The de Broglie wave as an undulatory distortion induced in the moving particle by the failure of simultaneity

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ABSTRACT. I present an ontologically parsimonious de Broglie wave in which this superluminal phenomenon is simply a modulation - essentially a distortion of undulatory form - induced in the structure of the moving particle by the failure of simultaneity. I show that the emergence of this modulation would explain the wave-like manner in which a massive particle evolves and interacts, whilst the underlying structure would define the physically realistic particle trajectories favoured by de Broglie-Bohm theories. As I will also demonstrate, significant support for this interpretation may be found in de Broglie's own writings, including in particular his 1926 work, *Ondes et Mouvements* [1].

RÉSUMÉ. Je présente une onde de Broglie ontologiquement parcimonieuse dans laquelle ce phénomène superluminal est simplement une modulation - essentiellement une distorsion de la forme ondulatoire - induite dans la structure de la particule en mouvement par l'échec de la simultanéité. Je montre que l'émergence de cette modulation expliquerait la manière ondulatoire selon laquelle une particule massive évolue et interagit, tandis que la structure sous-jacente définirait les trajectoires de particules physiquement réalistes privilégiées par les théories de Broglie-Bohm. Comme je le démontrerai également, un soutien significatif pour cette interprétation peut être trouvé dans les propres écrits de de Broglie, notamment dans son travail de 1926, Ondes et Mouvements [1].

Keywords: matter wave · simultaneity · Planck-Einstein relation · de Broglie-Bohm theories · wave function · phase modulation

1 Introduction

As papers presented at this conference[1] have shown very clearly, a massive particle evolves and interacts in the wave-like manner predicted by Louis de Broglie in his celebrated thesis of 1924 [2].

What might yet be debated is the origin of this wave-like effect - and the meaning, therefore, of the wave functions that emerge as solutions of the Schrödinger and other equations of quantum mechanics for massive particles, all of which were conceived as equations for the de Broglie wave (see Bloch [3] and Dirac [4]).

Schrödinger was himself concerned at the enigmatic nature of these wave functions. As he reported to the Solvay Conference of 1927, the "ψ function" seems to describe, not a single trajectory, but a "snapshot .... with the camera shutter open" of all possible classical configurations (see Bacciagaluppi and Valentini [5], p. 411).

The apparent inability of these ψ functions to identify physically realistic particle trajectories was a significant factor in the eventual ascendancy of Born's probabilistic interpretation of quantum mechanics [6]. But the probabilistic Born rule has in turn been the source of those various difficulties referred to in the literature as "the measurement problem".

What then is this wave-like effect that de Broglie referred to as a phase wave and is known today as the de Broglie or matter wave? Considered as a true wave, the velocity implied by its frequency and wavelength would be anomalous. It has a frequency (the Einstein frequency ω_E) directly related to the energy E of the particle through the Planck-Einstein relation,

$$E = \hbar\omega_E = \hbar\gamma\omega_o, \tag{1}$$

and a wave number (the de Broglie wave number κ_{dB}) that is similarly related to the particle's momentum p through the de Broglie relation,

$$p = \hbar\kappa_{dB} = \hbar\gamma\omega_o\frac{v}{c^2}, \tag{2}$$

[1]This paper is based in part on my presentation at the conference, *100 Years of Matter Waves*, held in honour of Louis de Broglie at the Sorbonne in July 2023

where v is the velocity of the particle, c that of light, ω_o, the characteristic frequency of the particle at rest, $\hbar$, the reduced Planck constant, and γ, the Lorentz factor,

$$\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}. \tag{3}$$

The de Broglie wave thus has the form,

$$\psi_{dB} = e^{i(\omega_E t - \kappa_{dB} x)},$$

which implies, from Eqns. (1) and (2), that its velocity,

$$v_{dB} = \frac{\omega_E}{\kappa_{dB}} = \frac{c^2}{v}, \tag{4}$$

exceeds the limiting velocity c of light.

De Broglie was able to recover the classical velocity of the particle from the group velocity of a superposition of de Broglie waves of differing frequencies (de Broglie [2], Chap. I, Sect. II). But such a wave packet spreads rapidly with time and very soon the particle could be almost anywhere at all. Nor is the range of frequencies required for such a wave packet consisent with the precision of the spectra observed in atomic and molecular processes.

Yet there is a clue in this anomalous velocity as to what this wave-like phenomenon might actually be. According to Eqn. (4), the velocity v_{dB} is not only superluminal but increases inversely with the velocity of the particle and becomes infinite as the particle comes to rest. While this is not the behaviour of any true energy or information carrying wave, it is typical of the phase modulation of an underlying carrier wave (see for example Feynman et al [7], Vol. I, Chap. 48).

I will argue that this is indeed the true nature of the de Broglie wave, that this superluminal phenomenon is not something ontologically distinct from, or in any way separate from, the particle, but simply a phase modulation of sinusoidal or undulatory form, induced in the structure of the moving particle by the failure of simultaneity. Considered in this way, the de Broglie wave becomes, as is the Fitzgerald-Lorentz contraction, a relativistically induced distortion observed in the structure of the particle as it moves.

It is not at all an original thought that the de Broglie wave might simply be the modulation of an underlying particle structure (see the literature listed in Ref. [8]). Indeed, I will show that an anticipation of this very effect appeared as early as de Broglie's thesis of 1924, and was given mathematical form in his small book[2], *Ondes et Mouvements* [1], published shortly after the thesis.

It is possible to discern in those early writings of de Broglie, the evolution of the de Broglie wave from its initial appearance as a "fictitious" wave of unexplained origin in a short paper [9] that preceded the thesis [2], and from thence to the thesis itself in which the particle is encompassed in its rest frame by a "periodic phenomenon", and finally to *Ondes et Mouvements* [1], where the periodic phenomenon is clearly now a standing wave from which the de Broglie wave emerges as what is also very clearly now, not a true wave, but the relativistically induced phase modulation of the underlying wave structure.

Since 1924, various interesting structures have been proposed as possible origins for the de Broglie wave, some pursuant to the idea that the wave is the modulation proposed here, others in furtherance of quite different interpretations of the wave, notably including de Broglie's own "double solution" theory (de Broglie [10]-[13]) in which the particle comprises a singularity or region of increased amplitude in the wave function[3].

A difficulty with some of these proposals is that they seem in danger of trespassing upon the province of the standard model of particle physics. What is required I suggest is a conception of the nature of matter that is sufficiently general as to explain the de Broglie wave whatever the species of particle being considered, and yet at the same time, so clearly consistent with already well-established principles of physics, that it should survive whatever conclusions might ultimately be reached regarding those as yet unexplained parameters of the standard model.

An objective of this paper will be to investigate what those aforesaid "well-established principles of physics" might be. As to the origin of the

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[2]I thank Dr. Aurélien Drezet for drawing this work to my attention and providing me with an English translation.

[3]For reviews of the double solution theory, see Fargue [14], and Colin et al [15], and for other interesting discussions, Matzkin [16], Drezet [17] - [19], and Durt [20]. Mechanical analogues have also been proposed, see recently, Borghesi [21], Drezet et al [22], and Jamet et al [23].

de Broglie wave, I will argue that in 1926 in *Ondes et Mouvements*, de Broglie had already given what is essentially the correct explanation of that wave.

I will begin by explaining - in the next two sections - how the de Broglie wave emerges as a relativistic distortion of underlying particle structure. In Sects. 4 and 5, I will discuss the support for that explanation to be found in de Broglie's writings. I will then say something in Sect. 6 regarding the circumstances, that in the years 1926-1927, appear to have led de Broglie to pursue instead his double solution theory. I will conclude with a brief summary in Sect. 7.

2 The de Broglie wave as phase modulation

Consider, as a general description of a massive particle in its rest frame, the structure,

$$f(x, y, z)\, e^{i\omega_0 t}, \tag{5}$$

where nothing has been assumed of the particle other than that it is spatially extended, at least though its fields, and as implied by the Planck-Einstein relation (1), is oscillating at the characteristic frequency ω_0 of the species of particle in question.

Now suppose that this particle is observed to be moving in the x-direction at the velocity v. Applying the Lorentz transformation,

$$\begin{aligned} x- &> \gamma\,(x - vt)\,, \\ y- &> y, \\ z- &> z, \\ t- &> \gamma\left(t - \frac{vx}{c^2}\right), \end{aligned}$$

wave (5) becomes the travelling wave,

$$f\,[\gamma(x - vt), y, z]\, e^{i\omega\gamma(t - vx/c^2)}, \tag{6}$$

in which the spatial factor $f(x, y, z)$ has become the relativistically-contracted carrier wave,

$$f[\gamma\,(x - vt)\,,\, y,\, z], \tag{7}$$

which is moving in the x-direction at the velocity v of the particle, while as a consequence of the failure of simultaneity, the oscillation $e^{i\omega_0 t}$ has acquired a modulation of phase (a progressive dephasing or beating effect) and now has the form of a transverse plane wave,

$$e^{i\omega\gamma(t-vx/c^2)}, \tag{8}$$

which is evolving through the carrier wave (7) at the superluminal velocity c^2/v.

With the assistance of Eqns. (1) and (2), wave factor (8) can be rewritten in terms of the Einstein frequency,

$$\omega_E = \gamma\omega_0,$$

and de Broglie wave number,

$$\kappa_{dB} = \gamma\omega_0\frac{v}{c^2},$$

as,

$$e^{i(\omega_E t-\kappa_{dB}x)}, \tag{9}$$

and is now more readily recognizable as the de Broglie wave. But it is not here a wave in its own right, but as contemplated above, a modulation of the carrier wave (7), defining the progressive dephasing of that wave (and the failure of simultaneity) in the direction of travel. Combining wave factors (7) and (9), the full modulated wave is,

$$f\left(\gamma(x-vt), y, z\right) e^{i(\omega_E t-\kappa_{dB}x)}. \tag{10}$$

What modulated wave (10) is saying in effect is that provided only that a massive particle is oscillating and spatially extended, it acquires when observed from an inertial frame in which it is moving, a modulation of phase having the velocity, frequency and wave length of the de Broglie wave. Thus whatever those as yet unexplained parameters of the standard model might eventually tell us regarding the structures of the elementary particles, a distortion with the characteristics of a de Broglie wave will necessarily be observed in a massive particle when it moves. I suggest that this consequence of the Lorentz transformation effectively precludes all possibility of any other explanation of the de Broglie wave.

However, the more interesting question is why such a wave-like distortion should occur in a particle that might more usually be thought of as solid or point-like. The answer to that question is, I believe, that even in the rest frame of the particle, where there is no de Broglie wave, a massive particle must be wave-like, this being precisely what two of those afore-mentioned "well-established principles of physics", namely the Lorentz transformation and the Planck-Einstein relation (1), seem to be saying.

The Lorentz transformation implies that matter is constituted by underlying effects - forces, influences, topologies, whatever they might be - that evolve at the velocity c of light. If there were some fundamental force or effect that evolves at a velocity other than c, it would have its own Lorentz factor γ and corresponding Lorentz transformation, and neither the structure of matter, nor the laws of physics, could survive unchanged from one inertial frame to the next.

That c has that fundamental significance is implicit in Einstein's various thought experiments involving moving trains and railway platforms and the like, in which the Lorentz transformation is derived from a comparison of light signals propagating longitudinally and transversely with respect to moving and stationary observers. In effect, Einstein assimilated the changes observed in the moving objects to the corresponding variations in superpositions of counter-propagating light waves (see Shanahan [25] and [26]).

There are of course velocities that differ from c, those for example of massive objects, sound waves, and refracted light. But in each case, the velocity in question must be considered the net effect of underlying influences that *do* evolve at velocity c. Unlike c, such a velocity does not remain unchanged on a change of inertial frame, but as Einstein explained in 1905 [27], transforms in accordance with the relativistic formula for the composition of velocities.

Meanwhile, the Planck-Einstein relation,

$$E = \hbar\omega,$$

implies that whatever these consistory influences of velocity c might be, they have in the rest frame of the particle, the characteristic frequency ω_o of the species of particle in question.

3 Illustration: the relativistic transformation of a standing wave

The nature of a phase modulation and some of its consequences for quantum mechanics may be understood from the relativistic behaviour of the simple standing wave shown in Fig. 1.

In its rest frame, every part of the wave is oscillating in unison. But to an observer for whom the frame of the standing wave is moving to the right at a relativistic velocity, the standing wave is experiencing the changes described by the Lorentz transformation. These include, as shown in the second drawing, the Fitzgerald-Lorentz contraction. But special relativity also predicts the failure of simultaneity and, to the stationary observer, those parts of the wave to the right will be rising and falling later than those to the left.

If the frame of the wave is moving sufficiently faster relative to the stationary observer, as in the bottom drawing, the modulation of phase induced by the failure of relativity will take the form of a sinusoidal wave advancing through the underlying wave structure. In accordance with Eqn. (10) above, this sinusoidal modulation will have the wave length, frequency and superluminal velocity c^2/v of a de Broglie wave, and indeed for this particular structure, it will be its de Broglie wave.

As these drawings demonstrate, an advantage of the interpretation of the wave as a mere modulation is that its otherwise anomalous velocity becomes consistent with special relativity. A phase modulation is simply a beating effect, well understood from classical wave theory (see again Feynman et al [7], Vol. I, Chap. 48).

And unlike the de Broglie wave considered alone, the full modulated wave structure (10) is a manifestly covariant relativistic object, capable in principle of taking its place in the tensor equations of relativistic physics. The Fitzgerald-Lorentz contraction appears in the carrier wave (7), while the dilation of time and failure of simultaneity predicted by the Lorentz transformation are described by the modulation, that is to say, by the de Broglie wave (9).

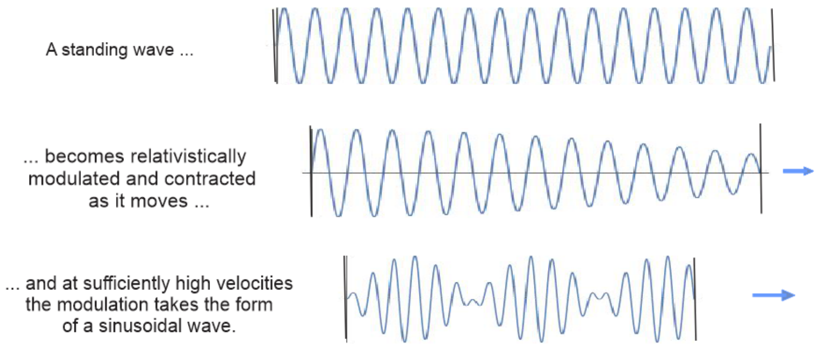

Fig. 1: The relativistic behaviour of a simple standing wave

A further consequence is that the de Broglie wave becomes exactly coextensive with the structure of the particle itself. It has no amplitude independently of the structure it modulates. The wave *is* the particle, and the paradox of wave-particle duality is resolved in favour of a wave structure that acts like a particle. For an explanation of the cohesion and stability of that structure, we must defer to the standard model, but it is the modulation that will explain the wave-like behavior that the moving particle exhibits in interference, diffraction and refraction.

It may be apparent from the lowermost drawing, for instance, that the manner in which a massive particle interferes will depend on the wavelength of the modulation. It will be in its sinusoidally distorted form that the moving particle interacts with a stationary beam splitter, and whether its interference at that beam splitter with another such wave is constructive or destructive or somewhere in between will depend on the degree to which their modulations are in or out of phase at the beam splitter.

Above all, these drawings illustrate the ontological parsimony of this understanding of the de Broglie wave. It requires no further structure. It is simply, as stated above, a distortion predicted in well known manner by the Lorentz transformation.

4 The thesis

In this section, I will consider de Broglie's thesis of 1924 [2] with the objective of showing where and why the interpretation of the wave that he adopted in the thesis came to differ from that proposed in this paper. I will then show in the next section (Sect. 5) that these differences disappeared in his 1926 work, *Ondes et Mouvements* [1].

De Broglie commenced by taking two equations, both associated with Einstein, the Planck-Einstein relation in the form,

$$E = h\nu, \tag{11}$$

(which prior to de Broglie had been known only for the photon), and Einstein's famous,

$$E = mc^2,$$

for the equivalence of mass and energy, and by eliminating E between these equations, was able to associate with a massive particle, a frequency,

$$\nu_0 = \frac{mc^2}{h},$$

directly related to its rest mass m, where ν and ν_0 are natural frequencies ($\nu = \omega/2\pi$) and h is the (unreduced) Planck constant.

But what could be oscillating at this frequency? De Broglie supposed that the particle must be surrounded in its rest frame by what he termed a "periodic phenomenon". What he said about this phenomenon is important:

> The frequency ν_0 is to be measured, of course, in the rest frame of the energy packet .... Must we suppose that this periodic phenomenon occurs in the interior of the energy packet? This is not at all necessary .... [I]t is spread out over an extended space .... What makes an electron an atom of energy is not that it occupies only a small region in space .... it occupies all space, but the fact that it is indivisible, that it constitutes a unit (de Broglie [2], Chap. I, Sect I).

So far so good: up to this point in his derivation, de Broglie has not departed significantly from the derivation of the wave as a modulation presented in Sect. 2 of this paper. He has assumed the existence of a spatially extended phenomenon that he might well have said was a standing wave, and which he did identify as a standing wave in *Ondes et Mouvements*.

However, in the thesis itself, de Broglie provided no further details of this "periodic phenomenon", and in the final paragraph he explained that this omission was intentional. Because, as he said, his theory was "not entirely precise" he "left intentionally vague .... the definitions of phase waves and the periodic phenomena for which such waves are a realization". And having failed to give mathematical form to the periodic phenomenon, he had nothing that he could conveniently subject to a Lorentz transformation when he went on to consider the form that the periodic phenomenon might take in another inertial frame.

He considered instead the differing conclusions that might be reached concerning the frequency of a moving particle. To a stationary observer, the moving particle has the frequency $\gamma\omega_0$, but that same observer believes that in the inertial frame of the particle itself, it must have the lower frequency $\gamma^{-1}\omega_0$. With these frequencies in mind, de Broglie asked what form a periodic phenomenon might take if it were to maintain consistency of phase with a moving particle. According to his "theorem of the harmony of phases", that periodic phenomenon would be the phase wave, not however the modulation discussed above, but according to his derivation, an independent wave.

But this is a part of the thesis that should be read very cautiously. De Broglie was no longer considering the periodic phenomenon that he had earlier described as surrounding the particle in its rest frame. All that de Broglie was contemplating when he deduced the de Broglie wave in the thesis was an oscillation at a single point in that extended periodic phenomenon, namely the position of the particle, which he took to be point-like. Thus all that de Broglie actually derived in the thesis was the history or record through space and time of the evolution in phase of a moving and oscillating point. Yet that ironically was all that he needed in order to explain (de Broglie [2], Chap 3), the quantization of atomic orbits that Bohr had explained in terms of the action (Bohr [28]).

Before moving on to *Ondes et Mouvements* in the next section, I mention that in addition to his primary derivation based on the harmonizing of phases, de Broglie provided in the thesis two further demonstrations of his wave, in both of which the de Broglie wave emerges as what is clearly a modulation rather than the independent wave he supposed. One, which I will not discuss here (but see Shanahan [8]) involved an analysis in Minkowski spacetime. The other, which is intuitively compelling, and which I can describe here, comprised an array of oscillating springs.

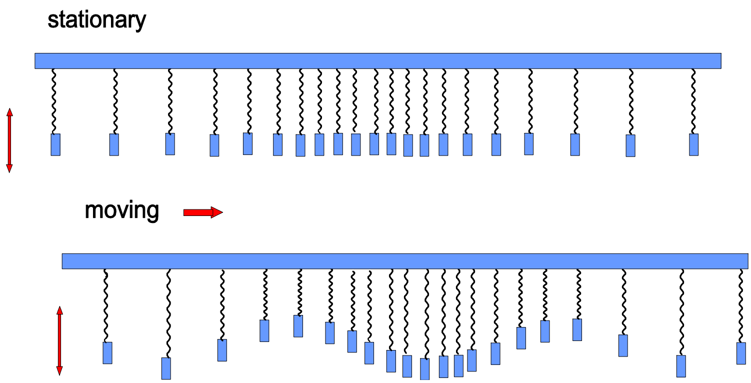

Fig. 2: De Broglie's array of oscillating springs. The sinusoidal effect in the moving array is a consequence of the failure of simultaneity. This sinusoidal "wave" is the de Broglie wave, not an independent wave as usually supposed, but as described by de Broglie himself, a dephasing of the standing wave modelled by the array of oscillating springs.

De Broglie provided no drawing of this array of springs, but it has been imagined from his description in Fig. 2. When the array is stationary, the springs oscillate in unison, but when it is observed to be moving at a relativistic velocity, there is again the curious effect of the failure of simultaneity. The springs no longer oscillate in unison, but in sequence, creating here again a sinusoidal effect moving through the array in the direction that the array is moving.

De Broglie's stated objective in describing this model was to show how an effect that evolves at a speed greater than that of light might yet be consistent with special relativity. And in this he succeeded very well. However, he expressly referred to the sinusoidal effect as an example of his phase wave and showed that it evolves through the array at the superluminal velocity c^2/v of that wave.

But why does this sinusoidal effect emerge in the moving array? It appears because phase is is being lost progressively from one spring to the next. What de Broglie has modelled is the phase modulation of the standing wave modelled by the springs.

5 *Ondes et Mouvements* (Waves and Motions)

In this "little volume" [1], published within two years of the thesis, de Broglie provided the mathematical treatment that was missing from the thesis. In *Ondes et Mouvements* (and see also de Broglie [29]) the periodic phenomenon is clearly a standing wave, and on subjecting this standing wave to a Lorentz transformation, he obtains the de Broglie wave, not as an independent wave, but as what is also very clearly now, the phase modulation of the underlying wave structure.

De Broglie does not expressly reject the possibility that the particle might comprise something small and solid or point-like within the wave, but his opening chapter refers in its heading to a "material point considered as a stationary wave", and he suggests that in view of the equivalence of relativistic mass and energy, radiation should be regarded as a continuous form of matter. He concludes this discussion by saying:

> Certainly, the concept of an isolated material point is only an abstraction. It is nonetheless legitimate and necessary to first study that simple case.

Consistently with the convenience of this "abstraction", he goes on to model the de Broglie wave as emerging from the Lorentz transformation of a standing wave organized around a singularity. But despite this modelling in which there is both wave and particle, he asserts that, as implied by the Planck-Einstein relation (1), the energy of the particle inheres in the wave. He says,

> The postulated frequency will be $\nu_0 = m_0 c^2/h$, and it will belong to a periodic phenomenon that prevails all around the material point, and which constitutes a singularity, just as the electron does for an electrostatic field. Since the frequency is unique and the material point is in a permanent state, the periodic phenomenon must be analogous[4] to a stationary wave, and we can represent it by an expression of the form:
>
> $$f(x_0, y_0, z_0)\, sin\,(2\pi\nu_0 t_0),$$

[4] Merely "analogous to a stationary wave" in this instance, but elsewhere a "stationary wave" or a "system of standing waves".

in which x_0, y_0, z_0, t_0 are the space-time coordinates of the proper system considered, and $f(x_0, y_0, z_0)$ is the amplitude of the phenomenon at each point.

On subjecting the stationary wave described above to a Lorentz transformation, he obtains the modulated wave,

$$f\,(x, y, \frac{z - vt}{\sqrt{1-\beta^2}}\, z,)\sin\frac{\nu_0}{\sqrt{1-\beta^2}}(t - \frac{\beta z}{c}),$$

which, when expressed in exponential form, is identical to Eqn. (6) of Sect. 2 of this paper, and can also be written as,

$$f\,(x, y, \frac{z - vt}{\sqrt{1-\beta^2}}\, z,)\exp i(\omega_E t - \kappa_{dB} x),$$

where the exponential term is the de Broglie wave.

De Broglie then does something very interesting. He suggests that the standing wave may be represented as a solution of the wave equation,

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}, \tag{12}$$

and considers a spherical wave in which the material point is the focus of incoming and outgoing waves having the velocity c of light. In its rest frame, this wave has the form,

$$\varphi(\mathbf{r},t) = \frac{A}{|\mathbf{r}|}\sin(2\pi\nu_0 t),$$

where,

$$\mathbf{r} = \sqrt{x^2 + y^2 + z^2}.$$

De Broglie shows that when this spherical structure moves at the velocity v, it acquires a distorted form,

$$\varphi(x,y,z,t) = \frac{A}{\sqrt{x^2 + y^2 + \gamma^2(z - vt)^2}}\sin(\kappa_0\sqrt{x^2 + y^2 + \gamma^2(z - vt)^2}\,)\sin(\omega_E t - \kappa_{dB} x$$

where I have expressed the final wave factor,

$$\sin(\omega_E t - \kappa_{dB} x),$$

is the usual form of the de Broglie wave, which is here again a modulation, rather than a wave in its own right.

De Broglie does not refer to that final wave factor as a modulation, but he does distinguish the differing roles of the two sinusoidal wave factors:

> When studied by an observer in uniform, rectilinear motion with respect to the atom-source, the stationary wave will take on a different aspect: Its amplitude will displace with the velocity of motion, and always with the same value on the surface of a flattened ellipsoid of revolution that is centered on the moving body, which is explained simply by the Lorentz contraction.
>
> The distribution of phases is much more curious and unexpected: They form wave planes that displace with a speed that is greater than that of light and becomes larger as the speed of the moving body becomes smaller. That is, one could say, the main point of my theory. It is derived directly from the Lorentz transformation and the relativity of time.

In this passage, the "flattened ellipsoid of revolution" is the relativistically contracted and modulated travelling wave of velocity v, while the "wave planes that displace with a speed that is greater than that of light" is the de Broglie wave evolving though the travelling wave as a phase modulation of superluminal velocity c^2/v.

Thus in *Ondes et Mouvements,* de Broglie provided an explanation for his phase wave in which it was very clearly not an independent wave, but rather the relativistically induced modulation that was described in the first three sections of this paper.

6 Solvay 1927

Ondes et Mouvements [1] bears the prefatory date of 20 February, 1926, while the Schrödinger equation was first presented to the world in a paper received by *Annalen der Physik* on 27 January of that same year (Schrödinger [30]).

Thus apart from a note added in proof, there no reference to the Schrödinger equation in *Ondes et Mouvements*, and although

Schrödinger, for his part, cited de Broglie's thesis and acknowledged very handsomely his debt to de Broglie, there is no reference to *Ondes et Mouvements* in any of the important papers on wave mechanics that Schrödinger submitted to *Annalen der Physik* during 1926 and 1927 (Schrödinger [31]). Nor therefore is there any reference in those papers to the possibility that the de Broglie wave might emerge from the relativistic transformation of a periodic phenomenon.

If inclined to counterfactual speculation, one might ponder whether the evolution of quantum mechanics might have proceeded a little differently had Schrödinger been able to begin his investigations of the de Broglie wave, not with the famous thesis of 1924, but with *Ondes et Mouvements.* He might have seen in the modulation the reconciliation he sought between the superluminality of the de Broglie wave and the existence of physically realistic particle trajectories. He might have also seen a path toward the thoroughly wave-theoretic explanation of matter that he himself seems to have favoured (see, regarding this, de Broglie [32]).

But that was not to be. Like those ships that pass unawares in the night, neither de Broglie nor Schrödinger seems to have been cognizant at the relevant time of the ideas being developed by the other.

Schrödinger did adopt - from the thesis - the notion that the velocity of the particle might be assimilated to the group velocity of a superposition of de Broglie waves. When he eventually realized that it was not possible to confine a particle to a non-dispersive wave packet, he argued instead that the wave function must be a description of the density of the electric field, and it was this interpretation of the wave function that he took unsuccessfully to the Solvay meeting in Brussels in October 1927 (see generally, Bacciagaluppi and Valentini [5]).

Notwithstanding the perplexing nature of its wave functions, the Schrödinger equation achieved an immense degree of explanatory success. Schrödinger was able to explain the observed energies of the Hydrogen atom and harmonic oscillator, as also the Stark and Zeeman effects. Importantly, he demonstrated the equivalence of his wave mechanics and the matrix mechanics of Heisenberg, Born and Jordan (Schrödinger [33]). The degree of interest engendered by Schrödinger's papers at the time is evident in his correspondence with colleagues, including in particular, Planck, Lorentz and Einstein [34].

The success of the Schrödinger equation does not itself explain why de Broglie did not develop further the interpretation of his wave that he described in *Ondes et Mouvements* [1]. But two factors seem relevant. One is that following the publication of Schrödinger's wave mechanics papers, de Broglie seems to have become very much involved with the interpretation of the Schrödinger wave functions, which are of course in a sense, de Broglie waves. The "double solution" theory that he proposed in 1927 is itself very much concerned with the Schrödinger equation and its relativistic sisters.

Another factor may have been de Broglie's (mostly) consistent view that particle and wave are physically distinct entities. In a discussion of the double solution theory (de Broglie [13]), dating from 1972, he stated that in 1923-1924, he "had no doubt whatsoever about the physical reality of waves and particles" and in a note published the following year [32], he confirmed his view that "a particle is a very small object that is localized and moves along a trajectory", while distancing himself at the same time from the attempt by Schrödinger in his papers of 1926 to develop a thoroughly wave-theoretic interpretation of the electron.

It is true that, as discussed in Sect. 5, de Broglie suggested in *Ondes et Mouvements* [1] that radiation might simply be a continuous form of matter and that the energy of the particle inheres in the wave. He also asserted that "the concept of an isolated material point is only an abstraction". However, by the time of his double solution theory of 1927, he had evidently taken a dualist position on the nature of waves and matter.

De Broglie encountered difficulties in explaining, as contemplated by the double solution theory, how the particle might persist as a singularity or small region of larger amplitude within the wave function. It was thus, as de Broglie put it, "an incomplete and diluted form" of that theory, which he referred to as "pilot-wave theory", that he presented without success to the Solvay conference of 1927 (see de Broglie [12], and generally, Bacciagaluppi and Valentini [5]).

Following that conference, and to briefly recount a well-known story, de Broglie eventually discontinued his efforts with the double solution theory, and it was not until he became aware of Bohm's own pilot wave approach [35] and [36], that he renewed those efforts, encouraged by the possibility that the particle might exist as a non-linear soliton within the wave function, a possibility that seems to be the main focus of current work on the theory (see again Refs. [14] - [20]).

7 Summary

I have presented a de Broglie wave that is ontologically parsimonious and consistent with both special relativity and classical wave theory. In support, of this interpretation, I have concentrated in this paper on just two arguments[5].

One was based on the nature of "solid" matter. I have shown that the Lorentz transformation induces an undulatory distortion with the characteristics of the de Broglie wave in any object that is both oscillatory and spatially extended. I have explained this as being a consequence of the nature of the elementary particles, which as implied by the Lorentz transformation must comprise underlying influences having the velocity c of light, while on the evidence of the Planck-Einstein relation, these influences of velocity c must be wave-like in nature and have the characteristic frequency of the species of particle in question.

My second argument was by way of an appeal to the authority of de Broglie himself, who in *Ondes et Mouvements* [1] described a wave that is essentially that presented in this paper, that is to say, an ontologically parsimonious de Broglie wave, consistent with well-established principles of physics, that leaves to the authority of the standard model, the difficult question of how these wave-like particles can actually exist.

References

[1] L. de Broglie, *Ondes et Mouvements*, Gauthier-Villars, Paris, 1926

[2] L. de Broglie, Doctoral thesis, *Recherches sur la théorie des quanta.* Ann. de Phys. (10) **3**, 22 (1925). English trans. by A. F. Kracklauer, *Researches on the Theory of Quanta*, Ann. Fond. Louis de Broglie 17, 92 (1992), and by J. W. Haslett (first chapter of thesis only) Phase waves of Louis de Broglie Am. J. Phys. **40**, 1315 (1972)

[3] F. Bloch, Heisenberg and the early days of quantum mechanics. Physics Today, Dec. 1976, adapted from a talk given on 26 April 1976 at the Washington D. C. meeting of The American Physical Society

[4] P. A. M. Dirac, The physical interpretation of the quantum dynamics. Proc. Roy. Soc. London A **113**, 621 (1927)

[5] For other arguments, see again the literature listed in Ref [8].

[5] G. Bacciagaluppi, and A. Valentini, *Quantum theory at the crossroads: Reconsidering the 1927 Solvay Conference.* Cambridge University Press. Cambridge, 2009

[6] M. Born, Zur Quantenmechanik der Stossvorgange. Z. Phys. **37**, 863 (1926), English trans., On the quantum mechanics of collisions, in J. A. Wheeler, and W. H. Zurek eds., *Quantum Theory and Measurement*, Princeton University Press, Princeton N. J.,1983

[7] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics.* Addison-Wesley, Reading, 1963

[8] D. Shanahan, Reverse Engineering the de Broglie Wave. IJQF **9,** 44-63 (2023)

[9] L. de Broglie, Ondes et Quanta. Comptes Rendus **177**, 507 (1923)

[10] L. de Broglie, La mécanique ondulatoire et la structure atomique de la matière et du rayonnement, J. Phys. Rad. **8**, 225 (1927)

[11] L. de Broglie, *The theory of measurement in wave mechanics.* Gauthier-Villars, Paris, 1957

[12] L. de Broglie, *Non-linear Wave Mechanics: a Causal Interpretation.* Elsevier, Amsterdam, 1960, a translation by A. J. Knodel and J. C. Miller of L. de Broglie, *Une Tentative D'Interprétation Causal et Non Linéaire de la Mécanique Ondulatoire (La théorie de la Double Solution)*, Gauthier-Villars, Paris, 1956

[13] L. de Broglie, Interpretation of quantum mechanics by the double solution theory. Ann. Fond. Louis de Broglie, **12**, 1 (1987)

[14] D. Fargue, Louis de Broglie's "double solution" a promising but unfinished theory. Ann. Fond. Louis de Broglie, **42**, 9 (2017)

[15] S. Colin, T. Durt, and R. Willox, L. de Broglie's double solution program: 90 years later. Ann. Fond. Louis de Broglie, **42**, 19 (2017)

[16] A. Matzkin, The theory of the Double Solution: Dynamical issues in quantum systems in the semiclassical regime. Ann. Fond. Louis de Broglie, **42**, 213 (2017)

[17] A. Drezet, The guidance theory of de Broglie. Ann. Fond. Louis de Broglie, **46**, 65-86 (2021)

[18] A. Drezet, Quantum Solitodynamics: Non-linear Wave Mechanics and Pilot-Wave Theory. Found. Phys. **53**, 31 (2023)

[19] A. Drezet, A time-symmetric soliton dynamics à la de Broglie. Found. Phys. **53**, 72 (2023)

[20] T. Durt, Testing de Broglie's Double Solution in the Mesoscopic Regime. Found. Phys. **53**, 2 (2023)

[21] C. Borghesi, Dualité onde-corpuscule formée par une masselotte oscillante dans un milieu élastique: étude théorique et similitudes quantiques. Ann. Fond. Louis de Broglie **42**, 161 (2017)

[22] A. Drezet, P. Jamet, D. Bertschy, A. Ralko, and C. Poulain, Mechanical analog of quantum bradyons and tachyons. Phys. Rev. E **102**, 052206 (2020)

[23] P. Jamet, and A. Drezet, A mechanical analog of Bohr's atom based on de Broglie's double-solution approach. Chaos **31**, 103120 (2021)

[24] D. Shanahan, The de Broglie Wave as Evidence of a Deeper wave Structure arXiv:1503.02534v2 [physics.hist-ph]

[25] D. Shanahan, What might the matter wave be telling us of the nature of matter? IJQF **5**,165 (2019)

[26] D. Shanahan, The Lorentz transformation in a fishbowl: a comment on Cheng and Read's "Why not a sound postulate?". Found. Phys. **53**, 55 (2023)

[27] A. Einstein, Zur elektrodynamik bewegter Korper. Ann. Phys. **17**, 891 (1905)

[28] N. Bohr, On the Constitution of Atoms and Molecules. Philos. Mag. **26**, 1-25; **26**, 476-502; and **26**, 857-875 (1913)

[29] L. de Broglie, Sur la fréquence propre de l'électron. C. R. Acad. Sc. **180**, 498 (1925)

[30] E. Schrödinger, Quantisierung als Eigenvertproblem. Ann. Phys. **79**, 361 (1926). For the English translations of this and related papers, see Ref. [31]

[31] E. Schrödinger, *Collected Papers on Wave Mechanics.* Minkowski Institute Press, Montreal, 2020

[32] L. de Broglie, On the true ideas underlying wave mechanics. C. R. Acad. Sci. **B277**, 71 (1973)

[33] E. Schrödinger, Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen. Ann. Phys. **79**, 734-756 (1926)

[34] *Letters on Wave Mechanics, Correspondence with H. A. Lorentz, Max Planck, and Erwin Schrödinger,* K. Przibram (ed.), Trans. by M. J. Klein, Philosophical Library, New York, 1963

[35] D. Bohm, A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables I. Phys. Rev. **85**, 166 (1952)

[36] D. Bohm, A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables II. Phys. Rev. **85**, 180 (1952)