From « La Nouvelle Théorie de la Lumière » to quantum relativistic equations for two particles

PIERRE PELCÉ

96 Chemin de Palama 13013 Marseille

pelcepierre@gmail.com

RESUMÉ. Les théories quantiques relativistes à deux particules de spin $\frac{1}{2}$ liées entre elles, initiées dans “La nouvelle Théorie de la Lumière“ et la “Théorie de la Fusion” de L.De Broglie, sont revues et complétées par l' étude d' équations quantiques relativistes à deux particules dans lesquelles les deux particules peuvent se mouvoir indépendamment. Nous focalisons ici l' étude sur des solutions simples de l' équation de Durt et Pelcé qui a l' avantage, par rapport à celle de Bohm et Hiley, d' être invariante par transformations de Lorentz. L' étude du corpuscule composé montre que les dynamiques déterminées par la méthode de fusion de L.De Broglie et par l' équation de Durt et Pelcé sont les mêmes. L' étude du mouvement relatif permet d' obtenir la correction relativiste à la corrélation de spins mesurée, avec les appareils A et B, à différents angles avec l' axe Oz, le long des vecteurs $\vec{a}$ et $\vec{b}$.

ABSTRACT. Relativistic quantum theories for two particles of spin $\frac{1}{2}$ bound together, initiated in “La nouvelle Théorie de la Lumière“ and la “Théorie de la Fusion” de L.De Broglie, are reviewed and completed by the study of relativistic quantum equations for two particles in which the two particles can move independently. Here we focus the study on simple solutions of the Durt and Pelcé equation, which has the advantage compared to the Bohm and Hiley equation, to be invariant under Lorentz transformations. The study of the motion of the compound particle shows that the dynamics determined from the L.De Broglie fusion method and the Durt and Pelcé equation are the same. The study of the relative motion allows to obtain the relativistic correction to the spin correlation measured with apparatus A and B, at different angles with the Oz axis, along the vectors $\vec{a}$ and $\vec{b}$.

1 Introduction

I thank the organizers of the “100 ans d’ onde de matière”, an international conference dedicated to the three historical papers of L.De Broglie of 1923, to invite me to give a lecture on the connection between fusion theories of L.De Broglie and recent developments of the quantum relativistic equations for two particles. It was an occasion to make the book “La nouvelle Théorie de la Lumière” bound with beautiful leather, from a faded copy that I bought 800FF, 25 years ago.

Quantum mechanics has had a considerable development after the determination of quantum equations for the wave function, first for one and many non relativistic particles with E.Schrödinger [1], then for relativistic particles with Klein and Gordon [2] and L.De Broglie [3], then for the electron of spin $\frac{1}{2}$ with the quantum relativistic equation of Dirac [4].

The development of relativistic quantum mechanics with many particles has quickly eclipsed the wave equations, with the essential argument that the number of particles cannot be conserved during a certain dynamics, since energy can be transformed in mass energy of a new particle. A new formalism has been developed from 1926 to 1950, which implies the concepts of identical particles, bosons, fermions, Fock space, operators annihilation and creation, second quantification, Feynman diagrams, leading to an exceptional agreement between theory and experiments [5].

However, the study of relativistic quantum equations with many particles, which remained marginal, is still active. As we explain for instance in section 3, it has been shown in the past [6] that the equation proposed by Bohm and Hiley to describe a pair of non-interacting Dirac fermions is not relativistically invariant, but it suggests another equation, the Durt and Pelcé equation [7], which is well invariant under Lorentz transformations. Contrary to the Bohm-Hiley equation, the density associated to the Durt-Pelcé equation is however not always positive. Actually, we were unable to find a two particle equations which is at the same time relativistically invariant and characterized by a positive conserved density. Despite of this frustrating result, the Durt-Pelcé equation presents however substantial advantages regarding the Bohm-Hiley equation :

-it is directly connected, as previously shown in [8], to the fusion theory of L.De Broglie, already mentionned in the abstract, and presented with more details in section 2 ; henceforth it can be used to study simple solutions, as plane waves of a compound particle, as we show here in section 4. The study

of the motion of the compound particle actually shows that the dynamics determined from the
L.De Broglie fusion method and the Durt-Pelcé équation are the same.

-the DP equation also allows us to solve the problem of spin correlation in the Bohm version of the EPR experiment in the relativistic range, as we also show in section 4, where we study the relative motion of two particles of spin $\frac{1}{2}$ emitted from a same source, moving in two opposite directions along a same Ox axis, providing a relativistic correction to the well-known non-relativistic formula.

2 Nouvelle Théorie de la Lumière et Théorie de la Fusion

One can again attribute to L.De Broglie, having initiated this field of research. After his great success in the 1920's on the wave mechanics of matter particles, L.De Broglie proposes, at the beginning of the 1940's, in "La nouvelle Théorie de la Lumière" to establish, along similar lines, a wave mechanics for photon [9]. However the problem appears more complex. The equation has to be invariant under Lorentz transformations, and at that time one only knows the Dirac equation for the electron. Then he proposes to consider the photon as a particle composed of two particles of spin $\frac{1}{2}$ and same masses. The two particles play a complementary role in the compound particle, since one of the particle of wave function φ satisfies the Dirac equation, the other of wave function ψ satisfying the conjugate of the Dirac equation. The photon wave function $\Phi_{ik}=\varphi_i\psi_k$ has 16 components. They satisfy two groups of 16 equations that one determines with the Dirac equation and its conjugate, both particles being coupled by the binding condition

$$\varphi_k\frac{\partial\Psi_i}{\partial q}=\frac{\partial\varphi_k}{\partial q}\Psi_i=\frac{1}{2}\frac{\partial(\varphi_k\Psi_i)}{\partial q}=\frac{1}{2}\frac{\partial\Phi_{ik}}{\partial q} \quad (1)$$

q being any of the four coordinates x, y, z, t.

L.De Broglie interprets this condition as the equality of energies and momenta of the two bound particles. As he told himself, this binding condition is in principle not correct, but succeeds. With his colleague J.Géhéniau [10], they show that one can build with the 16 components of the wave function, the components of an electromagnetic field which satisfy the Proca equation, the mass which appears in the equation being the common mass of the particles. In a remarkable way, when this mass tends to 0, these equations give the Maxwell equations. This theory raised the problem of an eventual possible mass for the photon, which will however be extremely small, less than the observation limit, of the order of 10^{-52} g.

Then, in 1943, L.De Broglie gives a new formulation of his theory in "La Théorie de la Fusion" in order to apply it to any particle composed of two particles of spin $\frac{1}{2}$ and same masses, the difference with the previous theory being that the wave functions of the particles satisfy both the same Dirac equation, playing now a symmetric role in the compound particle [11]. The same binding condition (1) is used, leading as previously to two groups of 16 equations for the wave function of the compound particle.

This reformulation of the theory takes on a more mathematical character, above all with G.Petiau, A.M.Tonnelat, then G.Lochak [12], since with the help of the two matrices Λ=-i$\gamma_2\gamma_4$ and Γ=-i$\gamma_3\gamma_1$, they decompose the matrices $\Psi\Lambda$ (Petiau et Tonnelat) and $\Psi\Gamma$ (Lochak) on the Clifford algebra, where Ψ is the 4-4 matrix built with the 16 components of the wave function, the decomposition factors being the components of an electromagnetic field. They satisfy Maxwellian equations, at the origin of the usual electric photon with the first matrix, and a magnetic photon with the second.

When we manipulate the 16 equations for the wave function of the compound boson, one can in particular obtain one group of 16 equations of which 8 evolution equations with a time derivative and 8 condition equations without time derivative. This group of equations, called the form III, determines the dynamics of a compound boson, which as already shown in [8], can be obtained more generally as a solution of the Durt and Pelcé equation for two particles. Here the two particles can move in general independently, but the two particles may be however eventually linked, when we equal the spatio-temporal coordinates of the two particles in the configuration space.

3 Quantum relativistic equations for two particles

Here, we take up the same problem, but in a broader context, since, in our approach, the two particles of spin $\frac{1}{2}$ can move independently. One has to determine the quantum relativistic equation for the wave function of the two particles in the configuration space, as the Schrödinger equation for two particles describes this dynamics in the non relativistic range. The first equation proposed is the Bohm and Hiley equation [13] which aligns the two spatial parts of the Dirac equation taken at the coordinates of each particle, with one common time, as the Schrödinger equation aligns the two Laplacien terms for the coordinates of each particle. Thus the wave function of the two particles system has 16 components, as the photon wave function introduced by L.De Broglie. The major flaw of the Bohm and Hiley equation is that is not invariant under Lorentz transformations, as we originally showed in [6]. To show that, one has to assign in the wave equation a different time to each particle, as did G.Wentzel in his “Quantum theory of fields” (1949) [14], since for instance a simple Lorentz transform along the Oz axis

$$t'=\frac{t-\frac{V}{c^2}z}{\sqrt{1-\frac{V^2}{c^2}}} \quad (2)$$

gives two different times to the two particles in the new frame, when they are at two different points at the same time in the old frame. We first recall the Dirac equation for a free particle of mass m and spin $\frac{1}{2}$ wich can be written as [4]

$$(P_0-\alpha_1 P_1-\alpha_2 P_2-\alpha_3 P_3-\alpha_0 mc)\Psi_k=0 \quad (3)$$

where

$$P_0=i\hbar\frac{\partial}{c\,\partial t} \quad P_r=-i\hbar\frac{\partial}{\partial x^r} \quad , \quad r=1,2,3 \qquad (4)$$

and where the Dirac matrices are expressed in the standard form as

$$\alpha_0=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad (5)$$

$$\alpha_1=\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad \alpha_2=\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \alpha_3=\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

This equation conserves a current

$$\vec{j}=\Psi^+\vec{\alpha}\,\Psi \qquad (6)$$

with the density

$$\rho=\Psi^+1\Psi \qquad (7)$$

definite positive, which can be associated to a probability density. One can also associate to this equation a spin density

$$\sigma_x=i\Psi^+\alpha_2\alpha_3\Psi,\ \sigma_y=i\Psi^+\alpha_3\alpha_1\Psi,\ \sigma_z=i\Psi^+\alpha_1\alpha_2\Psi \qquad (8)$$
$$\sigma_0=i\Psi^+\alpha_1\alpha_2\alpha_3\Psi$$

This equation has three important properties : It conserves a current with a definite positive density, it is invariant under Lorentz transformations and tends to the Schrödinger equation in the non relativistic limit.

The quantum relativistic equation of Bohm and Hiley for two particles, with one time for each particle, can be written as [6], [8]

$$((\mathbf{1}^A P_0^A-\overrightarrow{\alpha^A}.\overrightarrow{P^A}-m_A c\alpha_0^A)\otimes\mathbf{1}^B$$
$$+\mathbf{1}^A\otimes(\mathbf{1}^B P_0^B-\overrightarrow{\alpha^B}.\overrightarrow{P^B}-m_B c\alpha_0^B))\Psi_{AB}=0 \qquad (9)$$

where we notice that with the tensorial product, the 4-4 Dirac matrices become 16-16 matrices acting on the wave function for two particles Ψ_{AB} of 16 components.

For the Dirac equation, we apply the Lorentz transformation along Oz to the sub group of components Ψ_1, Ψ_3 [6,15]. Taking a linear combination of these equations, one can rebuild the original equation since the determinant of the associated homogeneous system vanishes [13].

For the Bohm and Hiley equation [6], we apply the Lorentz transformation along Oz to the sub group of components $\Psi_{11}, \Psi_{13}, \Psi_{31}, \Psi_{33}$. Taking a linear combination of these 4 equations, one cannot rebuild the original equation since the corresponding determinant does not vanish [6].

Another way to understand why the single particle Dirac equation is relativistically invariant is to note that it extermizes a Lagrangien that is invariant (scalar) under Lorentz transformations. It is out of the scope of the present paper to study this argument in details here, but it is based [16] on a well known mathematical property of spinors and spinorial representations of the Lorentz group : quadratic expressions of the type $\Psi^+\alpha_i\Psi$ transforms like the spatial components of a relativistic 4-vector, while $\Psi^+\Psi$ transforms like the time component of the same relativistic vector. Henceforth, $\Psi^+ P_0 \Psi + \sum_{i=1,2,3} \Psi^+\alpha_i P_i \Psi$ transforms like a Lorentz scalar. Moreover $\Psi^+\alpha_0\Psi$ transforms like a Lorentz scalar.

In the same line of thought, the Bohm and Hiley equation is not invariant under Lorentz transformation since the associated Lagrangien contains sums of products $(\Psi^+)^A\Psi^A$ and $(\Psi^+)^B\Psi^B$, where A and B refer to the particles A and B. Under a Lorentz boost of transformation matrix T [15], these products transform as $(\Psi^+)^A((T^A)^{-1})^+(T^A)^{-1}\Psi^A$ and $(\Psi^+)^B((T^B)^{-1})^+(T^B)^{-1}\Psi^B$, which are different from the initial ones since T is not unitary, showing that the Lagrangian is not invariant [7]. As noticed by Th.Durt [7], the introduction of the Dirac matrix α_0 in these products, for instance as $(\Psi^+)^A((T^A)^{-1})^+\alpha_0(T^A)^{-1}\Psi^A$ allows to recover the initial product because of the identity [16]

$$T^{+}=\alpha_0 T^{-1}\alpha_0 \quad (10)$$

making now the corresponding Lagrangian invariant by Lorentz transformations. This allows to dermine a new quantum relativistic equation for two particles, the Durt and Pelcé which can be written as [7],

$$((\mathbf{1}^A P_0^A-\overrightarrow{\alpha^A}.\overrightarrow{P^A}-m_A c\alpha_0^A)\otimes\alpha_0^B$$
$$+\alpha_0^A\otimes(\mathbf{1}^B P_0^B-\overrightarrow{\alpha^B}.\overrightarrow{P^B}-m_B c\alpha_0^B))\Psi_{AB}=0 \quad (11)$$

where the unity matrices of eqn.(9) are replaced by the Dirac matrix α_0.

The eqn.(11) has two important properties : It is invariant under Lorentz transformations and gives back the Schrödinger equation for two particles in the non relativistic limit. However it has the significant defect to conserve a current whose density is not definite positive, as the Klein-Gordon equation. Indeed, one can determine from eqn.(11) the conservation of the current

$$\rho_{AB}=\Psi^{+}_{AB}(\alpha_0^A\otimes 1^B+1^A\otimes\alpha_0^B)\Psi_{AB}$$
$$\vec{j}_A=\Psi^{+}_{AB}\vec{\alpha}^A\otimes\alpha_0^B\Psi_{AB},\quad \vec{j}_B=\Psi^{+}_{AB}\alpha_0^A\otimes\vec{\alpha}^B\Psi_{AB} \quad (12)$$

whose first expression is formed with square terms, some positive and other negative, because of the presence of the Dirac matrix α_0.

It has not been found up to now any quantum relativistic equation for two particles having these three properties, as it is for the Dirac equation. However we will use the Durt and Pelcé equation, checking in the solutions that we determine, that the density remains positive.

Note that, if we compare eqns.(6) and (7) with eqn.(12), we build density and current in the same way, but for the Durt and Pelcé equation, by distinguishing the quantities relating to each particle, in multiplying with a tensorial product the corresponding Dirac matrix by the matrix α_0, on the right or on the left. We can thus anticipate from eqn.(8), the form that will take the spin density in the Durt and Pelcé equation. One has to replace in the Dirac expressions, the Dirac matrices by the same matrices multiplied with tensorial product with the matrix α_0, on the right or on the left , for the particle A or the particle B. Thus, we obtain the spatial components of the spin components for the particles A and B,

$$\sigma^A_x = i\Psi^+_{AB}\,\alpha^A_2 \otimes \alpha^B_0 \alpha^A_3 \otimes \alpha^B_0 \Psi_{AB}\ ,\ \sigma^B_x = i\Psi^+_{AB}\,\alpha^A_0 \otimes \alpha^B_2 \alpha^A_0 \otimes \alpha^B_3 \Psi_{AB}$$

$$\sigma^A_y = i\Psi^+_{AB}\,\alpha^A_3 \otimes \alpha^B_0 \alpha^A_1 \otimes \alpha^B_0 \Psi_{AB}\ ,\ \sigma^B_y = i\Psi^+_{AB}\,\alpha^A_0 \otimes \alpha^B_3 \alpha^A_0 \otimes \alpha^B_1 \Psi_{AB}$$

$$\sigma^A_z = i\Psi^+_{AB}\,\alpha^A_1 \otimes \alpha^B_0 \alpha^A_2 \otimes \alpha^B_0 \Psi_{AB}\ ,\ \sigma^B_z = i\Psi^+_{AB}\,\alpha^A_0 \otimes \alpha^B_1 \alpha^A_0 \otimes \alpha^B_2 \Psi_{AB} \qquad (13)$$

The connection with L.De Broglie fusion theory [8] is that the second group of equations on the form III of the fusion theory is the same as the coalesced Durt and Pelcé equation, i.e. the equation obtained when the particles are at the same spatio- temporal coordinates in the configuration space. Because of Pauli principle, the two particles have to be different.

4 From compound boson to spin correlation

One looks for a solution of the Durt and Pelcé equation, whose time dependence is proportional to $\exp\text{-}i(E+m_A+m_B)\frac{t}{2}$, where 2t is the common time of the two particles in the laboratory frame considered here, and E the kinetic energy of the two particles. Notice that if in general the equation has two times, in order to solve the equation, we can choose the two times equal in the laboratory frame. For the compound boson, $x_A=x_B=x$, and we look for a solution of these 16 equations under the form of a plane wave,

$$\Psi_{11}=A\exp ip_x x\ ,\ \Psi_{12}=B\exp ip_x x\ ,\ \Psi_{21}=C\exp ip_x x\ ,\ \Psi_{22}=D\exp ip_x x \qquad (14)$$

From eqns.(DP.1-2) and (DP.5-6) of the appendix A, and the other components obtained from eqn.(A.2) of the appendix A, one determines

$$\Psi_{13}=\frac{E}{2p_x}\Psi_{12}\ ,\Psi_{14}=\frac{E}{2p_x}\Psi_{11},\Psi_{23}=\frac{E}{2p_x}\Psi_{22},\Psi_{24}=\frac{E}{2p_x}\Psi_{21}\quad(15)$$

They are compatible with the Durt and Pelcé equations if the total energy $W=E+m_A+m_B$ satisfies

$$W^2-4p_x^2=(m_A+m_B)^2\quad(16)$$

where one recognizes the relativistic dispersion relation for the energy of a particle of mass m_A+m_B and momentum $2p_x$. We determine the components of the currents from eqns.(12),

$$\begin{aligned}j_{0A}&=j_{0B}=(1-\alpha^2)(|A|^2+|B|^2+|C|^2+|D|^2)\\ j_{1A}&=j_{1B}=2(1-\alpha)\frac{E}{2p_x}(|A|^2+|B|^2+|C|^2+|D|^2)\quad(17)\end{aligned}$$

whose first the densities, are in this particular definite positive. The velocity of the compound particle is

$$u=\frac{j_{1A}+j_{1B}}{j_{0A}+j_{0B}}=\frac{2p_x}{W}\quad(18)$$

the same expression as the velocity of a free relativistic Dirac particle of momentum $2p_x$ [15]. If one does the same calculations with the Bohm and Hiley equation, the masses of particles A and B have to be equal in order to satisfy the equality of the currents of the two particles. Notice that this condition of same masses was taken at the beginning in the fusion theory of L.De Broglie [9,11].

Consider now, on the figure, as in the Bohm version of the EPR experiment, two particles of spin $\frac{1}{2}$ which move away from a common source along the Ox axis, with no potential interaction between them. One can measure the spin of each particle with the apparatus A and B

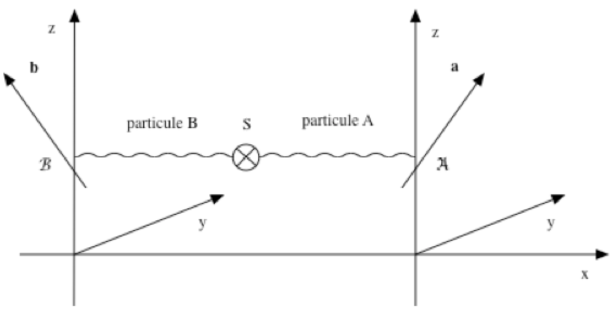

Figure : Simultaneous emission of two particles of spin $\frac{1}{2}$ moving along the Ox axis in opposite directions. The polarizer orientations $\vec{a}$ and $\vec{b}$ are shown on the two reception screens.

For this relative motion, as the source has no momentum, the particle momenta are opposite and of same magnitude p_x. By using the appendix A, the Durt and Pelcé equations become 16 differential equations of first order in the relative spatial coordinate $u=x_A-x_B$. We look for a solution of these 16 equations under the form of a plane wave,

$$\Psi_{11}=A\exp ip_x u\ ,\ \Psi_{12}=B\exp ip_x u\ ,\ \Psi_{21}=C\exp ip_x u\ ,\ \Psi_{22}=D\exp ip_x u \quad (19)$$

From eqns.(DP.1-2) and (DP.5-6) of the appendix A, and the other components obtained from eqn.(A.3) of the appendix A, one determines

$$\Psi_{13}=-\frac{E}{2p_x}\Psi_{12}\,,\Psi_{14}=-\frac{E}{2p_x}\Psi_{11},\Psi_{23}=-\frac{E}{2p_x}\Psi_{22},\Psi_{24}=-\frac{E}{2p_x}\Psi_{21} \quad (20)$$

They are compatible with the Durt and Pelcé equations if the total energy $W=E+m_A+m_B$ satisfies the dispersion relation for the energy (16).

Thus the Durt and Pelcé equation restores to a usual form, the relation obtained when we solved the Bohm and Hiley equation [17], but very unusual, probably due to the non invariance of this equation under Lorentz transformations. With the help of the general relation for the currents exposed before (12), we determine the components of the current associated to the plane wave,

$$\begin{aligned}&\rho_A=\rho_B=(1-\alpha^2)(|A|^2+|B|^2+|C|^2+|D|^2)\\&j_{1A}=-j_{1B}=2(1-\alpha)\frac{E}{2p_x}(|A|^2+|B|^2+|C|^2+|D|^2) \quad (21)\end{aligned}$$

whose densities are also here definite positive. One can deduce the velocities of the two particles as

$$u_A=-u_B=\frac{j_{1A}}{\rho_A}=\frac{2}{(1+\alpha)}\frac{E}{2p_x}=\frac{2p_x}{W} \quad (22)$$

the same expression as the velocity of a free relativistic Dirac particle of momentum $2p_x$ [15]. Notice the following paradox, the two particles have different masses, but same momenta, thus in classical mechanics they would have different velocities. In relation (22) they are opposite and of same magnitude. This is not the case with the Bohm and Hiley equation [17].

For the spin correlation, let us recall first the result obtained in non relativistic quantum mechanics for the spin correlation measured by the apparatus A and B. The quantum state of the two particles is an entangled state, the singlet state of total spin 0,

$$\Psi_0 = \frac{1}{\sqrt{2}}(|+,-\rangle - |-,+\rangle) \quad (23)$$

so that one easily determines the spin measurements correlation in this state,

$$\left\langle \sigma_A.\vec{a}\,\sigma_B.\vec{b} \right\rangle = -\vec{a}.\vec{b} \quad (24)$$

This expression became famous, with the Bell hypothesis of hidden variables [18], since it was always verified by experiments.

In the relativistic range, by using eqn.(13) and the Dirac matrices (5), one determines the spin correlation from the spin density of the Durt and Pelcé equation, applying first the spin operator for the particle B, followed by the spin operator of the particle A. Then, by using the components of the plane wave of the relative motion, obtained by eqns.(19-21) and (A.3), one obtains the three spatial components of the spin correlation (25),

$$\begin{aligned}
&\Psi^+_{AB} S_{Ax} S_{Bx} \Psi_{AB} = -\Psi^+_{AB}\, \alpha^A_2 \otimes \alpha^B_0 \alpha^A_3 \otimes \alpha^B_0 \alpha^A_0 \otimes \alpha^B_2 \alpha^A_0 \otimes \alpha^B_3 \Psi_{AB} \\
&= ((1+\alpha^2) + 2(\frac{E}{2p_x})^2)(AD^* + A^*D + BC^* + B^*C) \\
&\Psi^+_{AB} S_{Ay} S_{By} \Psi_{AB} = -\Psi^+_{AB}\, \alpha^A_3 \otimes \alpha^B_0 \alpha^A_1 \otimes \alpha^B_0 \alpha^A_0 \otimes \alpha^B_3 \alpha^A_0 \otimes \alpha^B_1 \Psi_{AB} \qquad (25) \\
&= ((1+\alpha^2) - 2(\frac{E}{2p_x})^2)(-AD^* - A^*D + BC^* + B^*C) \\
&\Psi^+_{AB} S_{Az} S_{Bz} \Psi_{AB} = -\Psi^+_{AB}\, \alpha^A_1 \otimes \alpha^B_0 \alpha^A_2 \otimes \alpha^B_0 \alpha^A_0 \otimes \alpha^B_1 \alpha^A_0 \otimes \alpha^B_2 \Psi_{AB} \\
&= ((1+\alpha^2) - 2(\frac{E}{2p_x})^2)(AA^* - BB^* - CC^* + DD^*)
\end{aligned}$$

If we want to get closer to the non relativistic singlet state, one has to take A=D=0, B=-C=λ, for the components Ψ_{12} and Ψ_{21}. so that, after determination of λ by normalization of the wave function of components $\Psi_{12},\Psi_{21},\Psi_{13},\Psi_{24},\Psi_{42},\Psi_{31},\Psi_{43},\Psi_{34}$, one obtains from eqn.(25) and the spin matrices of the appendix B, by using eqn.(22),

$$\Psi^{+}_{AB}\overrightarrow{S_A}\,\overrightarrow{S_B}\Psi_{AB}=-\frac{(1-\frac{1}{2}\frac{(1+\alpha)^2}{(1+\alpha^2)}u_A^2)}{(1+\frac{1}{2}\frac{(1+\alpha)^2}{(1+\alpha^2)}u_A^2)}(a_yb_y+a_zb_z)-a_xb_x \quad (26)$$

which gives back (24) in the non relativistic limit. The motion along Ox manifests itself relativistically, since the components of vectors $\vec{a}$ and $\vec{b}$ along Ox are particularized. There is an analogous symmetry breaking with the Bohm and Hiley equation (unpublished).

In the Lamehi-Richti and Mittig experiment [19], a beam of protons of 10Mev, delivered by the Saclay tandem accelerator, hits an hydrogen target of polyethylene, releasing two protons in different directions. In the analyzers, the protons are scattered by a carbon foil and the coïncidences between the detectors of one analyser and the detectors of the other are counted. This experiment was realized to test Bell inequalities resulting from the hypothesis of hidden local variables. The results are at 3% in agreement with the spin correlation (24) of non relativistic quantum mechanics. Let us evaluate the order of magnitude of the relativistic correction (26). The kinetic energy of the beam is

$$E_c=m_pc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right) \quad (27)$$

Taking $m_p c^2 = 940\text{Mev}$, $E_c = 10\text{Mev}$, one deduces from eqn.(27), $\frac{v}{c} = 0{,}15$.

Taking α of the order 0,01, the factor of $(a_y b_y + a_z b_z)$ in eqn.(26) is of the order of 0,98, compatible with the error of 3% of the Lamehi.Richti and Mittig experiment [19].

5 Conclusion

The Schrödinger equation for many particles is now well established, and provides a solid basis for quantum mechanics of many particles in the non relativistic range. This is not the case in the relativistic range. One has the essential Dirac equation for one particle with three essential properties : It conserves a current with a definite positive density, it is invariant under Lorentz transformations, it tends to the Schrödinger equation in the non relativistic limit. The case of two particles is still in progress. It appears that, for the problem of invariance under Lorentz transformation, one time has to be affected separately to each particle sothat, which is conceptually new, a quantum relativistic equation for two particles contains two times. One considers first the adapted Bohm and Hiley equation for two particles which has only two of the required properties, it conserves a current with a definite positive density and tends to the Schrödinger equation in the non relativistic limit. But it is not invariant under Lorentz transformations. The Durt and Pelcé equation, adapted from the two times Bohm and Hiley equation in replacing unity matrices by the Dirac matrices α_0, makes the previous equation invariant under Lorentz transformations, but the density of the conserved current is no more definite positive. Thus the real Dirac equation for two particles with the three required properties mentionned above is still unknown.

However, the Durt and Pelcé equation has interesting simple solutions like the compound boson and the relative motion, which present currents with a definite positive density. The more interesting one is the relative motion of two particles emitted in opposite directions from a single source along the same Ox axis, which can be measured by two distant polarizers, as in the Bohm version of the EPR experiment. With the 16 components of the corresponding wave function, we can compute the spin correlation measured with apparatus A and B, at different angles with the Oz axis, along the vectors $\vec{a}$ and $\vec{b}$. Of course, in the non relativistic limit, one finds again the well known scalar product of the vectors $\vec{a}$ and $\vec{b}$, which became famous with the Bell inequalities. More unexpected is the symmetry breaking of this formula obtained in the relativistic range with the Durt and Pelcé equation, the y and z part of the scalar product being multiplied by a factor less than one, decreasing when the particles velocity increases. It is suggested that the old experiment of Lamehi-Richti and Mittig [19] will be done another time to test this prediction.

Acknowledgment : I thank Th.Durt for valuable comments on the manuscript.

Références

[1] Schrödinger E. Mémoires sur la mécanique ondulatoire, Librairie Félix Alcan Paris (1933). Réimpression autorisée par les Presses Universitaires de France, Editions Jacques Gabay (1988).

[2] Klein O. Zeitschrift für Physik 37, 895 (1926), Gordon W. Zeitschrift für Physik 40, 117 (1926).

[3] De Broglie L. La mécanique ondulatoire et la structure atomique de la matière et du rayonnement, Journal de Physique, série IV, t.VIII, n°5, 1927, 225-241, (1927).

[4] Dirac P.A.M. The principles of quantum mechanics (1930), fourth edition (Oxford University Press) (1958).

[5] Bérestetski V., Lifchitz E. et Pitayevski L. Théorie Quantique Relativiste première partie, tome IV du cours de Physique Théorique de Landau et Lifchitz (Editions Mir Moscou) (1972).

[6] Pelcé P. Sur le problème de l' invariance par transformations de Lorentz de l' équation de Bohm et Hiley, Annales de la Fondation L.De Broglie 45, 71-85 (2020).

[7] Durt Th. and Pelcé P. Lorentz-covariant two-particles quantum relativistic equation in a Lagrangian approach, Annales de la Fondation L.De Broglie 45 163-181 (2020).

[8] Durt Th. and Pelcé P. On connections between L.De Broglie fusion theory and quantum relativistic two-body equations, Annales de la fondation L.De Broglie 47(1), 57-80 (2022).

[9] De Broglie L. Une nouvelle théorie de la lumière, Hermann et Cie Paris (1940).

[10] Géhéniau J. Mécanique ondulatoire de l'électron et du photon, La chimie mathématique, Vol.III, Gauthier-Villars et Cie (1938).

[11] De Broglie L. Théorie générale des particules à spin (méthode de fusion) Gauthier-Villars (1943).

[12] Lochak,G. A theory of light with 4 different photons : Electric and magnetic with spin 1 and 0, Annales de la Fondation L.De Broglie 35, 1-21 (2010).

[13] Bohm D. and Hiley B.J. The undivided universe, Routledge (1993).

[14] Wentzel G. Quantum theory of fields, Dover (1949).

[15] De Broglie L. L' électron magnétique, Hermann et Cie, Paris (1931).

[16] Messiah A. Mécanique quantique Tome 2, Dunod (1964).

[17] Pelcé P. Désintégration d 'une particule de spin 0 en deux ondes relativistes de spin $\frac{1}{2}$. Annales de la fondation Louis de Broglie 43(2),177-193 (2019).

[18] Bell J.S. Speakable and unspeakable in quantum mechanics, Cambridge University Press (1987).

[19] Lamehi-Richti M. and Mittig W. Quantum mechanics and hidden variables : A test of Bell's inequality by the measurement of the spin correlation in low energy proton-proton scattering, Physical Review D14, 2543-2555 (1976), in Quantum theory and measurement, edited by J.A.W. Wheeler and W.H. Zurek, 422-434, Princeton University Press (1983).

Appendix A : The 16 equations developed along the Ox axis of the Durt and Pelcé equation ($\hbar$ =c=1).

$$(P_{0A}+P_{0B}-(m_A+m_B))\Psi_{11}-P_{1A}\Psi_{41}-P_{1B}\Psi_{14}=0 \quad \text{(DP.1)}$$

$$(P_{0A}+P_{0B}-(m_A+m_B))\Psi_{12}-P_{1A}\Psi_{42}+P_{1B}\Psi_{13}=0 \quad \text{(DP.2)}$$

$$(-P_{0A}+P_{0B}+(m_A+m_B))\Psi_{13}+P_{1A}\Psi_{43}-P_{1B}\Psi_{12}=0 \quad \text{(DP.3)}$$

$$(-P_{0A}+P_{0B}+(m_A+m_B))\Psi_{14}+P_{1A}\Psi_{44}-P_{1B}\Psi_{11}=0 \quad \text{(DP.4)}$$

$$(P_{0A}+P_{0B}-(m_A+m_B))\Psi_{21}-P_{1A}\Psi_{31}-P_{1B}\Psi_{24}=0 \quad \text{(DP.5)}$$

$$(P_{0A}+P_{0B}-(m_A+m_B))\Psi_{22}-P_{1A}\Psi_{32}+P_{1B}\Psi_{23}=0 \quad \text{(DP.6)}$$

$$(-P_{0A}+P_{0B}+(m_A+m_B))\Psi_{23}+P_{1A}\Psi_{33}-P_{1B}\Psi_{22}=0 \quad \text{(DP.7)}$$

$$(-P_{0A}+P_{0B}+(m_A+m_B))\Psi_{24}+P_{1A}\Psi_{34}+P_{1B}\Psi_{21}=0 \quad \text{(DP.8)}$$

$$(P_{0A}-P_{0B}+(m_A+m_B))\Psi_{31}-P_{1A}\Psi_{21}+P_{1B}\Psi_{34}=0 \quad \text{(DP.9)}$$

$$(P_{0A}-P_{0B}+(m_A+m_B))\Psi_{32}-P_{1A}\Psi_{22}+P_{1B}\Psi_{33}=0 \quad \text{(DP.10)}$$

$$(-P_{0A}-P_{0B}-(m_A+m_B))\Psi_{33}+P_{1A}\Psi_{23}+P_{1B}\Psi_{32}=0 \quad \text{(DP.11)}$$

$$(-P_{0A}-P_{0B}-(m_A+m_B))\Psi_{34}+P_{1A}\Psi_{24}+P_{1B}\Psi_{31}=0 \quad \text{(DP.12)}$$

$$(P_{0A}-P_{0B}+(m_A+m_B))\Psi_{41}-P_{1A}\Psi_{11}+P_{1B}\Psi_{44}=0 \quad \text{(DP.13)}$$

$$(P_{0A}-P_{0B}+(m_A+m_B))\Psi_{42}-P_{1A}\Psi_{12}+P_{1B}\Psi_{43}=0 \quad \text{(DP.14)}$$

$$(-P_{0A}-P_{0B}-(m_A+m_B))\Psi_{43}+P_{1A}\Psi_{13}+P_{1B}\Psi_{42}=0 \quad \text{(DP.15)}$$

$$(-P_{0A}-P_{0B}-(m_A+m_B))\Psi_{44}+P_{1A}\Psi_{14}+P_{1B}\Psi_{41}=0 \quad \text{(DP.16)}$$

Summing all the plane wave equations 2 by 2, (1-16; 2-15; 3-14; 4-13; 5-12; 6-11; 7-10; 8-9), one obtains respectively for the compound boson and the relative motion

$$\Psi_{44}=\alpha\Psi_{11},\quad \Psi_{43}=\alpha\Psi_{12}$$
$$\Psi_{42}=\Psi_{13},\quad \Psi_{41}=\Psi_{14} \qquad (A.2)$$
$$\Psi_{34}=\alpha\Psi_{21},\quad \Psi_{33}=\alpha\Psi_{22}$$
$$\Psi_{32}=\Psi_{23},\quad \Psi_{31}=\Psi_{24}$$

and

$$\Psi_{44}=-\alpha\Psi_{11},\quad \Psi_{43}=-\alpha\Psi_{12}$$
$$\Psi_{42}=-\Psi_{13},\quad \Psi_{41}=-\Psi_{14} \qquad (A.3)$$
$$\Psi_{34}=-\alpha\Psi_{21},\quad \Psi_{33}=-\alpha\Psi_{22}$$
$$\Psi_{32}=-\Psi_{23},\quad \Psi_{31}=-\Psi_{24}$$

where

$$\alpha=\frac{E}{E+2(m_A+m_B)} \qquad (A.4)$$

Appendix B : Spin matrices.

$$\alpha^{A}_{2} \otimes \alpha^{B}_{0} \alpha^{A}_{3} \otimes \alpha^{B}_{0} \qquad \alpha^{A}_{0} \otimes \alpha^{B}_{2} \alpha^{A}_{0} \otimes \alpha^{B}_{3}$$

$$\begin{pmatrix}
0&0&0&0&i&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&i&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&i&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&i&0&0&0&0&0&0&0&0\\
i&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&i&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&i&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&i&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&i&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&i&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&i&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&i\\
0&0&0&0&0&0&0&0&i&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&i&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&i&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&i&0&0&0&0
\end{pmatrix}
\begin{pmatrix}
0&i&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
i&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&i&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&i&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&i&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&i&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&i&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&i&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&i&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&i&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&i&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&i&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&i&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&i&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&i\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&i&0
\end{pmatrix}$$

$$\alpha^{A}_{3} \otimes \alpha^{B}_{0} \alpha^{A}_{1} \otimes \alpha^{B}_{0} \qquad \alpha^{A}_{0} \otimes \alpha^{B}_{3} \alpha^{A}_{0} \otimes \alpha^{B}_{1}$$

$$\begin{pmatrix}
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0
\end{pmatrix}
\begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0
\end{pmatrix}$$

$$\alpha_1^A \otimes \alpha_0^B \alpha_2^A \otimes \alpha_0^B \qquad\qquad \alpha_0^A \otimes \alpha_1^B \alpha_0^A \otimes \alpha_2^B$$

$$\begin{pmatrix}
i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i
\end{pmatrix}
\qquad
\begin{pmatrix}
i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -i
\end{pmatrix}$$