (3D in this case) Helmholtz equation also plays a fundamental role. The rosette model predicts in accordance with the model of ref. [24] that when a dense distribution of droplets is located in a small region of space, an isolated droplet situated far away from this region will undergo an effective attractive force towards the dense "droplets" core. Due to the alternance of attractive and repulsive forces the trajectory will have the shape of a rosette as will be confirmed by our numerical treatment. As we discuss in section 2, two regimes can be distinguished: coherent and incoherent, depending on whether or not the droplets in the dense core interact "in phase" or independently (incoherently) with the isolated droplet.

In the coherent case the interaction is similar to the interaction with one central droplet but its intensity is multiplied by the number of droplets in the central region.

In the incoherent case the resulting interaction is still attractive but the intensity is quite weaker and we expect that it scales like the square root of the number of droplets in the core, in accordance with the law of large numbers.

Even if our treatment is not fully analytic, our model makes it possible :

1. to develop a simple intuition of the problem and to predict the appearance of an effective attractive force on the isolated droplet, directed towards the centre of the dense region.

2. to simulate thanks to a rather simple numerical treatment (section 3) the main features of this effective attractive force (intensity, escape velocity, and so on).

3. to infer (in the incoherent regime) the appearance of a generalized Kepler law for circular movements, characteristic of an effective force in $1/r$ in 3D (galactic velocity curves), and $1/r^{1/2}$ in 2D (droplets).

2 The rosette model.

2.1 Two droplets interaction: alternance of attraction and repulsion; Green function of Helmholtz's equation.

In ref. [22] one of us (TD) developed a model aimed at fulfilling de Broglie's double solution problem which led to the prediction of the appearance of a pseudo-gravitational field between two particles A and B proportional to the 3D Green function of the Helmholtz equation

$$g^3(|x_B - x_A|) = -\frac{1}{4\pi}\frac{\cos(k\,|x_B - x_A|)}{|x_B - x_A|} \tag{1}$$

with $|x_B - x_A|$ the distance between the two particles.

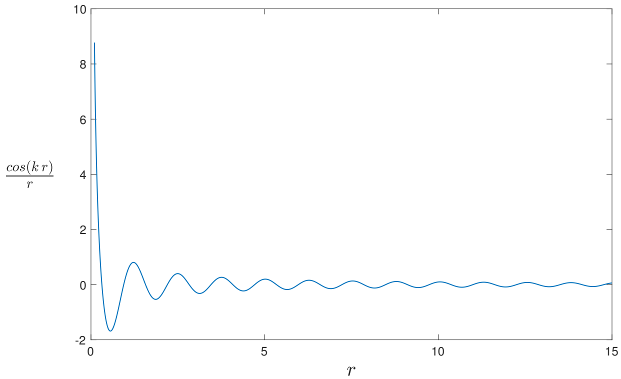

Figure 1: -4π times the Green function of the 3D Helmhotz equation.

This model explained the appearance of repulsive and attractive gravitation in droplets phenomenology in terms of alternating regions of pseudo-gravitational attraction and repulsion.

It also explains explains an apparent quantization rule of stable orbits in terms of the topology of attractive and repulsive basins. For instance, one can read in ref.[22] the following:

... In the same vein, we explain the appearance in the case of two interacting walkers of a pseudo-quantisation rule, self-adapting to the forcing frequency, and similar to the one observed in ref. [4] according

to which orbital radii obey $d^{orb} = (n/2 - \epsilon)\lambda_F$, with n a positive integer, where it was made use of the fact that the Faraday frequency is one half of the forcing frequency. From our point of view this effect is not specifically quantum, it is rather related to the very unfamiliar topology of the attractive and repulsive (!) gravitational basins ...

As we noted before, $g^3(r) = -\frac{1}{4\pi}\frac{\cos(k\,r)}{r}$ is (1) a Green function of the 3-D Helmholtz equation. However it is more appropriate to work with the Green function of the 2-D Helmholtz equation if we wish to describe bouncing oil droplets, in which case, instead of (1), it is more appropriate, following [23], to consider an effective potential of the form

$$\mathcal{I}m\left(g^2(r)\right) = -\mathcal{I}m\left(\frac{i}{4}\,H_0^{(1)}(k\,r)\right) \tag{2}$$

where $H_0^{(1)}$ is a Hankel function, which is proportional [8] to a Bessel function $J_0(kr)$. In our model the isolated droplet is situated far away from the dense central region where many droplets are located. It is then justified to replace the 2D Green function (2) by its asymptotic expression $g^2(r)$, proportional [23] to

$$-(1/4\pi)cos(kr - \pi/4)/\sqrt{kr}. \tag{3}$$

Note that the dephasing $-\pi/4$ does not modify the qualitative features of the dynamics so that we omitted it in the rest of the paper, as well as in the simulations, on order to simplify the treatment.

Although the 3-D potential can be considered as representing an abstract action-at-a-distance, for instance a generalised gravitational interaction, the situation is different in the case of droplets, where the potential represents the heigth of the bouncing fluid in the vicinity of the droplet. It possesses thus a direct reality that we can “touch with the finger”. The force is similar to the force undergone by a massive ball moving on an irregular surface. When the surface is flat, the gradient of the potential (height) is zero, otherwise, the ball undergoes a force proportional to the gradient of the potential. In the same vein, the parameter k represents 2π divided by the Faraday wavelength in the case of droplets. In the 3D case the value of k depends on the situation that we wish to represent in our model. For instance, as we explain in appendix, if we wish to represent a generalised Newtonian gravitational potential, k is equal to 2π divided by a length which is of the order of 5 to 6 lightyears.

2.2 N+1 droplets interaction: basic ingredients of the rosette model.

Let us from now on consider an isolated droplet orbiting around a dense "core" where many droplets are present, in analogy with the dynamics of a star orbiting around the dense centre of a galaxy. The interaction undergone by the external droplet is the sum of the interactions between this droplet and each droplet of the core.

$$V(x,y) = -C\ \Sigma_{i=1}^{N}\ g(\sqrt{(x-x_i)^2+(y-y_i)^2}) = -C\ \Sigma_{i=1}^{N}\ g(r_i) \quad (4)$$

with C an appropriate dimensional factor.

- In 2-D we get, replacing the Green function by its asymptotic expression and disregarding the dephasing $-\pi/4$

$$V^2(x,y) \sim -C\ \Sigma_{i=1}^{N}\ \frac{\cos(k\,r_i)}{\sqrt{kr_i}} \quad (5)$$

- In 3-D (which provides a model for an external star orbiting around the dense centre of a plane galaxy) we get

$$V^3(x,y) = -C\ \Sigma_{i=1}^{N}\ \frac{\cos(k\,r_i)}{k\,r_i} \quad (6)$$

These are the basic ingredients of the rosette model: in our numerical treatment (next section) we shall in a first time randomly generate N locations in a finite region of the 2D space. Then to predict the dynamics of an isolated droplet far away from the central region we shall integrate Newton's law with an effective potential which is equal to the sum of the two by two interaction potentials between the isolated droplet and each droplet in the central region.

At this level it is worth distinguishing two regimes, coherent and incoherent, in analogy with optics[1]:

[1] The analogy with optics is due to the fact that the effective force, at a distance quite larger than $1/k$ is dominated by the contribution $C\ \Sigma_{i=1}^{N}\ \frac{(-)\sin(k\,r_i)}{r_i}$ where each force is characterized by a phase. When all phases are the same, we will say that the regime is coherent, when all phases are independent from each other we will say that the regime is incoherent.

- the coherent one where the distance between the droplets in the central cluster is smaller than the inverse of k. Then, roughly

$$-C\Sigma_{i=1}^{N} g(\sqrt{(x-x_i)^2+(y-y_i)^2}) \approx -N \cdot C \cdot g(\sqrt{(x-x_0)^2+(y-y_0)^2}) \quad (7)$$

where (x_0, y_0) represents the position of the center of mass of the cluster; this is so because the attractions and repulsions are always "in phase".

- the incoherent regime occurs when the size of the central cluster is larger than the inverse of k. Then attractive and repulsive forces add to each other incoherently. As was discussed in ref. [24], even though the intensity of the global force is null in average, there should appear an effective force directed towards the central cluster, due to the fact that all attractions are always directed towards this centre, which induces a persistent curvature around it. This is not true for what concerns the repulsive contributions to the curvature which globally compensate each other. We represent in figure 2 a drawing representing a regular succession of attractive and repulsive forces; it helps to understand why despite of the fact that the force is null in average it however results into an effective curvature directed towards the densely populated region. These features explain why out model has been called the rosette model.

Remark that we observed by performing numerical simulations an orbit very similar to the one represent in the drawing of figure 2 as can be seen from figure 3 (of course it is very regular which corresponds to the coherent regime). In the simulation corresponding to figure 2 the parameter L represents the size of the central core inside which we chose at random the positions of the $N = 40$ central stars. Here, L was taken to be equal to 1, while k was taken to be equal to $\pi/4$. Obviously, the distance between the stars is small relatively to $2\pi/k$ (here $2\pi/k$=8) which corresponds to the coherent regime. In all our simulations the distance between the centre of the core and the external star is quite larger than the size of the core L and is also larger than $1/k$.

We expect that the effective force will scale in the incoherent regime like the square root of N. It should also scale like $1/R$ $(\frac{1}{\sqrt{(x-x_0)^2+(y-y_0)^2}})$ if we make use of the 3-D Helmholtz Green function ($g = g^3$) [24] and like $1/\sqrt{R}$ $(\frac{1}{\sqrt{(\sqrt{(x-x_0)^2+(y-y_0)^2})}}$ in the 2D case ($g = g^2$) [23, 8].

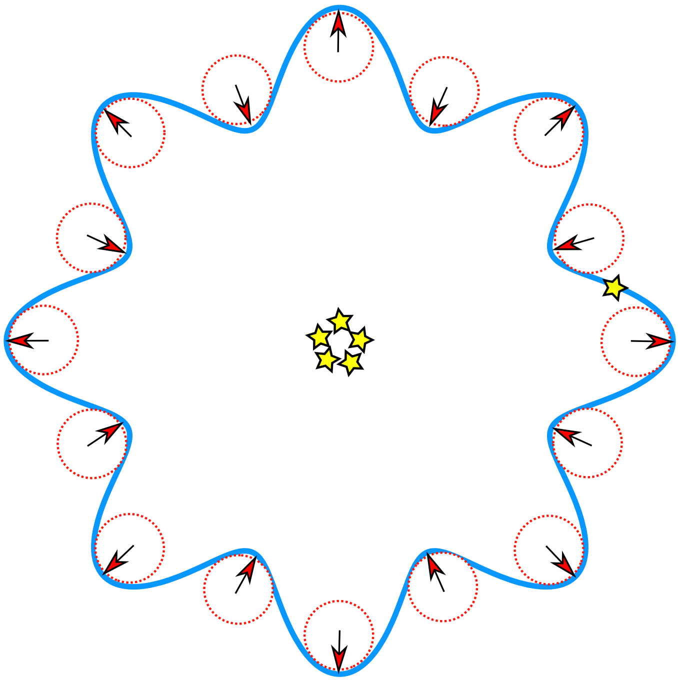

Figure 2: Drawing of a external droplet undergoing a regular succession of attractive and repulsive forces (with the radius of curvature respectively situated inside the orbit and outside from it).

This model is of course oversimplified:

- droplets are treated as 2-D classical material points; we do not associate a phase to each of them although it has been shown experimentally that in stable configurations droplets bounce either in phase or in anti-phase [3, 25] (see also [20, 21]);

- the positions of the droplets of the core are artificially frozen;

- the interaction is expressed through an instantaneous classical potential of interaction (no delay, no memory effect).

It is thus clear that many aspects of the complexity of the droplets dynamics are completely ignored here; nevertheless it leads, as we show in the present paper thanks to various numerical simulations, to the

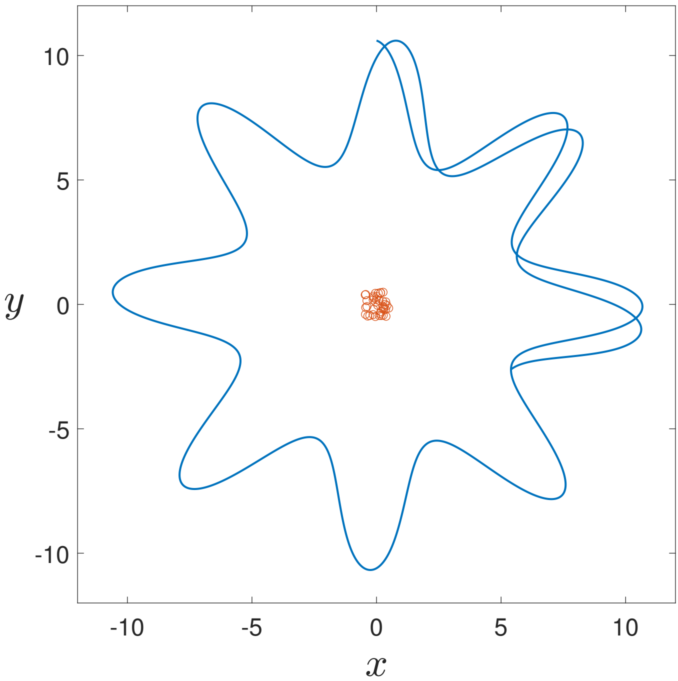

Figure 3: Simulation of an orbit in the coherent regime, very similar to the drawing of figure 2. Here we chose $k = \frac{\pi}{4}$, $L = 1$ and $N = 40$.

prediction of the existence of attractive orbits of quasi-circular shape, in the coherent and incoherent regimes as well. It also leads to the prediction in the incoherent regime of a generalised Kepler law, typical of a force in $1/R$ in 3D and in $1/\sqrt{R}$ in 2D (droplet case) which constitutes an experimental challenge in the case of droplets.

3 N+1 droplets interaction: numerical simulations.

3.1 Incoherent regime: 3D Green function.

In a first time we simulated the rosette model with the 3D Green function. Numerical simulations show that for a certain regime of velocities there exist stable nearly-circular orbits. This can be seen from figures 4, 5 and 6. In the simulation corresponding to figure 4 the parameter

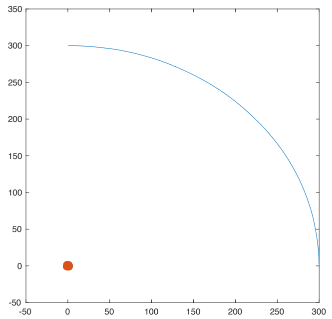

Figure 4: Plot of the trajectory of a droplet (Incoherent regime) with $k = 10$, $N = 40$, $L = 5$, initial position $(x_0 = 0, y_0 = 300)$ and initial velocity $(v_{x_0} = 0.3, v_{y_0} = 0)$.

L which represents the size of the central core was taken to be equal to 5, while k was taken to be equal to 10. Obviously, the typical distance between the stars is large relatively to $2\pi/k$ (here $2\pi/k \approx 0.6$) which corresponds to the incoherent regime.

Moreover for the same initial tangential velocities there also exist stable nearly circular orbits located at various distances from the central cluster which is reminiscent of a force in 1/R as can be seen[2] from figure 6. The analogy with a force in 1/R is only partially true however because,

[2]Indeed, for a force in $1/R$, circular orbits obey $\omega^2 R = C/R$ where ω is the angular velocity and R the distance to the attractive centre. This implies that rotation curves are flat: ωR is a constant.

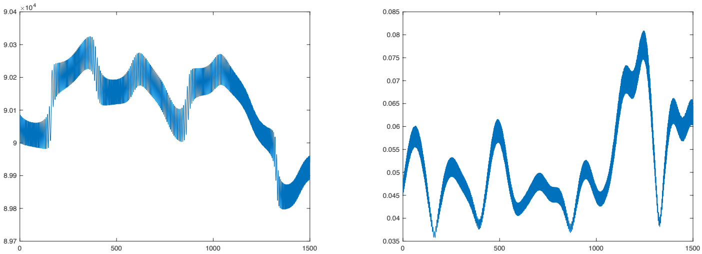

Figure 5: Plot of the square of the distance to the origin (left), and of the kinetic energy of the droplet plotted in the previous figure, illustrating the randomness of the force undergone by the external droplet in the incoherent regime. Relative fluctuations of the radius are relatively small, of the order of 0.3 percent, while fluctuations of the kinetic energy (and thus of the velocity) are relatively large, of the order of 50 percent.

as can be seen from the figure 5 (right), the statistical fluctuations of the kinetic energy are not negligible at all, as it would be the case with an effective force in 1/R in the case of a circular orbit. Amazingly, despite of these strong fluctuations, stable trajectories seemingly adopt a nearly circular shape and obey a generalised Kepler law through an adaptative process that we did not elucidate yet.

It is worth noting that the modified Kepler law corresponding to a force in 1/R has been observed in galactic rotation curves ("flat" rotation curves). This observation is at the origin of various speculative models, e.g. dark matter models, modified Newton dynamics (MOND) and so on (as explained in ref. [22] and in appendix). From this point of view, the (3D version of the) present model constitutes another explanative attempt of the modified Kepler law characterizing galactic rotation curves.

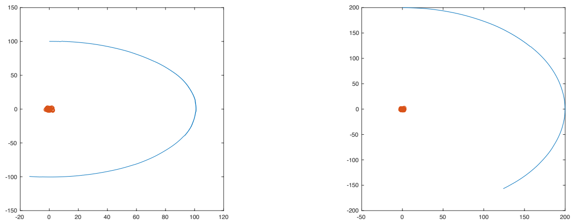

Figure 6: Generalised Kepler law-Plot of the trajectory of a droplet (Incoherent regime) with same parameters as in figure 5 but with initial position ($x_0 = 0, y_0 = 100$) and initial velocity ($v_{x_0} = 0.3, v_{y_0} = 0$) (left) and initial position ($x_0 = 0, y_0 = 200$) and initial velocity ($v_{x_0} = 0.3, v_{y_0} = 0$). The shapes of both trajectories were checked to be quasi-circular.

3.2 Coherent regime: 3D Green function.

As can be seen from figures 3 and 7, the trajectory in the coherent regime has still the shape of a rosette but quite more regular than in the incoherent case. What happens here is that the trajectory is trapped in a valley of the potential. As we discussed before, the potential is equal in good approximation to $-N\, C\, \Sigma_{i=1}^{N} \frac{\cos(k\sqrt{(x-x_0)^2+(y-y_0)^2})}{k\sqrt{(x-x_0)^2+(y-y_0)^2}}$, where (x_0, y_0) represents here the position of the center of mass of the droplets in the core. The external particle regularly oscillates inside this valley (see e.g. figure 1 for visualizing the succession of valleys of the effective potential in the 3D case). Randomness nearly disappears from the dynamics in the coherent case.

3.3 2-D Green function.

If we use the 2D Green function in place of the 3D one, the main features of the dynamics are preserved, excepted that the scaling in the incoherent

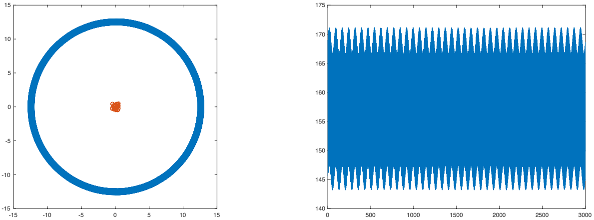

Figure 7: Plot of the trajectory of a droplet (Coherent regime) with $k = 1$, $N = 40$, $L = 1$, initial position $(x_0 = 0, y_0 = 13)$ and initial velocity $(v_{x_0} = 1, v_{y_0} = 0)$ (left) and plot of the square of the distance to the origin (right).

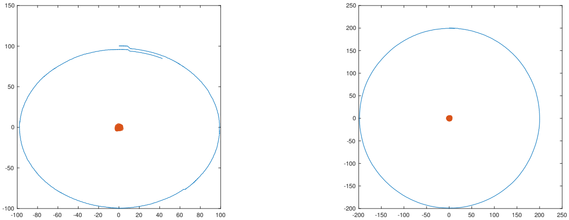

Figure 8: Plot of the trajectory of a droplet (Incoherent regime) with $k = 10$, $N = 40$, $L = 5$, potential=25 $cos(kr)/\sqrt{r}$, initial position $(x_0 = 0, y_0 = 100)$ and initial velocity $(v_{x_0} = 5, v_{y_0} = 0)$ (left), and new initial position $(x_0 = 0, y_0 = 200)$ and velocity $(v_{x_0} = 6, v_{y_0} = 0)$ (right).

regime is different. This can be seen from figure 8 where we found quasi-circular orbits associated[3] to a generalized Kepler law $\omega^2 \cdot R^{3/2} = constant$, to compare with the law $\omega \cdot R = constant$ associated to the 3D Green function or with the Kepler law $\omega^2 \cdot R^3 = constant$ in the case of a Hookian attractive force in $1/R^2$.

In the coherent regime, we find the same regular rosette-shape trajectories as can be seen from figure 9.

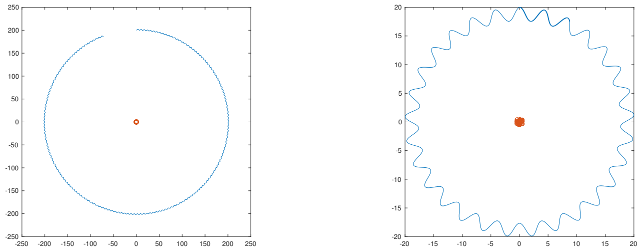

Figure 9: Plot of the trajectory of a droplet (Coherent regime) with $k = 1$, $N = 40$, $L = 1$, initial position $(x_0 = 0, y_0 = 200)$ and initial velocity $(v_{x_0} = 6, v_{y_0} = 0)$ (left) and initial position $(x_0 = 0, y_0 = 20)$ and initial velocity $(v_{x_0} = 6, v_{y_0} = 0)$ (right). Here we used the D=2Greenfunction, not the D=3 one as before, with an effective potential equal to 25 $cos(kr)/\sqrt{r}$.

4 Conclusion

In this paper we confirmed through numerical simulations the main features of the rosette model, originally aimed at explaining the non-standard (flat) distribution of velocities exhibited by stars orbiting in the periphery of galaxies, far away from the galactic cluster. In the incoherent regime, the rosette model leads to the prediction according to which an alternance of repulsive and attractive forces, always directed in the same direction, does not result into a global null accelaration, but results into an effective attraction towards the centre of the random force, here a cluster of bouncing oil droplets confined in a small region

[3]Indeed, for a force in $1/\sqrt{R}$, circular orbits obey $\omega^2\sqrt{R} = C/R$ where ω is the angular velocity and R the distance to the attractive centre. This implies that rotation curves obey $\omega R^{3/4}$ is a constant, so to say the velocity varies like $R^{1/4}$, in good agreement with the plots of figure 8 where $6/5 \approx 2^{1/4}$.

of space (in analogy with the dense core of a galaxy as was considered in the original formulation of the rosette model [24]).

The numerical simulations presented in section 3 show that indeed there exist stable trajectories with a rosette shape, in the coherent and incoherent regimes as well, a prediction that deserves to be investigated in the lab....

The next step would thus be to test experimentally the predictions of the rosette model in an experiment involving a similar configuration of bouncing oil droplets. For instance the generalized Kepler law $\omega^2 \cdot R^{3/2} = constant$ mentioned in section 3.3 could be scrutinized in experiments.

Various types of collective effects have been reported recently, for instance analogies with spin arrays [26, 27]. Here we propose a new type of dynamics, where the collective effect of a dense community of droplets results into a rosette shape trajectory for an isolated droplet located far away from this dense core. This trajectory is characterized in the incoherent regime by a generalized Kepler law which constitutes a prediction that could be tested experimentally.

Experimental confirmation of our simplified model would reveal a new facet of bouncing oil droplets, which were shown in the past to constitute a vivid source of inspiration regarding our description of the micro world. Here we propose to mimic thanks to droplets intriguing properties of the macro world at the galactic scale e.g. flat rotation curves [24]. In this approach, bouncing oil droplets would provide an analogical computer aimed at simulating (speculative) non-standard models of gravity. From this point of view, they would deserve to be considered as a bridge between the micro, quantum, world and the macro world at galactic scale, an unexpected surprise.

Last but not least it could be interesting to observe the dynamics of an isolated droplet orbiting around a regular lattice of bouncing oil droplets. The existence of such lattices has for instance been reported in refs. [28, 29].

Developing the analogy with optics such a lattice would have properties similar to those of a periodic optical array which implies the appearance of very strong interferences at the level of the dynamics of the isolated droplet.

Acknowledgements

TD thanks C. Borghesi for fruitful remarks and discussions. MH (now at OIST in Okinawa) acknowledges support from Aix-Marseille University for an ATER contract in 2019-2020.

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Appendix: MOdified Newton Dynamics (MOND) versus Vacuum Influenced Newton Dynamics (VIND), an attempt to explain flat galactic rotation curves.

Since the 30's, astronomical observations accumulated showing that Kepler's third law was not respected in the case of isolated stars orbiting far away from a dense galactic core. Several models aim at explaining this anomaly (see ref.[24] and references therein) among which dark matter models according to which invisible massive particles explain why the effective gravitational potential does not scale in $1/R^2$. Another succesful model is the MOND model proposed by Milgrom according to which at very large distance from the galactic core, when the Newtonian acceleration a_N gets very weak, it should get replaced by a modified acceleration equal to $\sqrt{a_N \cdot a_M}$ which is the geometric mean between a_N and a phenomenological constant introduced by Milgrom. This quantity, from now on denoted a_M has the dimensions of an acceleration ($a_M = 1,2 \cdot 10^{-10}$m s^{-2}).

The 3D rosette model described in the present paper constitutes another alternative explanation. The gravitational potential undergone by an orbiting star of mass M is then equal to $-\Sigma_{i=1}^{N}\ GMm_i\frac{\cos(k\,r_i)}{r_i}$ where G is Newton constant and m_i represents the mass of a star belonging to the central cluster. At large distance, $R >> 1/k$, we get a nearly radial force the intensity of which is equal, in good approximation, to $\Sigma_{i=1}^{N}\ GMm_i\frac{k\sin(k\,r_i)}{r_i}$. In the incoherent regime, the phases are randomly distributed, so that $\sin(k\,r_i)$ is randomly distributed too, between -1 and +1. Making use of the law of large numbers, we expect the effective gravitational acceleration to be gaussian distributed around zero, with a variance of the order of $\sqrt{N}\ G < m_i > \frac{k}{R}$, where $< m_i >$ represents the average mass of a star inside the central cluster and R the distance to this core. In a previous work [24], one of us (TD) calibrated the value of k based on the astronomical observations realized with the Pioneer probes and found that $2\pi/k^P$ (where k^P represents the value of k infered from the Pioneer anomaly) is of the order of 5 to 6 lightyears ($k \approx 3 \cdot 10^{-15}$m^{-1}). Estimating $< m_i >$ to be of the order of the mass of the sun, we find that the effective acceleration towards the core of the galaxy is of the order of $\sqrt{N}G < m_{sun} > \frac{k^P}{R}$. Amazingly, this prediction is of the same order of magnitude as the prediction made in the MOND model. In fine, this is so because Pioneer's anomalous acceleration is comparable to Milgrom's acceleration a_M. Indeed, $a_P \approx 8 \cdot 10^{-10}$m

s^{-2} and $a_M = 1,2 \cdot 10^{-10}$m s^{-2}. Making use of the fact that a_P, the acceleration of the Pioneer probe(s) can be shown [24] to be equal to $Gm_{sun}(k^P)^2$, the rosette model leads to the prediction that the effective centripetal acceleration of a far away star is of the order of

$$\sqrt{N}G\frac{<m_i>}{m_{sun}}m_{sun}\frac{k^P}{R}=\frac{<m_i>}{m_{sun}}\sqrt{N}Gm_{sun}\frac{k^P}{R}$$

$$=\frac{<m_i>}{m_{sun}}\sqrt{NGm_{sun}\frac{(k^P)^2}{R^2}}\sqrt{Gm_{sun}}=\sqrt{\frac{<m_i>}{m_{sun}}}\sqrt{Gm_{sun}(k^P)^2}\sqrt{\frac{NG<m_i>}{R^2}}$$

$$=\sqrt{\frac{<m_i>}{m_{sun}}}\sqrt{a_P}\sqrt{\frac{NG<m_i>}{R^2}}.$$

$\sqrt{\frac{<m_i>}{m_{sun}}}$ being of the order of 1 and a_P being of the order of a_M, the predictions made in the framework of the rosette model qualitatively fit with the predictions of the Milgrom MOND model (the modified acceleration being equal to $\sqrt{a_N \cdot a_M}=\sqrt{a_M}\sqrt{\frac{NG<m_i>}{R^2}}$) as already discussed in ref.[24]. Note that our model links Pioneer anomaly to the Milgrom constant in a natural manner, which is not the case with the MOND model. One could object that the ratio $\sqrt{a_P/a_M}$ is still of the order of 2,5 and not equal to unity, but we should not forget that with a probability fifty percent the interaction between the external star and the core is repulsive in the rosette model, which tends in the practice to compensate this factor 2,5.