De Broglie Scientific Research Program at Lisbon with a comment on the Wave-memory interpretation of Quantum Mechanics

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ABSTRACT. An historical review of de Broglie complex nonlinear realistic research program developed at Lisbon will be presented, including the efforts to design an empty quantum wave detection experiment, and holding a comment on the Wave-memory interpretation of quantum mechanics.

Keywords : De Broglie realistic research program in Lisbon, ontic nature of the quantum waves, quantum yes-no experiments to test the physical reality of de Broglie waves, nonlinear quantum physics, orthodox quantum mechanics, Wave-memory interpretation of quantum mechanics.

1 Introduction

In his Ph. D. thesis of 1924, de Broglie [1] advanced the revolutionary idea that electrons and other quantum particles had a dual nature, having properties of corpuscular localized entities and also, extended wavelike properties. In his subsequent work [2] he had the dared to propose explicitly, for the first time, the *guidage* principle also known as the pilot wave principle. A nonlinear relational complex process, in which a minor action may, in adequate conditions, give rise to a huge reaction. All this, against the general accepted Cartesian linear dogma, in which the whole is equal to the sum of the parts and where action is proportional to or equals reaction.

Still, due to mathematical difficulties, namely von Neumann impossibility theorem [3], he devoted his main activity to traditional quantum mechanics. Only after the very important papers [4a, 4b] of David Bohm in 1952, showing the non-generality of von Neumann theorem, he directed his main activities to develop the *double solution theory* [5].

Just after this change, in 1953, with his wife Maria Helena, João Andrade e Silva went to Paris and started working with de Broglie and his collaborators, namely, Jean-Pierre Vigier and George Lochak. Fully integrated in this research group he worked along the development of de Broglie research project, writing a book with de Broglie years later [6].

In 1972 Andrade e Silva returned to Lisbon to continue de Broglie scientific research project, and on a causal complex nonlinear approach for explaining the quantum phenomena, pioneering this approach in Portugal. This was the beginning of Lisbon research group devoted to the development of these ideas.

The basic difference between the de Broglie's causal realistic approach and the traditional one, still pursued by the orthodox Copenhagen school interpretation, was and is related with problem of how to understand the double behaviour of the quantum entities, which in certain experimental conditions manifest itself as waves and in others, as corpuscles.

2 Proposals for understanding quantum phenomena

2.1 Quantum Orthodoxy

To solve the problem of the observable double nature of the quantum entities Niels Bohr proposed his complementarity principle. According to this principle, any quantum entity manifests itself either as wave or as a corpuscle, the two complementary properties never manifesting at the same time. The wave property of the quantum entity, ψ, assumed to contain in it all information about the system, albeit being a solution of the Schrödinger equation, is no more than a mere helping predictive tool devoid of any physical meaning, thus normalized at will.

2.2 De Broglie causal double solution

For de Broglie and his school, a quantum entity, ϕ, a particle, may be understood as a dual entity, thus the name double solution theory, being both a wave, θ, practically devoid of energy and a very well localized and hight energetic kernel or corpuscle, ξ. In the linear approximation we have, $\phi = \theta + \xi$. In this approach there is hence a real quantum wave, θ, also named by the Lisbon group, theta wave, being also known in the literature by guiding wave, pilot wave and empty wave. Furthermore, the quantum particle complies a localized corpuscle within the theta wave, named by the Lisbon group, *acron* (from the Greek, ἄχρον *ákron*, high peak, from where the word acropolis also comes). This singularity is also referred to in the literature by nucleus, kernel, (sometimes confusingly) by particle and basic oscillator. A macroscopic analogue of a quantum particle equating to a theta wave plus an *acron*, could, for instance, be metaphorized by a hurricane in which the vortex represents the localized kernel and the remaining extended perturbation atmospheric represents the wave.

Thus, we are before two opposite positions :

Orthodox quantum mechanics, claiming that quantum waves are mere probability waves devoid of any physical meaning.

Causal de Broglie complex nonlinear quantum physics, maintaining that quantum waves have a real physical reality.

These two opposite ways of trying to understand quantum phenomena have deep implications in the description and predictive capacity of the quantum measurement process.

3 Quantum measurement

In quantum mechanics, due to the impossibility of evaluating the behaviour of a single quantum entity while interacting with the medium, the measurement process, different from most measurement processes in classical physics, is always of a statistical nature. In such conditions, the protocol for a quantum measurement decision process is basically the following :

1- First it is necessary to consider the actual state of the physical system evolving both the system to be evaluated and the measuring devices, that is, the interacting medium. This happens because, due to the reciprocal interaction, the medium modifies the system to be evaluated and, reciprocally, it is modified in a more or less degree by the system.

2 - Evaluate, as careful as possible, the universe of all present and future possible outcomes of the measurement, resulting from the reciprocal interaction between system and medium. These possible future outcomes need to be compatible with the physical interacting conditions that were set thru, relating with the problem.

3 - To each possible outcome, attribute a probability that it will occur. The sum of all such probabilities will naturally equal to one.

4 - Decision process. This final step makes one and only one of all these possibilities actual, that is, real.

This complex relational nonlinear process has two main possible descriptions :

3.1 The Orthodox description

1 – Initially, the physical system is described by the probability wave function, ψ, solution of the Schrödinger equation, and assumed to contain all possible information about the system, and thus, about all possible behaviours of the corpuscle.

2 - To the physical quantity, $\mathcal{A}$, we want to measure, a quantum operator, A, is then assigned.

3 - A set of eigenfunctions, φ_i, of operator A are determined.

4 – The description then expresses the state function as a linear combination of eigenfunctions φ_i of the quantum operator corresponding to the concrete physical property $\mathcal{A}$, we want to measure, such that, $\psi = c_1\varphi_1 + c_2\varphi_2 + \ldots + c_i\varphi_i + \ldots$. These eigenfunctions, φ_i represent the possible outcomes of the measurement process and $|c_i|^2$ their associated probabilities.

5 – The collapse of the wave function is postulated, corresponding to the quantum measurement decision process. Furthermore, claiming that when the actual measurement takes place, from all possible outcomes only one becomes real, say, $\psi \rightarrow \varphi_k$, all other vanishing into nothing.

6 - After the collapse, after the measurement, all information relative to present and future possible states, gathered on the physical concrete problem is consequently completely lost.

It is worth mentioning that even if orthodox quantum mechanics is assumed to be a linear theory, a most basic ingredient of it, the collapse of the wave function, is in fact a nonlinear complex process. This collapse may occur even when there are no observable physical modifications in the measuring apparatus, as can be inferred from Renninger type experiments. Precisely from this, somehow extreme interpretation of quantum mechanics, where reality is said to change even without a physical interaction with the system, comes claim that the collapse of the wave function is due to the Observer's consciousness.

3.2 de Broglie's realistic description

1 - It is assumed that a quantum measurement is indeed a very complex inter-relational process co-involving the observer, the system to be evaluated and the measurement apparatus, that is, the medium.

2 - Since, the quantum wave, θ, corresponds to the description of an entity having physical reality, it cannot be normalized at will. To allow the statistical predictions this real wave is related with an information abstract probability wave, through a normalizing constant, C, so that, $\psi = C\theta$.

3 - Evaluate the possible concrete future physical outcomes, θ_i, and assign to it a probability, c_i. This operational procedure is, in all, very much similar to the macroscopic traditional measuring way. So, the possible outcomes may be determined in a protocol much similar to the operatorial orthodox process, only now the possible states, θ_i, correspond to the description of a real physical situation. Consequently, such states can be normalized at will, a step that is done in a similar way as in any classic statistical process, with the introduction of adequate constants.

4 – When the concrete measurement takes place, just as in a statistic classic process, there occurs the collapse of the probabilities, all turn out zero except one, c_k. This is a purely informational process that does not involve any potential states.

5 – Extending de Broglie's ideas, one might think that, as the theta wave guides the corpuscle, the so-called probability wave collapse may be understood as representing a "guiding information" reading process, where the corpuscle reads momentum and position information from the wave, actualizing them, the wave acting as a physical memory. And if indeed the quantum wave is an encoding physical structure, then the claim that it contains all information about the system becomes quite natural. More so if one relates such information with the corpuscle possible behaviours. It should also be mentioned that what then physically exists in a state of superposition are *not the real physical states themselves*, but the representations of such states that are *physically encoded* in the quantum wave. Finally, in the context of this wave-memory hypothesis, de Broglie's double solution research program regains its strength, albeit requiring development. It so happens that there are always two descriptions involved about a quantum phenomenon. The one that deals with the spacetime evolution of the memory tridimensional wave carrier and the other, representing the reading process undergone by the corpuscle, to acquire and actualize the encoded information in the wave. We will say more about this in the final section of this article.

Let us recapitulate the two paradigmatic descriptions of quantum phenomena.

The orthodox quantum mechanics description :

Once the act of measurement is accomplished, after the collapse decision process occurs, nothing more remains of the previous possible states, all previous gathered information is lost. Since φ_i are mere probability waves, all became zero, except one, $\psi \to \varphi_k$.

de Broglie causal complex nonlinear description (extended) :

As in classical statistics, when the decision collapse process occurs, all probabilities, assigned to each possible future state, θ_i, except one, turn to be zero. Nevertheless, and here is the important point, since there is a real physical quantum wave and since it may encode the representations of all real physical possible future states, θ_i, such information persists.

Now the important question, how are we to decide between these two opposite visions of reality ?

According to the scientific method, the answer must be provided by praxis, empirically. More concretely, the discussion between the Copenhagen School approach and de Broglie assessment would be highly clarified if the physical reality of quantum waves could be settled experimentally.

It was precisely to try and solve this problem, in collaboration with the Italian group of Franco Selleri, that the Lisbon School of nonlinear quantum physics, initiated by Andrade e Silva and following de Broglie research program, devoted some of its main activities.

4 The search for experiments to test the physical reality of de Broglie waves

Many efforts were devoted to devise some concrete experiments that could decide on the eventual reality of the quantum waves, still, as far as we know, only in 1972 a first germ for an eventual experiment was proposed, in a scholar work [7], at the University of Lisbon by Paulo Neves, then a young student of Andrade e Silva.

A little later, in 1980, Andrade e Silva and his wife, Maria Helena Andrade e Silva explored the idea in the paper "*Une expérience possible concernant la nature du dualisme onde-corpuscule*" [8].

In 1983, in collaboration with Franco Selleri and Jean-Pierre Vigier, Andrade e Silva, in sequence of Paulo Neves initial idea, published the work, "*Some possible experiments on quantum waves*" [9].

These initial proposals, were naturally, mainly of a conceptual nature, with no clear indication for a concrete practical experiment.

The first concrete proposal of an experiment, although still lacking the technological means to do it, was presented at the International Conference Microphysical Reality and Quantum Formalism, Urbino, Italy in 1985 and later, in 1988, published at the proceedings of the conference by one of us, *Can the existence of de Broglie's empty waves be proven experimentally ?* [10]

Nevertheless, the first concrete proposal for a feasible experiment, with the then available technology was presented in 1990 again by one of us in collaboration with A. Garuccio, V. L. Lepore and R. N. Moreira in the paper Q*uantum-Optical predictions for an experiment on de Broglie waves* [11].

This proposal of experiment was performed with some modifications at the University of Rochester, in 1992, by X.Y. Zou, T. Grayson and L. Mandel [12].

The results of this experiment, according to the author, dismissed the physical existence of quantum waves. However, a careful analysis on the experiment clearly shows that the experimental results are inconclusive, something that we discuss in a previous work [13].

At about the same time Lucien Hardy published an interesting paper intitled "On the existence of empty waves in quantum theory" [14], where he presents some logical and physical arguments favoring the existence of empty waves.

Unfortunately, and since then, no more experiments with the sole objective to clear this important question, on the nature of quantum waves, has been performed. The authors can only report about a single experiment indirectly related with the subject, done in 2012 by R. Menzel and his research group [15a, 15b]. The results of Menzel's experiment may be interpreted as evidence for the existence of real physical de Broglie pilot waves, as discussed here [16].

More recently, in 2022, refereeing to the present technological means available, we have also published a paper proposing several updated setups for a quantum wave detection experiment [17].

5 A comment about the Wave-memory interpretation of Quantum Mechanics

The Wave-memory interpretation gets its name from the wave-memory hypothesis that can be summarily formulated, saying that a quantum wave is a physical structure that encodes probabilistic information about all corpuscle behaviors. In an initial, not so strong version, the hypothesis claims that the wave encodes probabilistic information about the corpuscle all possible position and momentum values. Hence, and using a concept from the Philosophy of Science, the quantum wave is a *nomological memory* (from the Greek : *nomos*, law, and *logos*, reason, order). A tridimensional physical structure, with a role metaphorically comparable to that of the genetical code in Biology or to that of a Turing machine's tape in Computer Science. The *guidage* process will then amount to a transfer of information from the wave to the corpuscle, that the corpuscle actualizes in a form of actual measurable states. The corpuscle behavior changing in accordance with the information transfer dynamics, and also altering the state of the wave, and thus reencoding probabilistic information in the field. It must be noted that the corpuscle only behaves, in an average way, consistently with the probabilistic information encoded in the quantum wave. This means two things : first, that the *guidage* process is not deterministic, as de Broglie initially thought in the spirit of classical physics. The corpuscle assumes different position and momentum values in a random way, according to the probability information that is associated with each value. Second, it means that we will only observe the behaviors that have an encoded probability high enough for such behaviors to be averagely expressed by the corpuscles. This introduces a degree of regularity or "law-like" degree in Nature at the quantum level, while excepting many states and favoring another. All this, it must be said, also resonates somehow with the "implicate order" ideas put trough by David Bohm and Basil Hiley [18], although they have taken a much more idealistic path with their version of the pilot wave.

The Wave-memory interpretation corresponds to a pilot-wave framework approaching quantum phenomena. As such, even if the trajectories of corpuscles are of a discontinuous nature, the corpuscles exist at all times, along with the quantum wave. Regarding the uncertainty principle, this may be interpreted, not as an assertion conditioning the actual physical existence of the position or momentum states of the corpuscles, but mainly as an assertion about the degree of order in the system. It can thus be thought as saying that in a very low degree of "law-like" order, to any possible position value it may correspond a very large set of momentum values. In fact, one of us as derived a generalized version of the uncertainty relations [19], that the other of us has identified as containing all possible degrees of law-like or nomological order, from the extreme Heisenberg case to the Newtonian deterministic one [20]. These generalized uncertainty relations were obtained using Morlet wavelets, going through the very same derivation process that Niels Bohr pursued in 1927 to obtain the usual ones. The main reason why such quantum wavelet solitons were used is that they are the natural solutions of a non-linear Master equation, containing the quantum potential, that one of us has derived from the usual Hamilton-Jacobi and continuity equations [19]. For elucidative purposes, we leave here the mathematical expressions of the above.

It should be emphasized that as the spatial spreading of the wavelet σ (sigma) becomes very large, one regains correspondently, the usual Schrödinger linear differential equation, the usual Fourier type plane wave, as well as the standard Heisenberg relations, these being a particular case of the generalized version. The wavelet soliton quantum wave having the form :

$$\theta = Ae^{-\frac{(x-vt)^2}{2\sigma_x^2}+i(kx-\omega t)} \underset{\sigma_x \to \infty}{\longrightarrow} Ae^{i(kx-\omega t)} \Rightarrow (\theta\theta^*)^{\frac{1}{2}} \underset{\sigma_x \to \infty}{\longrightarrow} A \text{ (const.)} \tag{1}$$

Where the velocity of the wavelet v depends on the momentum last value the particle happened to have extracted from the wave.

The nonlinear Master equation reading :

$$-\frac{\hbar^2}{2m}\nabla^2\theta + \frac{\hbar^2}{2m}\frac{\nabla^2(\theta\theta^*)^{\frac{1}{2}}}{(\theta\theta^*)^{\frac{1}{2}}}\theta + V\theta = i\hbar\theta_t \underset{(\theta\theta^*)^{\frac{1}{2}}=\text{const.}}{\longrightarrow} -\frac{\hbar^2}{2m}\nabla^2\theta + V\theta = i\hbar\theta_t \tag{2}$$

And the generalized uncertainty relations, using Morlet wavelet solitons, resulting in the following formulae :

$$\Delta p_x = \frac{\hbar}{\Delta x}\sqrt{1-\Delta x^2/\sigma_x^2} \tag{3}$$

$$\Delta E = \frac{\hbar}{\Delta t}\sqrt{1-\Delta t^2/\sigma_t^2} \tag{4}$$

We should now address the main difficulties, besides those invoked by the uncertainty relations (already dealt with in this work), that standard quantum mechanics usually posits before alternative proposals. Starting by the superposition objection, that a fully actualized multi-reality cannot be in a state of superposition in a four-dimensional world, one can argue quite naturally, that what is in a state of superposition are not mutual exclusive instantiations of reality itself, but the informational representations of such instantiations. More to the point, a wave carrier, acting as a nomological memory, can contain information about all possible states and behaviors of the corpuscle, including those states and behaviors that are mutually exclusive. It is, of course, notable that the sum of wavelet solutions to the Master equation are not in general a solution of the Master equation, since it is a nonlinear differential equation. This apparently insurmountable difficulty, from a theoretical point of view, perhaps again suggests that the linearity owed to the superposition principle strictly concerns the information dynamics, and not the four-dimensional dynamics of the quantum wave itself. Somehow vindicating the double solution hypothesis, that in this sense should be generalized into the necessity of having a double dynamics description, with the linear description focusing on the evolution of nomological information and the nonlinear focusing on the evolution of the carrier wave. Of course, that the relation between the two dynamics should be determined, being a very important research topic in the future.

Entanglement, on the other hand, is a much more difficult objection to surpass, even in the context of the wave-memory picture. The justification that, for the moment, comes to mind, can be one of two. Either the entangled corpuscles are somehow able to exchange information at superluminal velocities trough the quantum wave, or all corpuscles somehow acquire each one the right state from reading configurational information from the common quantum wave field where they happen to be. Both hypotheses require careful thinking, perhaps suggesting that there is a deeper connection between quantum waves and the fabric of spacetime itself.

To end up this comment, we would like to offer some motivation supporting the wave memory interpretation. Surprisingly, that motivation comes from the field of Hydrodynamics. It so happens that in 2005 Yves Couder, S. Protière, Emmanuel Fort and A. Boudaoud [21, 22] found that a millimetric oil droplet could self-propel, bouncing on the surface of a vibrating liquid bath. A trajectory composed of bouncing positions was formed, each bounce resulting from the interaction between the droplet and the quasi-monochromatic wave field originated and reinforced by that same droplet, while propagating on the oil surface. This physical scenario corresponds to a hydrodynamic pilot wave phenomenon, where the wave generated by the droplet will in turn, constrain or guide the droplet to follow a certain trajectory. The oil bath must be shaken from below by a vibrating force, producing an amplitude acceleration value, below what is called the Faraday threshold. This is the acceleration value above which the pilot wave phenomenon will no longer occur, giving way to standing Faraday waves on the oil surface.

During the last two decades a Hydrodynamics Quantum Analogs (HQA) international community has settled down. Very much due to the enthusiastic efforts of John Bush at the MIT, along with important work done in France, physicists have tested in an "analog type approach" the virtues of a possible pilot-wave picture of quantum mechanics. People have model, either experimentally or with the use of computational resources, several situations analogous to the ones found in quantum reality [23].

Much to the point in this work, the most striking feature coming from the HQA field, already foreseen by Yves Couder and his team, is that there seems to be a relation between the resulting trajectories stability and a sort of path memory encoded in the wavefield. This dependence had been noted by Antonin Eddi and co-workers in 2011 [24], and also by Stéphane Perrard and *et al*, even comparing the droplet-wavefield system to a natural Turing machine [25], something quite crucial, as an analogy, to the ideas we have been putting forward.

As was soon realized, the overall pilot wavefield, guiding the corpuscle, is composed of several waves, generated by the corpuscle in prior bounces. The interference of the waves produces an overall field that in turn affects the corpuscle next bouncing, acting as a "natural path memory storage device", and holding kinematic information about the droplet's behavior. The storage process is correlated with the dynamic evolution of the field. The decaying time of the prior waves define the wavefield memory storage capacity, controlled experimentally by the acceleration impressed on the bath. In the case where the decaying time is very rapid (low acceleration) the pilot wave effect will tend to disappear. If the decaying time is extended (using high acceleration forcing on the oil bath, either near or above the Faraday threshold), the pilot wave phenomena will give place to other effects, including some that are analogous to quantum chaos, with a droplet bouncing unpredictably [23]. All this suggests that the wave-memory, that in this particular case is indeed a nomological structure, has a finite storing capability, defining a memory capacity interval, something that one would expect to find in any realistic memory structure.

The HQA experimental domain is a very rich one, holding several insights and surprises, deserving a much wider description than the one presented here. The key point, however, is that the generated wavefield seems to encode possible position and momentum values for the droplet. The wavefield acting as a memory, while the droplet decodes such information, actualizing its behavior and reinforcing the wavefield. Significantly, it is even possible to make the droplet invert its movement and retrace its previous path [25], thus showing that it can write and read position and momentum information. For a more graphic illustration of the phenomena one can access some filmed footage here [26, 27].

The HQA experimental field shows that pilot-wave phenomena do exist, although in this particular variant being deterministic and hence being describable using Newtonian mechanics. However, the existence of chaotic regimes in some conditions, as stated before, seem to suggest that there is a powerful relation between a natural memory storing structure, the wavefield, and the behavior of the reading agent, the droplet. Given the conceptual resonance one encounters between HQA and the ideas of de Broglie, along with several hydrodynamically modeled cases of quantum mechanics, it seems reasonable to extend the analogy to a fully formulated hypothesis, thus conveying the wave-memory assumption. Namely, that a quantum wave encodes probabilistic information about all possible position and momentum values of the corpuscle, that in turn, extracts such information in a random way from the wave, actualizing those values along its movement.

The wave-memory interpretation [20], along with several other articles covering HQA phenomena, as well as some other important aspects of Pilot-wave theories, can be found in a recently published book [28] about the topic.

We wish to express our public acknowledgement for the efforts of Professor João Andrade e Silva (1928-2017), a co-worker of Louis de Broglie, that allowed us to go further in the exploration of an alternative pathway to standard quantum mechanics, trying to reach further scientific intelligibility about the world.

Quantum Mechanics is not a closed topic.

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