A Simple Fundamental Estimate of Quantum Tunneling Time

H. Razmi

Department of Physics, University of Qom, 3716146611, Qom, I. R. Iran

ABSTRACT. The quantum tunneling time (QTT) is estimated based on the uncertainty principle. The result is model independent and can be considered as a lower bound for different models and definitions for the QTT. The result found leads to the exclusion of superluminal effects claimed in some quantum tunneling (QT) phenomena.

Keywords: Quantum tunneling; Quantum tunneling time; Uncertainty Principle; Superluminality

Introduction

Just after introducing the Schrodinger equation, quantum tunneling (QT) (barrier penetration) was notified by Hund [1] in a calculation of the splitting of the ground state in a double-well potential (e.g. the ammonia molecule). In a few years, application of QT to nuclear physics (α radioactive decay of nuclei) [2-4] and to solid state physics (the field emission of electrons from a metallic surface) were discovered [5]. In 1934, and then in 1957 tunneling diodes invented by Zener and Esaki based on the QT idea [6-7]. Newer applications of the QT were known by the discovery of the Josephson effect in 1962 [8] and the invention of the scanning tunneling microscope by Binnig and Rohrer in 1982 [9].

Although QT, as a pure quantum mechanical effect, has a well-known theory with a number of applications, it is still under fundamental study and investigation. Among other things, one of the most important problems corresponding to this effect is its time duration. In spite of a wide variety of theories and research works, there isn't a standard definition of the QTT and a well-known method for experimental measurement of it yet. In this letter, an estimate for the tunneling time is given by a fundamental argument based on the uncertainty principle. The result which is found as a lower bound for QTT is model independent and thus can be considered in all different definitions and theories used in calculating QTT.

Quantum tunneling time (QTT)

Since the first time of raising the question on the subject by MacColl in 1932 [10], there have been a number of sometimes conflicting theories and definitions for the QTT [11-14]. Among others, one can introduce three QTTs named as the Wigner time (i.e. phase time or group delay) [15], the Büttiker -Landauer time (i.e. semiclassical time) [16], and the Larmor time [17] (with Büttiker 's modification [18]). The Wigner time is the time interval between the arrival time of the peak of the incident wave packet and the exit time of the peak of the transmitted wave packet which is equal to the derivative of the phase of the tunneling amplitude with respect to the energy of the particle. The Büttiker -Landauer time and its modification the Larmor time is defined based on a thought experiment and as an inverse of a characteristic frequency with a special scenario. The basic reason for such a wide variety of definitions and interpretations for QTT may be because of the non-availability of a quantum-mechanical time operator.

A simple fundamental estimate of QTT

Let see how one can simply estimate the QTT in terms of the particle mass and the barrier width irrespective of other things and without any other pre-assumption, particular definition, and special model. For a quantum particle of mass m with the total energy E which is to be penetrated through a potential barrier of height V ($E<V$) and width d, using the fundamental uncertainty principle, the minimum observable time corresponding to this phenomenon should satisfy the following inequality:

$$\tau_{\min}^{obs} \approx h/\Delta E \approx \frac{h}{(\Delta p)^2/m} \approx \frac{h}{(h^2/md^2)} = \frac{md^2}{h} \tag{1}$$

in which we have considered two following facts:

1. The measured value of a quantity is experimentally reliable/observable when its value is greater than its uncertainty.

2. Under the barrier (the black unobservable box!), all quantities can take a value at most of the order of their corresponding uncertainties satisfying the Heisenberg relation and thus we have used $\Delta E = \Delta((p^2/2m) + V) = (2p\Delta p/2m) \approx (\Delta p)^2/m \approx ((h/\Delta x)^2)/m \approx (h^2/md^2)$.

Therefore, for all different definitions and theories, QTT should satisfy the following estimate:

$$(QTT)_{\min} \approx \frac{md^2}{h} \tag{2}$$

If the tunneling happens, the corresponding time cannot be less than the above expression which depends only on the barrier width d and the mass of the particle. Less massive, more rapidly tunneled, and more massive with more delay. The above relation can be considered as a lower bound test for the QTT.

Is QT a superluminal effect?

Based on some definitions, QT seems to deal with the superluminal effect problem (e.g. the Wigner time in the Hartman effect [19]); but of course, some modern experiments reason on this fact that QT is a subluminal phenomenon [e.g. 20-21].

Although there isn't a definite velocity\speed under the barrier, an average velocity\speed may be simply defined as the ratio of the barrier width to the QTT. Using (2), one can have an upper bound on the average speed of the particle during tunneling the barrier as in the following:

$$\bar{v}_{QT} \leq \frac{d}{(QTT)_{\min}} \leq \frac{h}{md} \tag{3}$$

As we know, fundamentally, the minimum distance about which one can do quantum mechanical calculation for a particle of mass m, is its corresponding Compton wavelength $\lambda_{compton} = \frac{h}{mc}$; in other words, QT is observable only if the barrier width satisfies:

$$d \geq \lambda_{compton} \Rightarrow d \geq \frac{h}{mc} \tag{4}$$

Using (3) and (4), it is found that:

$$\bar{v}_{QT} \leq c \tag{5}$$

Considering (2) and (5), it is found that QT takes a finite time and doesn't happen instantaneously, but with a speed less than the speed of light in vacuum.

Numerical estimate for two real experimentally well-confirmed phenomena

Among a number of real experimentally confirmed phenomena which are either designed based on QT or justified by QT in different fields of physics and even other branches of modern science as in quantum biology, let have an estimate for cold emission as one of the old well-confirmed experiences [22]. The width of the barrier in cold emission is:

$$d = \frac{W}{eE} \tag{6}$$

where W is the work function of the metal under consideration and E is the strong electric field strength. Considering a real case for which the work function W is at the order of $1eV$ and the electric field is at the order of 100 *V/microns* and thus d is about 10^{-2} *microns* (10 *nanometers*) and using (2) for an electron tunneling through such a distance, it is found:

$$(QTT)_{coldemission} \approx \frac{(9.1 * 10^{-31}kg)(10^{-8}m)^2}{6.63 * 10^{-34}j.s} \approx 1.3 * 10^{-13}\sec. \tag{7}$$

Clearly, this a lower bound for QTT of the problem under consideration.

Using (3), one can have an upper bound on the average speed of the tunneled electrons as:

$$\bar{v}_{QT(coldemission)} \leq \frac{6.63 * 10^{-34}j.s}{(9.1 * 10^{-31}kg)(10^{-8}m)} \approx 7.3 * 10^{4}m/s \tag{8}$$

This is smaller than %0.025 speed of light reasoning on this fact that the tunneling occurs completely non-relativistic.

As a modern recently reported experiment in which rubidium atoms (^{87}Rb) tunnel through a barrier of thickness 1.3 micrometers [21]. According to (3), the average speed of the atoms should satisfy:

$$\bar{v}_{QT} \leq \frac{h}{md} \approx \frac{6.63 \times 10^{-34}}{(87 \times 1.66 \times 10^{-27} \times 1.3 \times 10^{-6})} \approx 3.5 \times 10^{-3}m/s \tag{9}$$

thus, the corresponding QTT should be greater than $\frac{(1.3\times10^{-6}m)}{(3.5\times10^{-3}m/s)} \approx (3.7\times10^{-4})s$.

An observable boundary value of 0.61(7) milliseconds has been reported in [21] which is about 1.67 greater than the fundamental minimum value (0.37 milliseconds) estimated here.

What about the massless particles?

Theoretical considerations of QT for photons and their corresponding modern experiments are already well-known with controversial deductions on the possibility of faster than light signaling [23-25]. Here, in continue of the above-noted calculations for massive particles, let check what can we say about the massless particles. Although the standard non-relativistic quantum mechanics cannot be applied to photons and there isn't any standard relativistic generalization of the uncertainty principle for massless particles, we can simply use the energy-time relation as in the following:

$$\tau^{obs}_{massless} \approx h/\Delta E \approx \frac{h}{(\Delta p)c} \approx \frac{h}{(h/d)c} = \frac{d}{c} \tag{10}$$

This is an acceptable natural result; it not only gives us a finite lower bound for QTT but also confirms this fact that QT for photons happens with a speed of at most c.

Conclusion and discussion

What has been found in this letter is a fundamental lower bound for QTT irrelevance of different possible definitions, models, and theories which are usually considered in describing QT effect. The estimate found in (2) may be considered as a criterion to test different proposed theories for QT. Indeed, the criterion (2) is a lower bound for QTT based on the uncertainty relation which ensures the observability\measurability of the QT. Moreover, this criterion keeps the consistency with the special theory of relativity because it excludes superluminal effects.

At last, let discuss about another point about the restriction (maybe condition) on the potential height , and the barrier width value d (the black box). In fact, both fundamentally and because of technical restrictions in observing quantum objects, one deals with a non-localized object whose non-localizability cannot be fundamentally removed down to a fundamental least value well-known as the Compton wavelength corresponding to the "particle" under consideration $\lambda_c = \frac{h}{mc}$; thus, for a particle (quantum object) of mass m, one cannot "speak" any quantum phenomenon (e.g. quantum tunneling effect) for spatial scale smaller than the Compton wavelength. This means that in quantum tunneling effect the barrier width satisfies $d > h/mc$ which is simply satisfied in practice (because of the great value of the speed of light c). Moe clearly, this is because the quantum object under consideration is spread over a distance at least larger than its Compton wavelength and therefore for barriers of width less than this value it is "instantaneously" cited at the left and the right parts of the tunnel. About the potential height , in order to tunneling happens, the maximum value of the barrier height should satisfy:

$$V_{\max.} - E \approx (\Delta p)^2/m \Rightarrow V_{\max.} = E + (\Delta p)^2/m \qquad (11)$$

This is because for the case of a particle (quantum object) at the least possibility of "success" to tunnel the potential barrier it should have an "energy" value equal to $(\Delta p)^2/m$ or h^2/md^2 which is about $h/\tau_{\min.}$ value ; this is a reasonable estimate in agreement to the energy-time uncertainty relation. From other point of view, considering the well-known transition probability amplitude for the quantum tunneling effect based on the standard quantum mechanics $T \sim e^{-\sqrt{\frac{2m}{\hbar^2}(V-E)d}}$ [26], it is obvious that tunneling doesn't occur for large values of $\frac{2m}{\hbar^2}(V-E)d$; thus, even for a very low energy particle (quantum object), if the potential barrier doesn't exceed the h^2/md^2 value, quantum tunneling happens. This proposes us a scenario for finding out not only an upper bound on the potential barrier but also a method in such a case that with varying (lowering) the kinetic energy of the particle (quantum object) so that just at the condition of the minimum value of the energy for which tunneling occurs one can get more information about the value of the potential barrier and possible restrictions on it.

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References

[1] F. Hund, *Zeitschrift für Physik* **43**, 805 (1927).

[2] J. R. Oppenheimer, *Phys. Rev.* **31**, 66 (1928); *Proc. Nat. Acad. Sci.* **14**, 363 (1928).

[3] G. Gamow, *Zeitschrift für Physik* **51**, 204 (1928); **52**, 510 (1928).

[4] R. W. Gurney and E. U. Condon, *Nature* **122**, 439 (1928); Phys. Rev. **33**, 127 (1929).

[5] R. H. Fowler and L. Nordheim, *Proc. Roy. Soc.* **A 119**, 173 (1928).

[6] C. Zener, *Proc. Roy. Soc.* **145**, 523 (1934).

[7] L. Esaki, Y. Kurose, T. Suzuki, *Proc. Phys. Soc. of Japan.* **12** (5), 85 (1957).

[8] B. D. Josephson, *Phys. Lett.* **1** (7), 251 (1962).

[9] G. Binnig, et. al. *Appl. Phys. Lett.* **40**, 178 (1982).

[10] L. A. MacColl, *Phys. Rev.* **40**, 621 (1932).

[11] E. H. Hauge and J. A. Støvneng, *Rev. Mod. Phys.* **61**, 917 (1989).

[12] V. S. Olkhovsky and E. Recami, *Phys. Rep.* **214**, 339 (1992).

[13] R. Landauer and Th. Martin, *Rev. Mod. Phys.* **66**, 217–228 (1994).

[14] https://www.sciencedirect.com/science/article/pii/S007966380870341X#!R. Y.Chiao and https://www.sciencedirect.com/science/article/pii/S007966380870341X# M. Steinberg, https://www.sciencedirect.com/science/journal/00796638*Progress in Optics* https://www.sciencedirect.com/science/journal/00796638/37/supp/C**37**, 345 (1997).

[15] E. P. Wigner, *Phys. Rev.* **98**, 145 (1955).

[16] M. Büttiker and R. Landauer, *Phys. Rev. Lett.* **49**, 1739 (1982).

[17] A. I. Baz', *Sov. J. Nucl. Phys.* **4**, 182 (1967); **5**, 161 (1967).

[18] M. Büttiker, *Phys. Rev.* **B 27**, 6178 (1983).

[19] T. E. Hartman, https://en.wikipedia.org/wiki/Journal_of_Applied_Physics*Journal of Applied Physics*. **33** (12), 3427 (1962).

[20] R. S. Dumont, T. Rivlin, and E. Pollak, https://iopscience.iop.org/journal/1367-2630*New Journal of Physics*, https://iopscience.iop.org/volume/1367-2630/22**22**, 093060 https://iopscience.iop.org/issue/1367-2630/22/9(2020).

[21] javascript:;R. Ramos, et. al., https://www.nature.com/nature*Nature* **583**, 529 (2020).

[22] R. H. Fowler and L. Nordheim, *Proc. R. Soc. London* **119**, 173 (1928).

[23] A. Enders and G. Nimtz, *J. Phys. I* **2**, 1693 (1992).

[24] A. M. Steinberg, P. G. Kwiat, and R. Y. Chiao, *Phys. Rev. Lett.* **71**, 708 (1993).

[25] C. Spielmann, et. al., *Phys. Rev. Lett.* **73**, 2308 (1994).

[26] S. Gasiorowicz, **Quantum Physics** (3^{rd} edition), Wiley, Chapter 5, Page 86 (2003).