On connections between L. de Broglie fusion theory and quantum relativistic two-body equations.

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In a previous paper one of us (P.P.) showed that the multitime generalization of the (two-fermion) Bohm-Hiley (BH) equation was not Lorentz covariant. We described in another paper a way to cure this problem which enabled us to generate another equation (the DP equation), close to Bohm-Hiley equation, which is well relativistically covariant. Here we propose a new approach to de Broglie's fusion theory, denoted spatio temporal (ST) coalescence which consists of identifying the temporal and spatial coordinates of a pair of particles in a two-body multitime equation of evolution. We remark here that the coalesced BH and DP equations were already considered by de Broglie in his theory. Moreover, we show that they are intimately related with each other, together with a complementary, relativistically covariant equation (the complementary DP equation). These equations are in one to one correspondence with the Maxwellian vectorial boson (photon) derived by de Broglie in his fusion theory. We also study the connections with a non-Maxwellian (but relativistic) pseudo-vectorial boson that is connected to G. Lochak's work on magnetic monopoles.

1 Introduction.

Integer spin particles play an essential role in quantum field theory, consider e.g. the photon, in QED, as well as the Higgs-Brout-Englert, W and Z bosons, in the standard model [1, 2]. L. de Broglie proposed in his fusion model [3, 4, 5, 6, 7] to associate a relativistic wave equation to the photon, that he built by "concatenating" the equations of two Dirac fermions [8] by a method called the fusion method. Passing from the two fermion equation to Maxwell equation was made possible thanks to the de Broglie-Géhéniau transformation. This model, which is unfortunately nearly fogotten today, presented interesting features such as the possibility to give a nonzero mass to the photon without breaking Lorentz invariance.

There also exist several spin 0 particles which are composed of two fermions such as mesons which consist of a quark-antiquark pair, while α particles are made of four fermions, two neutrons and two protons. It is thus interesting to study in detail the fusion process because it also presents potential applications in several physically relevant situations.

In the present work we propose an alternative approach to the fusion process of de Broglie, denoted here the single time (ST) coalescence process. This work was *in fine* motivated by the recent work of one of us [9] who studied a multitime and multiparticle generalization of the single particle Dirac equation which reduces, when all particles are independent, to a system of Dirac equations originally studied by Wentzel [10]. This multitime equation generalizes the single time Bohm-Hiley equation [11] which is itself a multiparticle generalization of the (single particle) Dirac equation. As is shown in [9], this multitime equation (from now on denoted the BH equation) is not Lorentz invariant. We explained in [12] the lack of relativistic invariance in terms of Lagrangian densities, and proposed a new multitime Dirac equation which derives from an invariant Lagrangian density, from now on denoted the DP equation. In the present paper we shall in a first time systematize this technique in order to generate several relativistic two-particles multitime Dirac equations. In a second time we shall relate these equations to the model of de Broglie by applying the ST coalescence procedure to two-body multitime equations (section 2).

As we shall show, this approach provides an alternative to de Broglie's fusion "recipy" and makes it possible to rederive the fundamental two-fermion single time-single space equations considered by de Broglie in his book *Particules à spin* (section 3). We also briefly recall here the essential ingredients of de Broglie's fusion theory, and show how Maxwell's equations are associated to the fusion of two fermions, when their mass is equal to zero.

Finally (section 4) we reconsider the quasi-duality relation connecting the DP and the complementary DP equation. This quasi-duality is shown here to constitue a fundamental discrete symmetry of de Broglie's fusion theory. When the mass of the Dirac fermions is equal to zero, this quasi-duality becomes an exact duality relation which is in one-to-one correspondence with the duality relation invoked by G. Lochak in his theory of the magnetic monopole[1]. The ideas of George Lochak are seen here to constitue a very natural, nearly "inconturnable", extension of de Broglie's fusion theory, and it is our pleasure to develop these ideas in the present issue, aimed at commemorating the life and work of George Lochak.

2 ST Coalescence: an alternative to de Broglie's fusion recipe.

2.1 de Broglie fusion theory.

To begin with, let us consider two Dirac fermions of masses m_A and m_B. If we consider them as independent, isolated particles, their evolution obeys Dirac equation:

$$(\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c \alpha_0^A)\mathbf{\Psi}_A = 0, \tag{1}$$

$$(\mathbb{1}^B P_0^B - \overrightarrow{\alpha}^B \cdot \overrightarrow{P}^B - m_B c \alpha_0^B)\mathbf{\Psi}_B = 0, \tag{2}$$

where $c \cdot P_0^{A(B)}$ represents the operator $i\hbar\frac{\partial}{\partial t}$ acting on the $A(B)$ particle: $P_0^{A(B)} = i\frac{\hbar}{c}\frac{\partial}{\partial t_{A(B)}}$.

In the same vein[2], $P_{x_i}^{A(B)} = \frac{\hbar}{i}\frac{\partial}{\partial x_i^{A(B)}}$, $i = 1, 2, 3, x_1 = x, x_2 = y, x_3 = z$. The matrices $\alpha_x, \alpha_y, \alpha_z$ and α_0 are the Dirac 4 times 4 matrices. These Dirac α matrices are here defined as follows[14]:

$$\alpha_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \alpha_x = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\alpha_y = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \alpha_z = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{4}$$

[1] The connection between Lochak's pseudo-vectorial boson and magnetic monopoles would deserve a full paper in its own; it has for instance been studied in depth in reference [13].

[2] In what follows, the notation $\overrightarrow{v}$ is always associated to a vector in the 3D space of cartesian components v_x, v_y, v_z.

Following the de Broglie, let us impose that

(1) both particles share the same spatiotemporal coordinates:

$$x_i{}^A = x_i{}^B = x_i;\ i = 1, 2, 3, \tag{5}$$

$$t^A = t^B = t. \tag{6}$$

In other words the two fermions do not live in the configuration space but they share the same "physical" spacetime; this also implies that $P_k^A = P_k^B = P_k$.

(2) the two particles are linked to each other in such a way that they share a same energy and a same momentum, through the constraints

$$\mathbf{\Psi}_m^A P_k \mathbf{\Psi}_l^B = \mathbf{\Psi}_l^B P_k \mathbf{\Psi}_m^A = \frac{1}{2} P_k (\mathbf{\Psi}_m^A \mathbf{\Psi}_l^B), \tag{7}$$

$$\forall m, l \in \{1, 2, 3, 4\}, \forall k \in \{0, 1, 2, 3\}$$

Making use of the aforementioned properties, and introducing a 16 components two fermions wave function $\mathbf{\Psi}^{AB}$ defined through $\Psi_{ij}^{AB}(t, x, y, z, t) = \Psi_i^A(t, x, y, z) \cdot \Psi_j^B(t, x, y, z)$, it is easy to fuse the single fermion Dirac equations of the isolated particles into two-fermion equations which read

$$(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c \alpha_0^A) \otimes \mathbb{1}^B \mathbf{\Psi}_{AB} = 0, \tag{8}$$

$$\mathbb{1}^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c \alpha_0^B) \mathbf{\Psi}_{AB} = 0, \tag{9}$$

de Broglie considered symmetric and anti-symmetric combinations of these equations.

The symmetric combination constitutes a group of 16 equations, each of them containing a temporal derivative of a particular component of $\mathbf{\Psi}_{AB}$:

$$\{(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c \alpha_0^A) \otimes \mathbb{1}^B +$$
$$\mathbb{1}^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c \alpha_0^B)\}\mathbf{\Psi}_{AB} = 0, \tag{10}$$

This group was henceforth called by de Broglie the group of **evolution** equations.

The antisymmetric combination constitutes a groupe of 16 equations, without any temporal derivative of any component of $\mathbf{\Psi}_{AB}$:

$$\{(-\overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c \alpha_0^A) \otimes \mathbb{1}^B - \mathbb{1}^A \otimes (-\overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c \alpha_0^B)\}\mathbf{\Psi}_{AB} = 0. \tag{11}$$

This second group was henceforth called by de Broglie the group of **condition** equations.

In his book *Particules à spin*, de Broglie also considered the symmetric and antisymmetric groups of equations below:

$$\{(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c \alpha_0^A) \otimes \alpha_0^B +$$
$$\alpha_0^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c \alpha_0^B)\}\mathbf{\Psi}_{AB} = 0, \tag{12}$$

$$\{(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c \alpha_0^A) \otimes \alpha_0^B -$$
$$\alpha_0^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c \alpha_0^B)\}\mathbf{\Psi}_{AB} = 0, \tag{13}$$

about which he wrote that they are very important concerning light-matter interaction; we shall come back to them very soon.

2.2 Interesting non-relativistic and relativistic multitime multispace two body equations.

The multitime generalisation of Bohm and Hiley's multiparticle single time equation reads [9]

$$\{(\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c \alpha_0^A) \otimes \mathbb{1}^B$$
$$+\mathbb{1}^A \otimes (\mathbb{1}^B P_0^B - \overrightarrow{\alpha^B} \cdot \overrightarrow{P^B} - m_B c \alpha_0^B)\ \}\mathbf{\Psi}_{AB} = 0, \tag{14}$$

where the matrices $\alpha_x, \alpha_y, \alpha_z$ and α_0 are the Dirac 4 times 4 matrices, while $c \cdot P_0^{A(B)}$ represents the operator $i\hbar\frac{\partial}{\partial t}$ acting on the $A(B)$ particle only: $P_0^{A(B)} = i\frac{\hbar}{c}\frac{\partial}{\partial t_{A(B)}}$; similarly, the operator $\overrightarrow{P}^A$ represents the 3-components operator $\frac{\hbar\overrightarrow{\nabla}}{i}$ acting on the $A(B)$ particle only:

$P_{x_i}^{A(B)} = \frac{\hbar}{i}\frac{\partial}{\partial x_i^{A(B)}}$, $i = 1, 2, 3, x_1 = x, x_2 = y, x_3 = z$.

As we have shown [12], the equation (18) is not Lorentz invariant. The reason therefore is that its solution extremizes an action built with the non-relativistic Lagrangian density

$$\mathcal{L}^{AB} = \mathbf{\Psi}_{AB}^{\dagger}\{(i\hbar\mathbb{1}^A\partial_t^A - \frac{\hbar c}{i}\overrightarrow{\alpha^A} \cdot \overrightarrow{\nabla} - m_A c^2\alpha_0^A) \otimes \mathbb{1}^B$$
$$+\mathbb{1}^A \otimes (i\hbar\mathbb{1}^B\partial_t^B - \frac{\hbar c}{i}\overrightarrow{\alpha^B} \cdot \overrightarrow{\nabla} - m_B c^2\alpha_0^B)\}\mathbf{\Psi}_{AB}. \quad (15)$$

There is a simple way to cure the problem however which consists [12] of replacing the non-relativistic Lagrangian density (15) by the relativistic, Lorentz covariant density

$$\mathcal{L}^{AB} = \mathbf{\Psi}_{AB}^{\dagger}\{(i\hbar\mathbb{1}^A\partial_t^A - \frac{\hbar c}{i}\overrightarrow{\alpha^A} \cdot \overrightarrow{\nabla} - m_A c^2\alpha_0^A) \otimes \alpha_0^B$$
$$+\alpha_0^A \otimes (i\hbar\mathbb{1}^B\partial_t^B - \frac{\hbar c}{i}\overrightarrow{\alpha^B} \cdot \overrightarrow{\nabla} - m_B c^2\alpha_0^B)\}\mathbf{\Psi}_{AB} \quad (16)$$

This is how we found the DP equation

$$\{(\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c\alpha_0^A) \otimes \alpha_0^B +$$
$$\alpha^A \otimes (\mathbb{1}^B P_0^B - \overrightarrow{\alpha^B} \cdot \overrightarrow{P^B} - m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} = 0, \quad (17)$$

We could as well have introduced an anti-symmetric Lagrangian density of the form

$$\mathcal{L}^{AB} = \mathbf{\Psi}_{AB}^{\dagger}\{(i\hbar\mathbb{1}^A\partial_t^A - \frac{\hbar c}{i}\overrightarrow{\alpha^A} \cdot \overrightarrow{\nabla} - m_A c^2\alpha_0^A) \otimes \alpha_0^B$$
$$-\alpha_0^A \otimes (i\hbar\mathbb{1}^B\partial_t^B - \frac{\hbar c}{i}\overrightarrow{\alpha^B} \cdot \overrightarrow{\nabla} - m_B c^2\alpha_0^B)\}\mathbf{\Psi}_{AB} \quad (18)$$

which leads to the complementary DP equation

$$\{(\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c\alpha_0^A) \otimes \alpha_0^B$$
$$-\alpha^A \otimes (\mathbb{1}^B P_0^B - \overrightarrow{\alpha^B} \cdot \overrightarrow{P^B} - m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} = 0, \tag{19}$$

which is well Lorentz covariant, as is the associated density (19).

2.3 ST Coalescence: passing from multitime multispace two body equations to single time single space equations.

At this level, we remark that there exists an alternative recipe for concatenating the two fermions, which consists of "coalescing" the spatio-temporal coordinates in the Lagrangians and equations derived in the previous paragraph. This is nothing else that the ingredient (1) of de Broglie's fusion recipe already introduced before (5,6):

$x_i{}^A = x_i{}^B = x_i$; $i = 1, 2, 3$, $t^A = t^B = t = x_0$ and $P_k^A = P_k^B = P_k$.

Now, the second ingredient of ST coalescence consists of making use of the first ingredient to reduce (coalesce) multitime and multispace two fermion evolution equations of the type considered in the previous paragraph. It is easy to check that applying this procedure to equations (14,18,20) leads to the de Broglie equations (10,12,13).

The antisymmetric non-relativistic equation (11) is obtained in a similar fashion by applying the ST coalescence procedure to the antisymmetric version of the Bohm-Hiley equation (14) which gives

$$\{(\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c\alpha_0^A) \otimes \mathbb{1}^B$$
$$-\mathbb{1}^A \otimes (\mathbb{1}^B P_0^B - \overrightarrow{\alpha^B} \cdot \overrightarrow{P^B} - m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} = 0 =$$
$$\{(-\overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c\alpha_0^A) \otimes \mathbb{1}^B + \mathbb{1}^A \otimes (\overrightarrow{\alpha^B} \cdot \overrightarrow{P^B} + m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} \tag{20}$$

equivalent to (11) .

In summary, applying the ST coalescence procedure, we rederived a pair of non-relativistic matricial equations (10,11) and a pair of relativistic matricial equations (12,13). We realized in the meanwhile that these systems had already been considered by de Broglie in his book *Particules à spin.* Each of these matricial equations consists of a system of 4 times 4, 16, equations.

The non-relativistic pair consists of a group of 16 non-relativistic evolution equations associated to a group of 16 non-relativistic condition equations. In appendix we considered a simplified version of the relativistic equations obtained by assuming that there is no dependence of the wave function to x and y. We checked however that our analysis is still valid if we restore the dependence to x and y, but the computations are too long to be reproduced here. As can be checked from the appendix, each group of 16 relativistic equations consists of a group of 8 evolution equations plus a group of 8 condition equations. Taken together, and one by one, these 32 relativistic equations (80-111) to are strictly equivalent to the 32 non-relativistic equations (48-79). Actually, the evolution equations (48) to (63) can be split into two groups, each of them containing 8 equations.

In the first group the mass terms only appear via the symmetric combination $m_A + m_B$. The 8 equations of the first group also belongs to the coalesced DP equations (80) to (95).

In the second group the mass terms only appear via the antisymmetric combination m_A - m_B. The 8 equations of the secondgroup also belong to the coalesced complementary DP equations (96) to (111). The same can be said regarding the condition equations (64) to (79).

Following de Broglie, the two groups of 16 relativistic equations (each of them containing 8 evolution equations plus 8 condition equations) must be considered as independent, each of them describing a relativistic boson.

2.4 Reformulation in terms of the von Neumann γ matrices.

As is well-known, we are free to reformulate Dirac equation in terms of von Neumann γ matrices, which are most often [14] defined as follows:

$\gamma_0 = \alpha_0$ and $\overrightarrow{\gamma} = \alpha_0 \overrightarrow{\alpha}$. It is worth noting that the gamma matrices anti-commute, square to unity, while $(\overrightarrow{\gamma})^\dagger = -\overrightarrow{\gamma} = \gamma_0 \overrightarrow{\gamma} \gamma_0$. Multiplying Dirac equation (1) at the left by α_0^A we get

$$(\gamma_0^A P_0^A - \overrightarrow{\gamma}^A \cdot \overrightarrow{P}^A - \mathbb{1}^A m_A c)\mathbf{\Psi}_A = ((\gamma^\mu)^A P_\mu^A - m_A)\mathbf{\Psi}_A = 0 \quad (21)$$

It derives from the action associated to relativistically invariant Lagrangian density $\bar{\mathbf{\Psi}}_A((\gamma^\mu)^A P_\mu^A - m_A)\mathbf{\Psi}_A$ where $\bar{\mathbf{\Psi}}_A = \mathbf{\Psi}^\dagger{}_A \gamma_0^A$. If we extremize this action under variations of $\bar{\mathbf{\Psi}}_A$ we get equation (22), while if we extremize it over variations of $\bar{\mathbf{\Psi}}_A$, we get the adjoint of (22).

This procedure directly generalizes when two particles are present. Defining $\bar{\mathbf{\Psi}}^{AB} = (\mathbf{\Psi}^{\dagger})^{AB}\gamma_0^A\gamma_0^B$ the Lagrangian densities (16,19) correspond to the symmetric and antisymmetric combinations

$$\mathcal{L}_{\pm}^{AB} = \bar{\mathbf{\Psi}}^{AB}\{(i\hbar(\gamma^{\mu})^A\partial_t^A - \frac{\hbar c}{i}\overrightarrow{\gamma^A} \cdot \overrightarrow{\bigtriangledown} - m_A c^2) \\ \pm(i\hbar\mathbb{1}^B\partial_t^B - \frac{\hbar c}{i}\overrightarrow{\gamma^B} \cdot \overrightarrow{\bigtriangledown} - m_B c^2)\}\mathbf{\Psi}_{AB}, \tag{22}$$

from which derive, after extremization under variations of $\bar{\mathbf{\Psi}}^{AB}$ the equation

$$((\gamma^{\mu})^A P_{\mu}^A - m_A) \pm ((\gamma^{\mu})^B P_{\mu}^B - m_B))\mathbf{\Psi}_{AB} = 0, \tag{23}$$

which in turn corresponds to the symmetric and antisymmetric bosonic (resp. DP and complementary DP equations) equations (18,20).

3 de Broglie's fusion theory and Maxwell photon

3.1 Two relativistic systems of 16 equations.

In order to link the boson evolution with Maxwell's equations, de Broglie (following Pétiau and Tonnelat) represents the sixteen components of $\mathbf{\Psi}_{AB}$ by a 4 times 4 matrix here denoted $(\mathbf{\Psi}^{AB})$ via the convention $(\mathbf{\Psi}^{AB})ij = \Psi_i^A \cdot \Psi_i^B$. One can check easily that if M^A (M^B) represents a 4 times 4 matrix acting on the A (B) spinor, then $M^A \otimes \mathbb{1}^B\mathbf{\Psi}_{AB}$ corresponds to the 4 times 4 matrix $M^A(\mathbf{\Psi}^{AB})$ (while $\mathbb{1}^A \otimes M^B\mathbf{\Psi}_{AB}$ corresponds to $(\mathbf{\Psi}^{AB})(M^B)^T$). Making use of this convention, the relativistic equations for the two relativistic bosons now read

$$0 = (\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c\alpha_0^A)(\mathbf{\Psi}^{AB})\alpha_0^B \tag{24}$$
$$\pm\alpha_0^A(P_0^B(\mathbf{\Psi}^{AB})\mathbb{1}^B - (\overrightarrow{P^B}(\mathbf{\Psi}^{AB})) \cdot \overrightarrow{(\alpha^B)^T} - m_B c(\mathbf{\Psi}^{AB})\alpha_0^B),$$

where the + determination of $\pm$ corresponds to the symmetric (DP) equation and the - determination corresponds to the antisymmetric (complementary DP) equation.

3.2 de Broglie's Maxwellian and non-Maxwellian bosons

3.2.1 Passing from the Ψ to the Φ representation.

Multiplying (25) at the left by γ_0^A and at the right by γ_0^B, and expressing $\mathbf{\Psi}_{AB}$ through the 4 times 4 matrix $(\mathbf{\Psi}_{AB})$, we arrive to the equations

$$0 = (\gamma_0^A i\partial_0^A + \overrightarrow{\gamma}^A \cdot i\overrightarrow{\nabla}^A - \frac{m_A c}{\hbar}\mathbb{1}^A)(\mathbf{\Psi}^{AB})$$
$$\pm(i\partial_0^B(\mathbf{\Psi}^{AB})(\gamma_0^B)^T + (i\overrightarrow{\nabla}^B(\mathbf{\Psi}^{AB}))\cdot\overrightarrow{(\gamma^B)^T} - \frac{m_B c}{\hbar}(\mathbf{\Psi}^{AB})\mathbb{1}^B), \quad (25)$$

where the symmetry (antisymmetry) between the A and B fermion is now apparently broken, because all γ matrices are not equal to their transposed[3]. One can check, however, that the bosonic equations "look" more symmetric and more elegant if we replace $(\mathbf{\Psi}^{AB})$ by $(\mathbf{\Phi}^{AB})$ via the transformation

$$(\mathbf{\Phi}^{AB}) = (\mathbf{\Psi}^{AB})\cdot\Gamma, \quad (26)$$

where $\Gamma = \gamma_2\cdot\gamma_0$ from which, making use of the fact that Γ commutes with γ_1 and γ_3 and anticommutes with γ_2 and γ_0, we get:

$$0 = (\sum_{\mu=0,1,2,3}\gamma_\mu i\partial_\mu^A(\mathbf{\Phi}^{AB}) - \frac{m_A c}{\hbar}\mathbb{1}^A)(\mathbf{\Phi}^{AB})$$
$$\mp(\sum_{\mu=0,1,2,3} i\partial_\mu^B(\mathbf{\Phi}^{AB})\gamma_\mu + \frac{m_B c}{\hbar}(\mathbf{\Phi}^{AB})\mathbb{1}^B). \quad (27)$$

Essentially, this is how de Broglie considered in his book Théorie de la lumière[4] the asymmetric fusion of a fermion (A) obeying the usual

[3] Actually, in the "modern" representation adopted at the beginning of this paper, γ_0 and γ_2 are equal to their transposed, while γ_1 and γ_3 are equal to minus their transposed. The definition of the gamma matrices adopted by de Broglie differs by a global i factor from the modern, standard convention in the case of the "spatial" gamma matrices ($\mu = 1,2,3$). Therefore all the gamma matrices are self-adjoint in de Broglie's convention, while the "modern" matrices obey $(\gamma^\mu)^\dagger = \gamma_0\gamma^\mu\gamma_0$ so that only γ_0 is hermitic while the spatial γ matrices are anti-hermitic. Here we stick to the modern conventions adopted by Géhéniau [7] and not to de Broglie's conventions.

[4] de Broglie also introduces the so-called universal time which is equal to $i\cdot t$ but we do not follow that approach here, in accordance with Géhéniau's formulation of the fusion theory [7] which is our major source of inspiration in writing this section.

Dirac equation (1) with a complementary fermion (B) obeying, instead of (2), the complementary equation

$$(\mathbb{1}^B P_0^B - \overrightarrow{\alpha}^B \cdot \overrightarrow{P}^B - m_B c\tilde{\alpha}_0^B)\mathbf{\Phi}_B = 0, \tag{28}$$

where $\tilde{\alpha}_l = (-)^{1+l}\alpha_l$, $l = 0, 1, 2, 3$.

3.2.2 Deriving Maxwell's equations: preamble.

To derive Maxwell's equation, let us firstly remark, following Géhéniau and others, that when $\mathbf{\Psi}_{AB}$ transforms as a tensor product of two spinors, the $\mathbf{\Phi}_{AB}$ states transform as the product of a A spinor $|\Psi^A>$ with a B bar spinor $<\bar{\Psi}^B|$. This must be put in relation with the fact that one can build a basis of the 4 times 4 matrices by choosing 16 convenient products of the γ matrices comprising the identity operator such that

-the trace of these matrices is zero excepted for the identity.

-the matrices are orthogonal to each other relatively to the trace norm.

-the average values of the type $<\bar{\Psi}|O|\Psi>$ have a well-defined transformation law under Lorentz transformations (boosts) and rotations: some of them transform as components of a Minkoskian 4-vector, some as scalars and some transform as elements of an antisymmetric second order tensor.

-Moreover they are eigenstates of the parity operation either for the eigenvalue +1 (they are then “true” scalars, vectors and so on) or -1 (in which case they are “pseudo” scalars, vectors and so on).

-Developing the matrix ($\mathbf{\Phi}^{AB}$) in such a basis delivers amplitudes which transform like $<\bar{\Psi}|O|\Psi>$ and can thus be classified in function of their transformation law (scalars, vectors and so on).

-Remarkably, the equations considered here, in the limit where $m_A = m_B = 0$,are such that the dynamics of the associated amplitudes can be split into separated, independent, systems of equations among which we find a system equivalent to Maxwell's equation.

-If we do not reach the limit case $m_A + m_B = 0$, the system of equations that we will obtain can be interpreted as a generalisation of Maxwell's equation to a massive photon of (small) mass $m_A + m_B$.

-We also derive another group of equations which can be associated to a pseudo-scalar particle, which is reminiscent of the work of G. Lochak on fusion theory [15].

As we shall now show, following [7], the equations (28) make it possible, up to a convenient reparametrisation, to derive Maxwell's equation in the limit where $m_A + m_B$ goes to zero.

3.2.3 Derivation of Maxwell's equations.

As is known (see e.g. Messiah), $< \bar{\Psi}|O|\Psi >$ transforms as a scalar when O is the identity and as a pseudo-scalar when O is $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. It transforms as the μ component of a 4-vector when $O = \gamma^\mu$ and like the μ component of a pseudo 4-vector when $O = \gamma_5\gamma^\mu$. Later on, we shall identify $< \bar{\Psi}|\gamma^\mu|\Psi >$ with the corresponding component A_μ of the electro-magnetic potential.

Finally, when $O{=}\sigma_{\mu\nu} = i\gamma_\mu\gamma_\nu$ (with $\mu \neq \nu$), $< \bar{\Psi}|O|\Psi >$ transforms as one of the 6 components of an antisymmetric tensor that we shall in what follows identify with the spatial components of the electric and magnetic fields.

Developing the 4 times 4 matrix $(\mathbf{\Phi}_{AB})$ in the orthonormal basis achieved by collecting these 16 operators leads to the known reduction formula

$4(\mathbf{\Phi}_{AB}){=}S\mathbb{1} + A_\mu\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu} + B_\mu\gamma_5\gamma^\mu + Pi\gamma_5,$

where $S = Tr.(\mathbb{1}(\mathbf{\Phi}_{AB}))$, $A_\mu = Tr.(\gamma^\mu)^\dagger(\mathbf{\Phi}_{AB}))$ and so on.

For instance, if we parameterize Dirac matrices following (3,4), in accordance with Messiah's conventions [14], we find that A_0 the 0-component of the 4-potential vector, that is to say, the electric potential is related to the bifermionic field through the relation (see ref.[16] for more details)

$A_0 = \Phi_{14} - \Phi_{23} - \Phi_{32} + \Phi_{41}.$

Similarly, we find $H_{01} = i(\Phi_{11} - \Phi_{22} + \Phi_{33} - \Phi_{44})$ while H_{01} is proportional [16] to the X component of the electric field: $E_x = \frac{(m_A+m_B)}{2}H_{01}$).

Taking the Trace of the first equation of (28) we derive (adopting $\hbar = 1$ and $c = 1$ as well as $m_A = m_B = m$) after reduction the following system of equations:

$$\begin{aligned}
mS &= 0\\
\partial^\nu H_{\nu,\mu} + mA_\mu &= 0\\
\partial_\nu A_\mu - \partial_\mu A_\nu &= mH_{\nu\mu}\\
\partial_\nu P &= mB_\nu\\
-\partial_\mu B^\mu &= mP
\end{aligned} \tag{29}$$

Proceeding in a similar fashion with the second equation of (28) we get

$$\begin{aligned}
\partial_\mu A^\mu &= 0\\
\partial_\mu S &= 0\\
\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha) &= 0\\
\partial^\alpha H^x_{\alpha\mu} &= 0\\
0 &= 0
\end{aligned} \tag{30}$$

where $H^x_{\alpha\mu} = \epsilon_{\alpha\nu\beta\mu}H^{\beta\mu}/2$ with $\epsilon_{\nu\mu\alpha\beta}$ the completely antisymmetrical tensor whose component $\epsilon_{0123} = -1$ $(\epsilon^{0123} = +1)$.

In order to prove this result it is worth noting [7] that if we rewrite the equation (8) in terms of $(\mathbf{\Phi}^{AB})$ and impose $m = m_A = m_B, \hbar = 1, c = 1$ we get

$$0 = (\sum_{\mu=0,1,2,3} i\gamma_\mu\partial_\mu(\mathbf{\Phi}^{AB}) - m\mathbb{1}^A)(\mathbf{\Phi}^{AB}) \tag{31}$$

Making use of the reduction formula this equation reads

$$\begin{aligned}
i\partial_\mu A^\mu &= mS\\
i\partial_\mu S - \partial^\nu H_{\nu,\mu} &= mA_\mu\\
\partial_\nu A_\mu - \partial_\mu A_\nu - i\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha)/2 &= mH_{\nu\mu}\\
\partial_\nu P - i\partial^\alpha H^x_{\alpha\nu} &= mB_\nu\\
-\partial_\mu B^\mu &= mP
\end{aligned} \tag{32}$$

where as before $H^x_{\alpha\mu} = \epsilon_{\alpha\nu\beta\mu}H^{\beta\mu}/2$ with $\epsilon_{\nu\mu\alpha\beta}$ the completely antisymmetrical tensor whose component $\epsilon_{0123} = -1$ $(\epsilon^{0123} = +1)$.

Similarly, if we rewrite the equation (9) in terms of $(\mathbf{\Phi}^{AB})$, imposing $m = m_B, \hbar = 1, c = 1$, we get

$$0 = (\sum_{\mu=0,1,2,3} i\partial_\mu(\mathbf{\Phi}^{AB})\gamma_\mu + m(\mathbf{\Phi}^{AB})\mathbb{1}^B). \tag{33}$$

Its reduction delivers a system which is the same as (33) excepted that all i factors in it must be replaced by -i:

$$
\begin{aligned}
-i\partial_\mu A^\mu &= mS \\
-i\partial_\mu S - \partial^\nu H_{\nu,\mu} &= mA_\mu \\
\partial_\nu A_\mu - \partial_\mu A_\nu + i\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha)/2 &= mH_{\nu\mu} \\
\partial_\nu P + i\partial^\alpha H^x_{\alpha\mu} &= mB_\nu \\
-\partial_\mu B^\mu &= mP
\end{aligned}
\tag{34}
$$

It is now easy to check that a symmetric (anti-symmetric) combination of the systems (33) and (35) results into the system (30) ((31)).

There appear in (30,31), in addition with the constraints $S = 0$ and $\partial_\mu S = 0$, two closed groups of equations respectively related to the "states" defined through $4(\mathbf{\Phi}^{Maxwell}_{AB})$=$A_\mu\gamma^\mu + \frac{1}{2}H_{\mu\nu}\sigma^{\mu\nu}$ and $4(\mathbf{\Phi}^{pesudo}_{AB})$=$B_\mu\gamma_5\gamma^\mu + Pi\gamma_5$. The first closed system can be shown [16] to be equivalent with Maxwell equations generalised to the case of a massive photon.

$$
\begin{aligned}
\partial_\mu A^\mu &= 0 \\
\partial^\nu H_{\nu,\mu} + mA_\mu &= 0 \\
\partial_\nu A_\mu - \partial_\mu A_\nu &= mH_{\nu\mu} \\
\partial^\alpha H^x_{\alpha\mu} &= 0 \\
0 &= 0
\end{aligned}
\tag{35}
$$

It is worth noting that this system is not gauge invariant excepted in the limit of vanishing photon mass, but gauge is fixed here through Lorentz gauge constraint $\partial_\mu A^\mu = 0$. The second system describes [7] a pseudo-scalar bosonic particle:

$$
\begin{aligned}
\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha) &= 0 \\
\partial_\nu P &= mB_\nu \\
-\partial_\mu B^\mu &= mP
\end{aligned}
\tag{36}
$$

4 Γ transformation versus Λ transformation and quasi-duality.

4.1 Quasi-duality.

At this level, it is worth noting that a quasi-duality symmetry makes it possible formally to pass from the DP equation to the complementary DP equation. We call this relation a quasi-duality relation because it is only in the limit of vanishing masses $m_A = m_B = 0$ that this symmetry is exact. It consists of replacing the spinor of the second (B) fermion by a new spinor $\mathbf{\Psi^{q.d.}}_B$ via

$\mathbf{\Psi}_B^{q.d} = \gamma_5^B \mathbf{\Psi}^B = i \cdot \gamma_0^B \cdot \gamma_1^B \cdot \gamma_2^B \cdot \gamma_3^B \mathbf{\Psi}^B = i \cdot \alpha_1^B \cdot \alpha_2^B \cdot \alpha_3^B \mathbf{\Psi}^B$.

As γ_5 commutes with the spatial α matrices $(\alpha_1, \alpha_2, \alpha_3)$ and anti-commutes with α_0, the new fermion obeys the evolution equation

$$(\mathbb{1}^B P_0^B - \overrightarrow{\alpha}^B \cdot \overrightarrow{P}^B + m_B c \alpha_0^B)\mathbf{\Psi}_B^{q.d} = 0, \tag{37}$$

which corresponds to a fermion with a negative mass $-m_B$. Alternatively this fermion can be seen to obey a Dirac equation where the sign of the mass is the same as before while all derivatives are multiplied by a minus sign:

$(-\mathbb{1}^B P_0^B + \overrightarrow{\alpha}^B \cdot \overrightarrow{P}^B - m_B c \alpha_0^B)\mathbf{\Psi}_B^{q.d} = 0$

Reexpressed in terms of the quasi-dual B spinor through the relation $(\mathbf{\Psi}^{AB}) = (\mathbf{\Psi}^{AB})^{q.d}(\gamma_5)^T$ the relativistic equations (25) now read

$$\begin{aligned} 0 =\ & (\mathbb{1}^A P_0^A - \overrightarrow{\alpha}^A \cdot \overrightarrow{P}^A - m_A c \alpha_0^A)(\mathbf{\Psi}^{AB})^{q.d} \alpha_0^B \\ & \mp \alpha_0^A (P_0^B (\mathbf{\Psi}^{AB})^{q.d} \mathbb{1}^B - (\overrightarrow{P^B} (\mathbf{\Psi}^{AB})^{q.d}) \cdot \overrightarrow{(\alpha^B)^T} + m_B c (\mathbf{\Psi}^{AB})^{q.d} \alpha_0^B). \end{aligned} \tag{38}$$

Expressed in terms of γ matrices, and making use of the fact that γ_5 anti-commutes with all gamma matrices, the equation (39) is equivalent to

$((\gamma^\mu)^A P_\mu^A - m_A) \mp ((\gamma^\mu)^B P_\mu^B + m_B))\mathbf{\Psi}_{AB}^{q.d} = 0$,

-now, the equation (24) is equivalent to

$((\gamma^\mu)^A P_\mu^A - m_A) \pm ((\gamma^\mu)^B P_\mu^B - m_B))\mathbf{\Psi}_{AB} = 0$,

which constitutes what we call a quasi-duality relation: in the limit of vanishing masses it is easy to check that this quasi-duality permutes the

DP boson and the complementary DP boson. If $m_B \neq 0$ however this is not true; the reason therefore is that the DP equation only contains the symmetric combination $m_A + m_B$ (see e.g. (80) to (95) for illustration) while the complementary DP equation only contains the antisymmetric combination $m_A - m_B$ (as illustrated by (96) to (111)). This is why we are talking about quasi-duality here.

4.2 Γ transformation versus Λ transformation.

As was emphasised by Lochak [15] the Γ transformation possesses a natural partner, the Λ transformation. To see this, let us consider instead of (27) the new transformation

$$(\mathbf{\Psi'}^{AB}) = (\mathbf{\Psi}^{AB}) \cdot \Lambda, \tag{39}$$

where $\Lambda = \gamma_1 \cdot \gamma_3$.

It can also be achieved by replacing the B-spinor $\mathbf{\Psi'}_B$ by $\mathbf{\Psi'}_B = \Lambda^B \mathbf{\Psi'}_B = \gamma_1^B \cdot \gamma_3^B = -\alpha_1^B \cdot \alpha_3^B$.

As $\gamma_1\gamma_3$ commutes with α_0 and α_2 and anti-commutes with α_1 and α_3, we get, instead of the Dirac equation (2), the complementary equation

$$(\mathbb{1}^B P_0^B - \overrightarrow{\alpha'}^B \cdot \overrightarrow{P}^B - m_B c(\alpha'_0)^B)\mathbf{\Psi'}_B = 0, \tag{40}$$

with again $\alpha'_l = (-)^l \alpha_l$, $l = 0, 1, 2, 3$.

It is worth noting [7] that if we rewrite the equation (8) in terms of $(\mathbf{\Psi'}^{AB})$ and impose $m = m_A, \hbar = 1, c = 1$ we get

$$0 = (\sum_{\mu=0,1,2,3} i\gamma_\mu \partial_\mu (\mathbf{\Psi'}^{AB}) - m\mathbb{1}^A)(\mathbf{\Psi'}^{AB}) \tag{41}$$

which is equivalent to (32). Similarly, if we rewrite the equation (9) in terms of $(\mathbf{\Psi'}^{AB})$, imposing $m = m_B, \hbar = 1, c = 1$, we get

$$0 = (\sum_{\mu=0,1,2,3} i\partial_\mu (\mathbf{\Psi'}^{AB})\gamma_\mu - m(\mathbf{\Psi'}^{AB})\mathbb{1}^B). \tag{42}$$

Henceforth, performing the transformation

$\mathbf{\Psi}_{AB} \rightarrow \mathbf{\Psi'}_{AB} = \mathbb{1}^A \otimes (-)\alpha_1^B \cdot \alpha_3^B \mathbf{\Psi}_{AB} = \mathbb{1}^A \otimes \gamma_1^B \cdot \gamma_3^B \cdot \mathbf{\Psi}_{AB}$ also formally "erases the dissymetry" between both fermions present at the level of evolution equations (26), and leads to the following pair of equations:

$$0 = (\sum_{\mu=0,1,2,3} i\gamma_\mu \partial_\mu^A (\mathbf{\Psi'}^{AB}) - m\mathbb{1}^A)(\mathbf{\Psi'}^{AB})$$
$$\mp(\sum_{\mu=0,1,2,3} i\partial_\mu^B (\mathbf{\Psi'}^{AB})\gamma_\mu - m(\mathbf{\Psi'}^{AB})\mathbb{1}^B), \tag{43}$$

In particular, the antisymmetric (symmetric) combination is similar to the antisymmetric combination in (28) but instead of having the sum (difference) of the masses we now get their difference (sum). In de Broglie's fusion theory it is assumed that $m_A = m_B$, which means that the mass terms now appear in the antisymmetric equation and no longer in the symmetric one as was the case in (28).

As already shown, making use of the reduction formula, the equation (42) is equivalent to the system (33) which reads

$$\begin{aligned} i\partial_\mu A^\mu &= mS \\ i\partial_\mu S - \partial^\nu H_{\nu,\mu} &= mA_\mu \\ \partial_\nu A_\mu - \partial_\mu A_\nu - i\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha)/2 &= mH_{\nu\mu} \\ \partial_\nu P - i\partial^\alpha H^x_{\alpha\mu} &= mB_\nu \\ -\partial_\mu B^\mu &= mP \end{aligned}$$

Making use of the reduction formula, the equation (43) is equivalent to the following system which is obtained by replacing all i factors by -i and all m factors by $-m$ which delivers

$$\begin{aligned} -i\partial_\mu A^\mu &= -mS \\ -i\partial_\mu S - \partial^\nu H_{\nu,\mu} &= -mA_\mu \\ \partial_\nu A_\mu - \partial_\mu A_\nu + i\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha)/2 &= -mH_{\nu\mu} \\ \partial_\nu P + i\partial^\alpha H^x_{\alpha\mu} &= -mB_\nu \\ -\partial_\mu B^\mu &= -mP \end{aligned}$$

It is easy to check that a symmetric (anti-symmetric) combination of the two systems above results into the system (45) ((46)).

$$\begin{aligned} 0 &= 0 \\ \partial^\nu H_{\nu,\mu} &= 0 \\ \partial_\nu A_\mu - \partial_\mu A_\nu &= 0 \\ \partial_\nu P &= 0 \\ \partial_\mu B^\mu &= 0 \end{aligned} \tag{44}$$

$$\begin{aligned} i\partial_\mu A^\mu &= mS \\ i\partial_\mu S &= mA_\mu \\ -i\epsilon_{\nu\mu\alpha\beta}(\partial^\alpha B^\beta - \partial^\beta B^\alpha)/2 &= mH_{\nu\mu} \\ -i\partial^\alpha H^x_{\alpha\mu} &= mB_\nu \\ mP &= 0 \end{aligned} \tag{45}$$

These systems describe a scalar and a pseudo-scalar particle, as was noted by Géhéniau [7].

4.3 Passing from the Φ to the Ψ' representation, making use of the quasi-duality relation.

Actually, combining equations (27) and (40) leads to the relation

$$(\mathbf{\Psi'}^{AB}) = (\mathbf{\Psi}^{AB}) \cdot \Lambda = (\mathbf{\Phi}^{AB}) \cdot \Gamma^{-1} \cdot \Lambda \tag{46}$$

Now, $\Gamma^{-1} \cdot \Lambda = \gamma_5$, up to a global phase, which means that $(\mathbf{\Psi'}^{AB}) = (\mathbf{\Phi}^{AB}) \cdot \gamma_5$, up to a global phase. This corresponds, up to an irrelevant global phase, to the quasi-duality symmetry considered by us in section 4.1. The quasi-duality relates at one side a vectorial and a pseudo-scalar field (systems (36) and (37)) and at the other side a pseudo-vectorial and a scalar field (systems (45) and (46)). Lochak interpreted them in Ref. [15] as, at one side, an electric, spin 1, photon and a spin 0 photon defined by an axial potential without field, and, at the other side, a magnetic, spin 1, photon and a spin 0 photon defined by a polar potential witout field. As was noted by Lochak, in the limit where m_A and m_B go to zero, the Maxwellian vectorial boson of de Broglie is the dual counterpart of Lochak's non-Maxwellian pseudo-vectorial boson

[15]. The duality associated to γ_5 implies, still according to Lochak [15], an exchange between electricity and magnetism (see also Ref.[13] for clarifying the link between Lochak pseudo-vectorial boson and the magnetic monopole).

5 Conclusions and discussion.

As we have shown here, the coalescence of multitime multiparticle relativistic equations studied by us in a previous paper [12] provides an alternative derivation of the basic equations considered by de Broglie in his fusion theory [3, 4, 5, 6, 7] . We have also shown that the asymmetric fusion theories studied by de Broglie at one side and Lochak at the other side are naturally related by a quasi-duality relation in which γ_5 plays a prior role. This quasi-duality also appears naturally if we consider the symmetric and anti-symmetric combinations of multiparticle relativistic equations obtained through the coalescence process. From this point of view, coalescence sheds a new light on the fusion “recipy” of de Broglie.

At this level, it is however not clear at this level how to interpret the equations (45,46) in terms of particles. For instance, how should they be coupled to matter, how should we be able to measure their presence and so on (see e.g. Refs. [17, 18] concerning these points). These equations look promising regarding new physics, and they are obviously related to the impressive work of Georges Lochak in relation with the magnetic monopole [15]. It is not our scope to study these questions in depth in the present paper, but we hope to have convinced the reader that these concepts naturally appear if we consider the fusion of two fermions in a relativistically covariant manner.

The present analysis brought us to consider what happens when the masses m_A and m_B of the two fermions are different from each other. As one of us (P.P.) will show in a forthcoming paper [16], we obtain generalized Maxwell equations in which the Maxwell fields are coupled to pseudo-fields with a coupling constant proportional to the mass difference.

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6 Appendix A: evolution equations and condition equation of two fused "demi-photons".

6.1 Coalesced Bohm-Hiley equations (evolution equations).

Equation (10) reads

$$\{(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c\alpha_0^A) \otimes \mathbb{1}^B +$$
$$\mathbb{1}^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} = 0$$

Assuming a solution which does not depend on y and z we get

$$(2P_0 - (m_A + m_B))\Psi_{11} - P_3(\Psi_{31} + \Psi_{13}) = 0 \tag{47}$$
$$(2P_0 - (m_A + m_B))\Psi_{12} - P_3(\Psi_{32} - \Psi_{14}) = 0 \tag{48}$$
$$(2P_0 - m_A + m_B)\Psi_{13} - P_3(\Psi_{33} + \Psi_{11}) = 0 \tag{49}$$
$$(2P_0 - m_A + m_B)\Psi_{14} - P_3(\Psi_{34} - \Psi_{12}) = 0 \tag{50}$$
$$(2P_0 - (m_A + m_B))\Psi_{21} + P_3(\Psi_{41} - \Psi_{23}) = 0 \tag{51}$$
$$(2P_0 - (m_A + m_B))\Psi_{22} + P_3(\Psi_{42} - \Psi_{24}) = 0 \tag{52}$$
$$(2P_0 - m_A + m_B)\Psi_{23} + P_3(\Psi_{43} - \Psi_{21}) = 0 \tag{53}$$
$$(2P_0 - m_A + m_B)\Psi_{24} + P_3(\Psi_{44} + \Psi_{22}) = 0 \tag{54}$$
$$(2P_0 + m_A - m_B)\Psi_{31} - P_3(\Psi_{11} + \Psi_{33}) = 0 \tag{55}$$
$$(2P_0 + m_A - m_B)\Psi_{32} - P_3(\Psi_{12} - \Psi_{34}) = 0 \tag{56}$$
$$(2P_0 + m_A + m_B)\Psi_{33} - P_3(\Psi_{13} - \Psi_{31}) = 0 \tag{57}$$
$$(2P_0 + m_A + m_B)\Psi_{34} - P_3(\Psi_{14} - \Psi_{32}) = 0 \tag{58}$$
$$(2P_0 + m_A - m_B)\Psi_{41} + P_3(\Psi_{21} - \Psi_{43}) = 0 \tag{59}$$
$$(2P_0 + m_A - m_B)\Psi_{42} + P_3(\Psi_{22} + \Psi_{44}) = 0 \tag{60}$$
$$(2P_0 + m_A + m_B)\Psi_{43} + P_3(\Psi_{23} - \Psi_{41}) = 0 \tag{61}$$
$$(2P_0 + m_A + m_B)\Psi_{44} + P_3(\Psi_{24} + \Psi_{42}) = 0 \tag{62}$$

6.2 de Broglie conditions equations linked to the coalesced Bohm-Hiley evolution equations.

Equation (11) (or (21)) reads

$$\{(-\overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c\alpha_0^A) \otimes \mathbb{1}^B - \mathbb{1}^A \otimes (-\overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} = 0$$

Assuming a solution which does not depend on y and z we get

$$(m_A - m_B)\Psi_{11} + P_3(\Psi_{31} - \Psi_{13}) = 0 \tag{63}$$
$$(m_A - m_B)\Psi_{12} - P_3(\Psi_{32} - \Psi_{14}) = 0 \tag{64}$$
$$(m_A + m_B)\Psi_{13} + P_3(\Psi_{33} - \Psi_{11}) = 0 \tag{65}$$
$$(m_A + m_B)\Psi_{14} + P_3(\Psi_{34} + \Psi_{12}) = 0 \tag{66}$$
$$(m_A - m_B)\Psi_{21} + P_3(\Psi_{41} - \Psi_{23}) = 0 \tag{67}$$
$$(m_A - m_B)\Psi_{22} + P_3(\Psi_{42} - \Psi_{24}) = 0 \tag{68}$$
$$(m_A + m_B)\Psi_{23} + P_3(\Psi_{34} + \Psi_{12}) = 0 \tag{69}$$
$$(m_A + m_B)\Psi_{24} - P_3(\Psi_{44} - \Psi_{22}) = 0 \tag{70}$$
$$(m_A + m_B)\Psi_{31} - P_3(\Psi_{11} - \Psi_{33}) = 0 \tag{71}$$
$$(m_A + m_B)\Psi_{32} - P_3(\Psi_{12} + \Psi_{34}) = 0 \tag{72}$$
$$(m_A - m_B)\Psi_{33} - P_3(\Psi_{13} - \Psi_{31}) = 0 \tag{73}$$
$$(m_A - m_B)\Psi_{34} - P_3(\Psi_{24} - \Psi_{42}) = 0 \tag{74}$$
$$(m_A + m_B)\Psi_{41} + P_3(\Psi_{21} - \Psi_{43}) = 0 \tag{75}$$
$$(m_A + m_B)\Psi_{42} + P_3(\Psi_{22} - \Psi_{44}) = 0 \tag{76}$$
$$(m_A - m_B)\Psi_{43} - P_3(\Psi_{23} - \Psi_{41}) = 0 \tag{77}$$
$$(m_A - m_B)\Psi_{44} - P_3(\Psi_{24} - \Psi_{42}) = 0 \tag{78}$$

6.3 Coalesced DP equations.

Equation (12) reads

$$\{(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c\alpha_0^A) \otimes \alpha_0^B + \\ \alpha_0^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c\alpha_0^B)\}\boldsymbol{\Psi}_{AB} = 0$$

Assuming a solution which does not depend on y and z we get

$$(2P_0 - (m_A + m_B))\Psi_{11} - P_3(\Psi_{31} + \Psi_{13}) = 0 \quad (79)$$
$$(2P_0 - (m_A + m_B))\Psi_{12} - P_3(\Psi_{32} - \Psi_{14}) = 0 \quad (80)$$
$$(m_A + m_B)\Psi_{13} + P_3(\Psi_{33} - \Psi_{11}) = 0 \quad (81)$$
$$(m_A + m_B)\Psi_{14} + P_3(\Psi_{34} + \Psi_{12}) = 0 \quad (82)$$
$$(2P_0 - (m_A + m_B))\Psi_{21} + P_3(\Psi_{41} - \Psi_{23}) = 0 \quad (83)$$
$$(2P_0 - (m_A + m_B))\Psi_{22} + P_3(\Psi_{42} - \Psi_{24}) = 0 \quad (84)$$
$$(m_A + m_B)\Psi_{23} + P_3(\Psi_{34} + \Psi_{12}) = 0 \quad (85)$$
$$(m_A + m_B)\Psi_{24} - P_3(\Psi_{44} - \Psi_{22}) = 0 \quad (86)$$
$$(m_A + m_B)\Psi_{31} - P_3(\Psi_{11} - \Psi_{33}) = 0 \quad (87)$$
$$(m_A + m_B)\Psi_{32} - P_3(\Psi_{12} + \Psi_{34}) = 0 \quad (88)$$
$$(2P_0 + m_A + m_B)\Psi_{33} - P_3(\Psi_{13} - \Psi_{31}) = 0 \quad (89)$$
$$(2P_0 + m_A + m_B)\Psi_{34} - P_3(\Psi_{14} - \Psi_{32}) = 0 \quad (90)$$
$$(m_A + m_B)\Psi_{41} + P_3(\Psi_{21} - \Psi_{43}) = 0 \quad (91)$$
$$(m_A + m_B)\Psi_{42} + P_3(\Psi_{22} - \Psi_{44}) = 0 \quad (92)$$
$$(2P_0 + m_A + m_B)\Psi_{43} + P_3(\Psi_{23} - \Psi_{41}) = 0 \quad (93)$$
$$(2P_0 + m_A + m_B)\Psi_{44} + P_3(\Psi_{24} + \Psi_{42}) = 0 \quad (94)$$

6.4 Coalesced complementary DP equations.

Equation (13) reads

$$\{(\mathbb{1}^A P_0 - \overrightarrow{\alpha}^A \cdot \overrightarrow{P} - m_A c\alpha_0^A) \otimes \alpha_0^B - \\ \alpha_0^A \otimes (\mathbb{1}^B P_0 - \overrightarrow{\alpha}^B \cdot \overrightarrow{P} - m_B c\alpha_0^B)\}\mathbf{\Psi}_{AB} = 0$$

Assuming a solution which does not depend on y and z we get

$$(m_A - m_B)\Psi_{11} + P_3(\Psi_{31} - \Psi_{13}) = 0 \quad (95)$$
$$(m_A - m_B)\Psi_{12} - P_3(\Psi_{32} - \Psi_{14}) = 0 \quad (96)$$
$$(2P_0 - m_A + m_B)\Psi_{13} - P_3(\Psi_{33} + \Psi_{11}) = 0 \quad (97)$$
$$(2P_0 - m_A + m_B)\Psi_{14} - P_3(\Psi_{34} - \Psi_{12}) = 0 \quad (98)$$
$$(m_A - m_B)\Psi_{21} + P_3(\Psi_{41} - \Psi_{23}) = 0 \quad (99)$$
$$(m_A - m_B)\Psi_{22} + P_3(\Psi_{42} - \Psi_{24}) = 0 \quad (100)$$
$$(2P_0 - m_A + m_B)\Psi_{23} + P_3(\Psi_{43} - \Psi_{21}) = 0 \quad (101)$$
$$(2P_0 - m_A + m_B)\Psi_{24} + P_3(\Psi_{44} + \Psi_{22}) = 0 \quad (102)$$
$$(2P_0 + m_A - m_B)\Psi_{31} - P_3(\Psi_{11} + \Psi_{33}) = 0 \quad (103)$$
$$(2P_0 + m_A - m_B)\Psi_{32} - P_3(\Psi_{12} - \Psi_{34}) = 0 \quad (104)$$
$$(m_A - m_B)\Psi_{33} - P_3(\Psi_{13} - \Psi_{31}) = 0 \quad (105)$$
$$(m_A - m_B)\Psi_{34} - P_3(\Psi_{24} - \Psi_{42}) = 0 \quad (106)$$
$$(2P_0 + m_A - m_B)\Psi_{41} + P_3(\Psi_{21} - \Psi_{43}) = 0 \quad (107)$$
$$(2P_0 + m_A - m_B)\Psi_{42} + P_3(\Psi_{22} + \Psi_{44}) = 0 \quad (108)$$
$$(m_A - m_B)\Psi_{43} - P_3(\Psi_{23} - \Psi_{41}) = 0 \quad (109)$$
$$(m_A - m_B)\Psi_{44} - P_3(\Psi_{24} - \Psi_{42}) = 0 \quad (110)$$