On the wave-particle duality of a Hertzian dipole

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RÉSUMÉ. Un dipôle hertzien est une petite antenne qui émet des ondes radio. Dans cet article, l'interaction d'un dipôle hertzien avec le champ réfléchi par l'environnement est analysée. L'étude montre qu'un seul dipôle hertzien peut se comporter comme un objet quantique : il peut notamment interférer avec lui-même au niveau d'une double fente, en raison d'une force pondéromotrice. Par conséquent, il ne peut pas atteindre tous les endroits derrière une double fente avec la même probabilité, et la distribution de probabilité pour l'emplacement d'un impact sur un écran derrière la double fente peut montrer un schéma d'interférence clair. Cet effet disparaît si l'une des fentes est fermée, si deux polariseurs orthogonaux sont placés devant les fentes ou si le système du dipôle hertzien, de la double fente et de l'onde stationnaire est perturbé par des influences extérieures. L'interférence des particules et l'effondrement de la fonction d'onde sont considérés comme des effets qui ne peuvent être expliqués par la physique classique. Cependant, les dipôles hertziens peuvent être des objets tangibles se déplaçant sur des trajectoires classiques, et leur position et leur vitesse peuvent être mesurées simultanément à tout moment. Ainsi, le dipôle hertzien fournit un exemple de la manière dont les caractéristiques centrales de la mécanique quantique peuvent être expliquées à l'aide de la physique classique.

ABSTRACT. A Hertzian dipole is a small antenna that emits radio waves. In this article, the interaction of a Hertzian dipole with the field reflected from the environment is analyzed. The study shows that a single Hertzian dipole can behave like a quantum object: in particular, it can interfere with itself at a double slit, owing to a ponderomotive force. Therefore, it cannot reach every location behind a double slit with the same probability, and the probability distribution for the location of an impact on a screen behind the double slit may show a clear interference pattern. This effect disappears if one of the slits is closed, two orthogonal polarizers are placed in front of the slits, or if the system of the Hertzian dipole, double slit and standing wave is disturbed by external influences. Both particle interference and the collapse of the wave function are *considered to be effects that cannot be explained by classical physics. However, Hertzian dipoles can be tangible objects moving on classical trajectories, and their location and velocity can be measured simultaneously at any time. Thus, the Hertzian dipole provides an example of how central features of quantum mechanics can be explained by means of classical physics.*

P.A.C.S.: 52.35.Mw, 84.40.Ba, 03.75.-b

1 Introduction

The term *wave-particle duality* indicates that objects in quantum physics simultaneously behave as waves and point particles. Both, wave and particle are concepts of everyday life and accordingly intelligible.

In the most general sense, a wave is a disturbance that propagates on or in a medium, and is caused by an anomaly at a specific location. Waves, by nature, are non-local phenomena and must be described by fields. If several waves are present, they can superimpose, thereby attenuating one another in some locations and amplifying one another at other locations.

A classical point particle, in contrast, is always located at only one defined location at any given time. Moreover, it cannot suddenly be found in a completely different place a short time later. Therefore, a point particle can be described with much less information than a wave, because only a continuous, time-dependent function – the trajectory – is necessary.

Quantum objects, such as electrons or photons, appear to be neither particles nor waves. When they hit a detector, they impact at a small point, just like point particles would. But they do not seem to move in the same way. The motion of classical objects, e.g., comets or billiard balls, can be described by trajectories, which in turn can be calculated precisely and deterministically with Newtonian mechanics, provided that all necessary parameters are known with sufficient accuracy. The vast majority of researchers believe that quantum objects cannot move along classical trajectories because the place of impact of a quantum object on a detector has to be described with a probability distribution.

Remarkably, this probability distribution can interfere with itself. Interference, however, is a property of waves. How is this possible? Classical point particles, if launched at exactly the same location with exactly the same velocity, and subjected to exactly the same force fields, will always hit a detector at the same location[1]. If all parameters are not known with infinite precision,

[1] Note that sufficiently complex classical systems can behave like pseudo-random number generators, i.e. a slight change in the seed value can lead to a completely different result.

probability theory can be used to predict where a classical point particle is more or less likely to arrive. However, such classical probability distributions do not have wave properties and do not show interference.

To physicists in the 1920s, the behavior of quantum objects therefore seemed very strange. Later generations did not succeed in explaining the wave-particle duality by using classical physics; therefore, most scientists currently are firmly convinced that such an explanation will never be possible. Some physicists have even warned against trying to find an explanation at all. For example, T. Hey and P. Walters – inspired by a famous statement from R. P. Feynman – write [1]:

> To avoid running into a frustrating psychological cul-de-sac, try to be content with the mere acceptance of the observed experimental facts. Try not to ask the question, "but how can it be like that?"

The prevailing interpretation of wave-particle duality is based on an assumption that quantum objects cannot have classical trajectories and that their motion must be described by a field of imaginary numbers, which can be calculated with the Schrödinger equation and is called the wave function, $\psi(\boldsymbol{r}, t)$. The probability of finding a quantum object at a given location $\boldsymbol{r}$ at a given time t corresponds to the square of the absolute value of this wave function, $|\psi(\boldsymbol{r}, t)|^2$, normalized to 1. According to the above quotation, the question of which mechanism causes the wave function is not recommended to be investigated.

Nevertheless, some scientists find Feynman's worldview too pragmatic and continue to raise this question. Interestingly, classical systems exist in which non-classical probabilities play a role [2]. However, the most remarkable model is that of Couder and Fort [3]. In this model, small droplets of oil are deposited onto the surface of a vibrating oil bath. When such an oil droplet touches the surface, a wave is created, which in turn interacts with the oil droplet. The motion and properties of such an oil bath/oil droplet system can be observed, thus providing an idea of how quantum mechanics might work. Some scientists have undertaken intensive research to further develop the corresponding theory, pilot-wave hydrodynamics [4–6]. They are also trying to determine how pilot-wave hydrodynamics relates to the de Broglie-Bohm theory [7–11], a deterministic interpretation of the Schrödinger equation [12–14].

In this article, another example of a classical model system is presented, which has even stronger similarities to quantum mechanics than that of Couder and Fort. The model discussed herein is the Hertzian dipole, a point-like electromagnetic elementary antenna, which is of particular interest in electrical engineering, because it is an important solution of Maxwell's equations.

The Hertzian dipole is a very simple but powerful model that predates quantum mechanics. It consists of a negative and a positive electric point charge of equal quantity, which are together located at the same place within a very small volume of space. From the outside, the object appears electrically neutral. However, the two charges can begin to oscillate with respect to each other, thereby producing a cyclically reversing dipole moment. Consequently, the electromagnetic force oscillates in the vicinity of the Hertzian dipole and propagates at the speed of light in the form of a wave into the surrounding space.

The importance of the Hertzian dipole in electrical engineering is that its field can be integrated along current paths, thus providing a way to calculate the radiated electromagnetic fields of arbitrary antenna configurations. However, a Hertzian dipole serves not only an abstract model in electrical engineering but also an excellent approximation for electromagnetic transmitters of arbitrary size and shape. For example, many objects, such as satellites, behave exactly as Hertzian dipoles from a distance; the same is also true for small dipole antennas or electronic devices that emit radio waves.

However, atoms have the greatest similarity to Hertzian dipoles, because they are small and almost point-like, and they also consist of two charge quantities that neutralize each other toward the outside. Moreover, the atomic nucleus and the electron shell of an atom can oscillate with respect to each other. Interestingly, a single Hertzian dipole from the perspective of ordinary classical electrodynamics must behave as a quantum object and notably can interfere with itself if other structures of Hertzian dipoles (or atoms) are present in the vicinity. These aspects are discussed and mathematically supported herein.

2 The Hertzian dipole

As described in the preceding section, a Hertzian dipole consists of two point charges, $+q_1$ and $-q_1$, which are always very close to each other because of the influence of internal forces. However, the Hertzian dipole as a whole can move, and this motion can be expressed with a classical trajectory, $\boldsymbol{r}_1(t)$, i.e., a three-dimensional vector describing the location $\boldsymbol{r}_1$ of the Hertzian dipole at time t.

The electromagnetic forces of two perfectly superposed electric point charges $+q_1$ and $-q_1$ obviously completely neutralize each other. However, in a Hertzian dipole, the two charges are moving slightly with respect to each other. Therefore, a time-dependent electric dipole moment is formed. The spatial separation of the point charges from the center of gravity $\boldsymbol{r}_1(t)$ of the Hertzian dipole can be described by the displacement vector $\boldsymbol{s}_1(t)$. The meaning of this displacement vector is that $\boldsymbol{r}_1(t)+\boldsymbol{s}_1(t)$ is the location of the positive point charge, whereas $\boldsymbol{r}_1(t)-\boldsymbol{s}_1(t)$ is the location of the negative point charge.

An oscillating Hertzian dipole radiates a classical electromagnetic wave. A classical electromagnetic wave differs from the interpretation of modern physics in that it is not quantized but is a continuous field in which the electromagnetic force is a continuous function without singularities except for the locations of the point charges. In other words, a classical electromagnetic wave has only wave properties and can be obtained by solving Maxwell's equations.

The field of a Hertzian dipole, which rests at the coordinate origin and oscillates sinusoidally there, has long been known. The corresponding solution of Maxwell's equations can be found in numerous textbooks [15, 16] but is usually divided into the magnetic and the electric component. However, one can also express the formula directly in terms of a force equation. Also the restriction to sinusoidal oscillations is not necessary. Ultimately, for a Hertzian dipole resting at location $\boldsymbol{r}_1$ and a test charge q_2 resting at location $\boldsymbol{r}_2$, the formula of the electromagnetic force $\boldsymbol{F}_p$ on the test charge is [17]

$$\boldsymbol{F}_p(\boldsymbol{r}_2,\boldsymbol{r}_1,t,q_2,q_1) = \frac{\mu_0\, q_2\, q_1}{2\,\pi\, r_{21}} \left(\frac{\boldsymbol{r}_{21}}{r_{21}} \times \left(\frac{\boldsymbol{r}_{21}}{r_{21}} \times \ddot{\boldsymbol{s}}_1\left(t-\frac{r_{21}}{c}\right)\right)\right), \tag{1}$$

if $\|\boldsymbol{r}_2-\boldsymbol{r}_1\|$ is assumed to be much greater than $\|\boldsymbol{s}(t)\|$. In this case, the far-field approximation is sufficient, and, according to practical experience, is a very good approximation at a distance of more than a few wavelengths. To shorten the notation, the distance vector $\boldsymbol{r}_{21} := \boldsymbol{r}_2-\boldsymbol{r}_1$ is defined. r_{21} is the magnitude of the vector $\boldsymbol{r}_{21}$, and $\ddot{\boldsymbol{s}}_1(t)$ is the second derivative of the displacement vector with respect to time.

Notably, fairly compact solutions of Maxwell's equations can also be provided for approximately uniformly moving Hertzian dipoles and test charges. These solutions contain additional terms describing mainly the Doppler effect and magnetic components [18]. This article refrains from using these somewhat more complex formulas, because the wave-particle duality of a Hertzian dipole can already be demonstrated by using the simple equation (1). However, future work should be based on the complete solution and, if possible, should even consider the near field, because interesting additional effects are to be expected.

3 Interaction of a Hertzian dipole with the environment

The previous section explained what a Hertzian dipole is and indicated that such an object emits a wave, in which not simply any field strength or potential oscillates, but a force. In this section, the effect of this electromagnetic force of a Hertzian dipole on the environment is analyzed in greater detail. For this purpose, we first consider the force on a distant single test charge q_2. According to equation (1), the equation of motion of the test charge q_2 at location $\boldsymbol{r}_2 + \boldsymbol{s}_2(t)$ is

$$m_2 \, \frac{\partial^2}{\partial t^2} \, (\boldsymbol{r}_2 + \boldsymbol{s}_2(t)) = \boldsymbol{F}_p(\boldsymbol{r}_2 + \boldsymbol{s}_2(t), \boldsymbol{r}_1, t, q_2, q_1). \tag{2}$$

Here, m_2 is the mass of the point charge q_2. $\boldsymbol{s}_2(t)$ denotes a small displacement of the charge q_2 from its time-averaged location $\boldsymbol{r}_2$. For $\boldsymbol{r}_2$ to be a time-independent constant, the time average of the force $\boldsymbol{F}_p$ must be assumed to be zero, and the force must be assumed to be sufficiently spatially homogeneous at location $\boldsymbol{r}_2$. In this case, the equation of motion (2) can be simplified to

$$m_2 \, \ddot{\boldsymbol{s}}_2(t) \approx \boldsymbol{F}_p(\boldsymbol{r}_2, \boldsymbol{r}_1, t, q_2, q_1). \tag{3}$$

Equation (3) shows that the point charge q_2 oscillates because of the emitted wave of the Hertzian dipole. Thus, it itself becomes the source of a secondary electromagnetic wave. If a second point charge of charge quantity $-q_2$ and mass m_2 is imagined to exist at the location $\boldsymbol{r}_2$, the two charges $+q_2$ and $-q_2$ together form a second Hertzian dipole with a total mass of $2\,m_2$. The electromagnetic force $\boldsymbol{F}_s$ of this second Hertzian dipole on a third test charge q_3 at location $\boldsymbol{r}_3$ can be expressed in an equivalent manner to formula (1) and reads with $\boldsymbol{r}_{32} := \boldsymbol{r}_3 - \boldsymbol{r}_2$ as follows:

$$\boldsymbol{F}_s = \frac{\mu_0 \, q_3 \, q_2}{2\,\pi \, r_{32}} \left(\frac{\boldsymbol{r}_{32}}{r_{32}} \times \left(\frac{\boldsymbol{r}_{32}}{r_{32}} \times \ddot{\boldsymbol{s}}_2 \left(t - \frac{r_{32}}{c} \right) \right) \right). \tag{4}$$

In equation (4), the displacement vector $\boldsymbol{s}_2(t)$ of the secondary dipole is needed and can be obtained from the equation of motion (3). Substitution into equation (4) yields

$$\begin{aligned} &\boldsymbol{F}_s(\boldsymbol{r}_3, \boldsymbol{r}_2, \boldsymbol{r}_1, t, q_3, q_2, q_1) = \\ &\frac{\mu_0 \, q_3 \, q_2}{2\,\pi \, m_2 \, r_{32}} \left(\frac{\boldsymbol{r}_{32}}{r_{32}} \times \left(\frac{\boldsymbol{r}_{32}}{r_{32}} \times \boldsymbol{F}_p \left(\boldsymbol{r}_2, \boldsymbol{r}_1, t - \frac{r_{32}}{c}, q_2, q_1 \right) \right) \right). \end{aligned} \tag{5}$$

Using equation (1), we finally obtain

$$\boldsymbol{F}_s(\boldsymbol{r}_3, \boldsymbol{r}_2, \boldsymbol{r}_1, t, q_3, q_2, q_1) = \frac{\mu_0^2\, q_3\, q_1}{4\,\pi^2\, r_{32}\, r_{21}} \cdot \frac{q_2^2}{m_2} \cdot \left(\frac{\boldsymbol{r}_{32}}{r_{32}} \times \left(\frac{\boldsymbol{r}_{32}}{r_{32}} \times \left(\frac{\boldsymbol{r}_{21}}{r_{21}} \times \left(\frac{\boldsymbol{r}_{21}}{r_{21}} \times \ddot{\boldsymbol{s}}_1\left(t - \frac{r_{32} + r_{21}}{c}\right)\right)\right)\right)\right). \tag{6}$$

Equation (6) is very interesting because it shows that an electrically neutral object, such as a Hertzian dipole, is affected by its own field if additional Hertzian dipoles are located in its vicinity. In this case, the force $\boldsymbol{F}_s(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_1, t, \pm q_1, q_2, q_1)$ acts on each of the two charges $+q_1$ and $-q_1$ in the primary dipole.

This is a type of self-interaction, and the oscillation $\boldsymbol{s}_1(t)$ and even the trajectory of a moving Hertzian dipole are obviously affected by the configuration and geometrical layout of the surrounding Hertzian dipoles. In particular, Hertzian dipoles that are moving past each other cannot be expected to move in straight lines.

4 Hertzian dipole in front of a reflective surface

We now want to calculate the field radiated by an infinitely extended reflective plane surface when a Hertzian dipole is in front of it, oscillating sinusoidally at angular frequency $\omega = 2\,\pi\, f$. In this case, in equation (1), we have

$$\boldsymbol{s}_1(t) = s_1\, \boldsymbol{e}_p\, \sin(\omega\, t), \tag{7}$$

where $\boldsymbol{e}_p$ is a unit vector, and s_1 the constant amplitude of $\|\boldsymbol{s}_1(t)\| = s_1$. Without loss of generality, let us further assume that the reflective surface is exactly on the y-z plane, and the Hertzian dipole is located at $\boldsymbol{r}_1 = x_1\, \boldsymbol{e}_x$, with $x_1 < 0$.

We model the reflecting plane surface with infinitely many secondary Hertzian dipoles located at $\boldsymbol{r}_2 = y_2\, \boldsymbol{e}_y + z_2\, \boldsymbol{e}_z$. The total field $\boldsymbol{F}_t$ produced by the reflecting surface at location $\boldsymbol{r}_3 = x_3\, \boldsymbol{e}_x + y_3\, \boldsymbol{e}_y + z_3\, \boldsymbol{e}_z$ at time t can be obtained by integrating over all the Hertzian dipoles that make up the surface. Use of equation (6) gives the following:

$$\boldsymbol{F}_t(\boldsymbol{r}_3, \boldsymbol{r}_1, t, q_3, q_1) = \sigma \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \boldsymbol{F}_s\left(\boldsymbol{r}_3, y_2\, \boldsymbol{e}_y + z_2\, \boldsymbol{e}_z, \boldsymbol{r}_1, t, q_3, q_2, q_1\right)\, \mathrm{d}y_2 \mathrm{d}z_2. \tag{8}$$

The coefficient σ is a material constant with physical units of $1/m^2$, whose quantification is discussed later in this article.

The integrals in equation (8) cannot yet be solved analytically. However, they can be approximated by sums:

$$\boldsymbol{F}_t(\boldsymbol{r}_3, \boldsymbol{r}_1, t, q_3, q_1) \approx \sigma\, d^2 \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} \boldsymbol{F}_s\left(\boldsymbol{r}_3, i\, d\, \boldsymbol{e}_y + j\, d\, \boldsymbol{e}_z, \boldsymbol{r}_1,\ t, q_3, q_2, q_1\right). \tag{9}$$

Herein, d is a parameter with physical units of m. For $d \rightarrow 0$, equation (9) becomes equation (8).

Of note, equation (9) also has a physical meaning, because it describes the field that would be reflected from a layer of atoms with a square lattice with lattice constant d. If d is smaller than half the wavelength of the incident electromagnetic wave, the discretization is scarcely important. This aspect can be verified by testing, because the equation (9) gives almost the same results for arbitrary values of $d < 1/2\, c/f$. This finding is quite interesting, and it indicates that the integral (8) could actually be considered as the approximation of equation (9), because matter is not a continuous substance but often consists of regular lattice structures, wherein each atom with its electron shell represents a Hertzian dipole.

The resulting fields of equation (9) can be visualized for specifically chosen parameters. The field emitted by the Hertzian dipole is found to be mirrored at the y-z plane as expected, and, except for a phase shift, the resulting field corresponds to the field that a Hertzian dipole would produce on the opposite side of the plane surface. The numerical value of the parameter σ must be chosen with respect to the unknown parameters m_2 and q_2, such that the reflected wave produces the same force on a test charge q_3 at a distance R from the reflecting surface as that produced by the primary wave on the test charge q_3 at a distance $2\,R$. This is the case when we choose for σ the relation

$$\sigma \approx \frac{\omega\, m_2}{c\, \mu_0\, q_2^2}. \tag{10}$$

Therefore, the time-consuming summation in equation (9) can be avoided, and the reflected field can be approximated by the model

$$\boldsymbol{F}_t(\boldsymbol{r}_3, \boldsymbol{r}_1, t, q_3, q_1) \approx \boldsymbol{S}_x \cdot \boldsymbol{F}_p\left(\boldsymbol{S}_x \cdot \boldsymbol{r}_3, \boldsymbol{r}_1, t + \frac{\pi}{2\,\omega}, q_3, q_1\right). \tag{11}$$

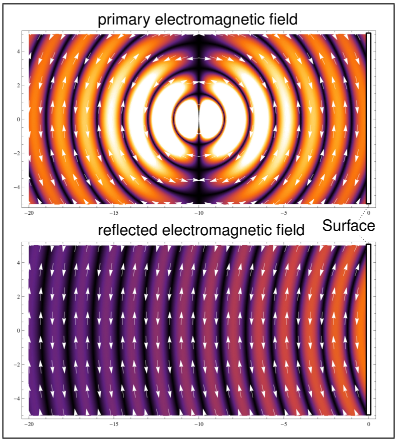

Figure 1: A Hertzian dipole oscillating parallel to a reflecting plane surface. The upper figure shows the radiated field of the Hertzian dipole. The reflected wave is shown below.

However, this approximation is valid only for the region in front of the mirror, i.e., where the sign of x_1 is equal to the sign of x_3. $\boldsymbol{S}_x$ is the mirror matrix

$$\boldsymbol{S}_x := \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{12}$$

Because the function $\boldsymbol{F}_p$ given by equation (1) is a simple expression, function (11) can be quickly visualized and mathematically analyzed, thus simplifying the reasoning that follows herein. Figures 1, 2 and 3 show examples of the reflected wave from a Hertzian dipole for different directions of polarization.

It is worth noting that the calculations can also be performed numerically with the software framework *OpenWME*. *OpenWME* also contains an example in *C*++ including a video on this subject.

In figure 1, the Hertzian dipole is 10 m in front of the reflecting surface, and oscillates parallel to it in the z-direction with a frequency of 100 MHz. The time corresponds to $t = 0$. The primary field emitted by the Hertzian dipole is shown in the upper part of the figure. In the lower part, the corresponding reflected wave is seen. Note that the reflected wave is approximately a plane transverse wave, particularly at the location of the Hertzian dipole.

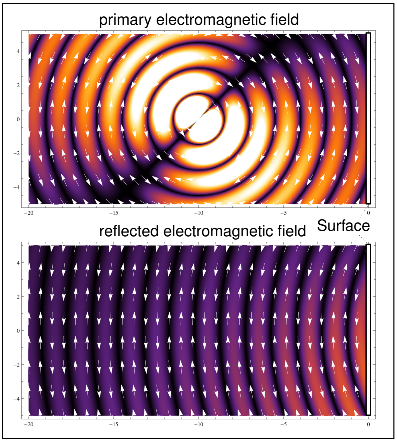

Figure 2: A Hertzian dipole oscillating diagonally to a reflecting plane surface. The upper figure shows the radiated field of the Hertzian dipole. The reflected wave is shown below.

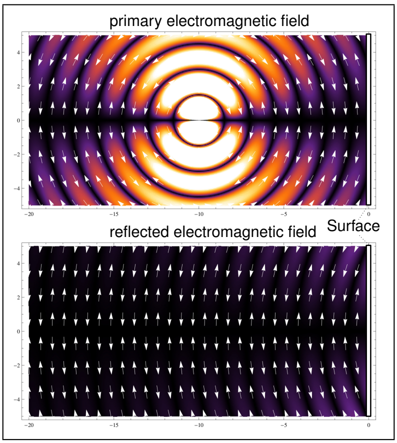

Figure 3: A Hertzian dipole oscillating perpendicular to a reflecting plane surface. The upper figure shows the radiated field of the Hertzian dipole. The reflected wave is shown below.

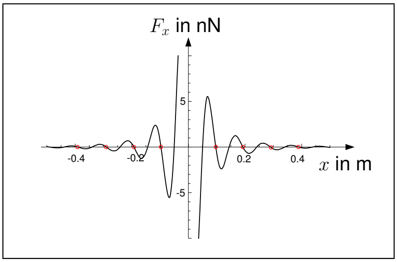

Figure 4: Attractive and repulsive force of a 3GHz Hertzian dipole (effective radiated power approx. 100 W) on itself when the antenna is located in front of an ideally reflecting surface in the y-z plane. The fixed points are marked in red.

The reflected wave obviously influences the Hertzian dipole, and the Hertzian dipole interacts with itself because of the existence of the reflecting surface. Furthermore, a freely moving Hertzian dipole, after a certain time, will clearly always be located where the reflected force is in agreement with the phase of the Hertzian dipole's oscillation. Thus, the Hertzian dipole prefers certain discrete distances to the reflecting plane.

What if the Hertzian dipole does not oscillate parallel to the reflecting surface? To answer this question, figure 2 shows a Hertzian dipole oscillating diagonally to the surface. As can be seen, the reflected wave is almost the same as that generated by a parallel oscillating Hertzian dipole. Therefore, a freely moving, diagonally oscillating Hertzian dipole gradually aligns itself parallel to the reflecting surface and ultimately has the same state as that shown in figure 1.

The only special case in which the reflected wave is not a plane wave is when the Hertzian dipole oscillates exactly perpendicular to the surface. The corresponding fields are shown in figure 3. However, even a small amount of noise is sufficient to destroy this unstable state. Consequently, in this case, the Hertzian dipole will be aligned parallel to the surface after a certain period of time.

In conclusion, freely moving Hertzian dipoles, which are located in front of a reflecting surface, prefer certain distances and directions of oscillation. The preferred direction of oscillation is always *parallel* to the reflecting surface and the preferred distances are *discrete* and depend on the wavelength.

Figure 4 shows a calculation of the force with realistic parameters. As one can see, a force arises that has an attractive or repulsive effect depending on the distance. This effect is of magnetic origin. It arises because, depending on the distance, the charges in the Hertzian dipole and reflector oscillate in phase or in antiphase. However, currents oscillating in phase attract each other, while an oscillation in antiphase causes repulsion. Figure 4 shows furthermore that the force is very small. Consequently, it plays no role in engineering, since transmitters with an effective radiated power (ERP) of 100 W (e.g. a magnetron) have a certain size and mass and the small forces can therefore not cause any noticeable accelerations. Nevertheless, the effect is of great theoretical interest, as it suggests that quantum mechanics might have its origins in classical electrodynamics.

5 Hertzian dipole in front of a double slit

We now extend the previous section to investigate which electromagnetic field $\boldsymbol{F}_d$ is radiated by a reflecting surface with two openings at $z = -L$ and $z = +L$ and $y = 0$. Let us assume that both openings have a length of $2\,l$ in the y-direction, and that the first opening at $z = -L$ has a width of $2\,b_-$, and the second opening at $z = +L$ has a width of $2\,b_+$.

The electromagnetic field emitted by the reflecting surface with the two openings can be calculated with the equation

$$\begin{aligned} \boldsymbol{F}_d(\boldsymbol{r}_3, \boldsymbol{r}_1, t, q_3, q_1) \; = \; & \sigma\, d^2 \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} \boldsymbol{F}_h(i, j) \; - \\ & \sigma\, d^2 \sum_{i=-l/d}^{+l/d} \left(\sum_{j=(-L-b_-)/d}^{(-L+b_-)/d} \boldsymbol{F}_h(i, j) + \sum_{j=(L-b_+)/d}^{(L+b_+)/d} \boldsymbol{F}_h(i, j) \right), \end{aligned} \tag{13}$$

where the auxiliary quantity is defined

$$\boldsymbol{F}_h(i, j) := \boldsymbol{F}_s\left(\boldsymbol{r}_3, i\, d\, \boldsymbol{e}_y + j\, d\, \boldsymbol{e}_z, \boldsymbol{r}_1,\, t, q_3, q_2, q_1\right), \tag{14}$$

to shorten the notation.

The basic idea of equation (13) is to first calculate the field of the entire closed reflecting surface and then to subtract the parts that would have been generated by the slits. The first term in equation (13) is therefore equal to the right side of equation (9). The related field, as shown in the previous section, corresponds to the mirror image of the Hertzian dipole and can be approximately described by equation (11).

The second term in equation (13) is more interesting. It can be approximately evaluated with the approximations

$$d^2 \sum_{i=-l/d}^{+l/d} \sum_{j=(-L-b_-)/d}^{(-L+b_-)/d} \boldsymbol{F}_h(i,j) \approx (2\,l)\,(2\,b_-)\,\boldsymbol{F}_h\left(0, -\frac{L}{d}\right) \tag{15}$$

and

$$d^2 \sum_{i=-l/d}^{+l/d} \sum_{j=(L-b_+)/d}^{(L+b_+)/d} \boldsymbol{F}_h(i,j) \approx (2\,l)\,(2\,b_+)\,\boldsymbol{F}_h\left(0, +\frac{L}{d}\right). \tag{16}$$

For these approximations to be applicable, however, l, b_- and b_+ must be ensured to be sufficiently small.

Using approximations (15) and (16), and equation (11), we then obtain for equation (13) the approximation

$$\begin{aligned} &\boldsymbol{F}_d(\boldsymbol{r}_3, \boldsymbol{r}_1, t, q_3, q_1) \;\approx \\ &\quad \boldsymbol{S}_x \cdot \boldsymbol{F}_p\left(\boldsymbol{S}_x \cdot \boldsymbol{r}_3, \boldsymbol{r}_1, t + \frac{\pi}{2\,\omega}, q_3, q_1\right) - \\ &\quad \sigma\, a_-\, \boldsymbol{F}_s\,(\boldsymbol{r}_3, -L\,\boldsymbol{e}_z, \boldsymbol{r}_1,\; t, q_3, q_2, q_1) - \\ &\quad \sigma\, a_+\, \boldsymbol{F}_s\,(\boldsymbol{r}_3, +L\,\boldsymbol{e}_z, \boldsymbol{r}_1,\; t, q_3, q_2, q_1)\,, \end{aligned} \tag{17}$$

where $a_- := (2\,l)\,(2\,b_-)$ is the area of the first opening, and $a_+ := (2\,l)\,(2\,b_+)$ is the area of the second opening. Of note, the model has no dependence on q_2 or m_2, because these parameters disappear because of equation (10).

Figure 5 (C) shows an example of the amplitude of the reflected wave of a 5 GHz dipole for $L = 15$ cm and $l = b_- = b_+ = 5$ cm. The Hertzian dipole oscillates parallel to the reflecting surface in Figure 5 because, as explained above, the axis of oscillation of a Hertzian dipole aligns parallel to the reflecting surface. The reflected wave at the double slit (C) differs considerably from that which would be emitted by a reflecting plane surface without openings (B). The presence of the two openings markedly changes the reflected electromagnetic field, and an interference pattern is observed; i.e. the radiated amplitude of the wave remains approximately the same overall but is now not homogeneously distributed in space. A similar effect occurs with only one opening (D). This effect is referred to as diffraction.

However, the reflected field also depends on the location of the Hertzian dipole relative to the two openings in the reflecting surface. In figure 6, the reflections for four different positions on the z-axis are shown. In partial figure (A), the Hertzian dipole is located exactly at the midpoint between the two

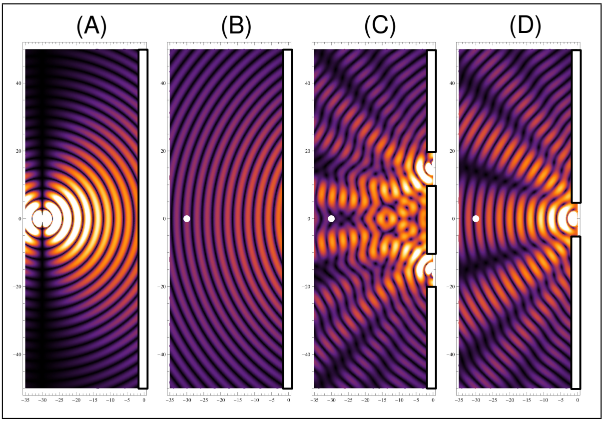

Figure 5: Figure (A) shows a snapshot of the absolute value of the electromagnetic field of a Hertzian dipole oscillating at a frequency of 5 GHz (6 cm wave). In (B), the absolute value of the wave that would be radiated by a perfectly reflecting surface is shown. In (C), the reflecting surface has two openings at −15 cm and +15 cm. In (D), the reflective surface has only one opening. For clarity, the location of the Hertzian dipole is marked each time with a white disc, and the reflecting surface is shown on the right side. All numerical values are in cm.

openings, as in figure 5 (C). In sub-figures (B) to (D), the Hertzian dipole is located at 10, 20 and 30 cm. Please note that in addition to the analytical calculation, there is also a numerical simulation with source code and video.

As clearly shown in figures 5 and 6 and as well in the video, the spatial structure of the reflected field is quite complicated when openings in the surface are present. Consequently, because the reflected wave is an electromagnetic force, the location and orientation of the Hertzian dipole may be affected. In other words, the Hertzian dipole generates a force on itself via the reflecting surface, and this force depends on where it is located and on the properties of the reflecting plane near which the Hertzian dipole is located. The force that plays a decisive role in this is referred to as the *ponderomotive force*.

The ponderomotive force [19] is a quite interesting effect, which was little known during the twentieth century and therefore was scarcely mentioned in textbooks. In a sense, the ponderomotive force, when acting on compound particles, represents the electrodynamic equivalent of the van der Waals force. Like the van der Waals force, it has only a very short range, but, owing to

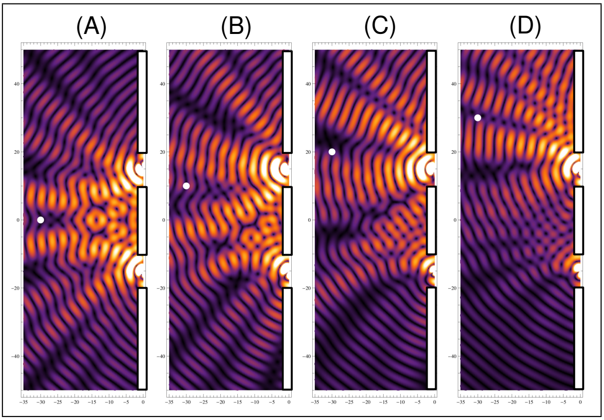

Figure 6: This figure, in addition to figure 5, shows the reflected fields at the double slit when the Hertzian dipole is shifted on the z-axis. The position of the Hertzian dipole is marked in each case with a white disk. The plots illustrate that the reflected field considerably depends on the location of the Hertzian dipole relative to the openings.

dynamic effects, it can become remarkably strong.

The ponderomotive force acts on charged particles such as electrons or protons; notably always in the same direction regardless of the sign of the electric charge. For these reasons, and because of its sometimes very high strength [20], this force is particularly important in plasma and laser physics [21–23]. In some very important technologies, such as the Paul trap [24], the free-electron laser [25] or laser cooling [26], the ponderomotive force plays a central role. However, in the context of this article, it is important to note that the ponderomotive force is also capable of accelerating electrically neutral particles, provided they contain electric charges. An example of such a particle is a single atom or Hertzian dipole.

The basic principle of the ponderomotive force is fairly simple. The effect can be explained by means of an inhomogeneous electric field in the x-direction, which periodically changes direction. If a compound particle exists in this field, the two charge quantities are pulled apart. If the field direction points to the right, the positive charge in the compound particle moves to the right, and the negative charge moves to the left. However, if the field strength is greater on the right side, then the positive charge feels a stronger force than the negative charge. Therefore, the center of gravity of the compound particle shifts slightly to the right. When the force reverses polarity, then the center of gravity would be pushed back again. However, the compound particle might have reversed its polarity by that time. In this case, the force would again pull the center of gravity of the particle even further to the right. Finally, the compound particle is accelerated gradually to the right. Similar considerations can be applied for non-neutral particles.

The ponderomotive force, like the van der Waals force, is an effect of classical physics. However, the mathematical details are complicated. In particular, many parameters affect the strength and direction of the ponderomotive force, such as the eigenfrequency of the compound particle or its direction of oscillation. The eigenfrequency in turn depends on the force acting between the charges of the Hertzian dipole. Often, a harmonic force is assumed. However, other models may also be reasonable. For example, a box potential can be imagined, if the charges inside the Hertzian dipole are assumed to be de facto free to move, as long as they do not move further than a certain maximum distance from each other. Such Hertzian dipoles do not have an eigenfrequency but oscillate with a frequency directly proportional to the velocity of the charges within the Hertzian dipole.

In the simplest case, the ponderomotive force can be expressed with a force law of the form

$$\boldsymbol{F}_q(\boldsymbol{r}) = -\nabla\,\phi, \tag{18}$$

where

$$\phi := \kappa_q\,\langle F_e(\boldsymbol{r})^2\rangle \tag{19}$$

is referred to as ponderomotive potential [19]. Notably, the ponderomotive potential should not be confused with the scalar potential of electrodynamics. The physical units of the ponderomotive potential are J. κ_q is a constant with physical units of s^2/kg, which must be either inferred from the model parameters or determined experimentally. Its sign can be positive or negative. $\langle F_e(\boldsymbol{r})^2\rangle$ is the time-averaged squared amplitude of the external force $\boldsymbol{F}_e(\boldsymbol{r},t)$ at location $\boldsymbol{r}$; i.e. $\langle F_e(\boldsymbol{r})^2\rangle$ is defined by

$$\langle F_e(\boldsymbol{r})^2\rangle := \frac{1}{T}\int_0^T \boldsymbol{F}_e(\boldsymbol{r},t)\cdot\boldsymbol{F}_e(\boldsymbol{r},t)\,\mathrm{d}t, \tag{20}$$

where $T = 1/f$ is the period of the oscillating external force $\boldsymbol{F}_e$ of frequency f.

As has already become apparent, when a Hertzian dipole is located in the vicinity of other Hertzian dipoles, it produces a force on itself, because its own field is reflected back from the environment. We now calculate this ponderomotive force of the Hertzian dipole on itself. For this purpose, we need the reflected field (17) only at the location where the Hertzian dipole resides. For this reason, we set $\boldsymbol{r}_3 = \boldsymbol{r}_1 = \boldsymbol{r} = r_x\,\boldsymbol{e}_x + r_y\,\boldsymbol{e}_y + r_z\,\boldsymbol{e}_z$ and $q_3 = q_1 = q$ and obtain for equation (19) by using definition (20) and the relation

$$\phi = \frac{\kappa_q}{T} \int\limits_0^T \boldsymbol{F}_d(\boldsymbol{r}, \boldsymbol{r}, t, q, q) \cdot \boldsymbol{F}_d(\boldsymbol{r}, \boldsymbol{r}, t, q, q)\, \mathrm{d}t. \tag{21}$$

To be able to calculate the integral, we must evaluate equation (17) by substituting equations (1) and (6). For the special case

$$\boldsymbol{s}(t) = s_1\ \sin(\omega\, t)\ \boldsymbol{e}_z, \tag{22}$$

we obtain

$$\boldsymbol{F}_d(\boldsymbol{r}, \boldsymbol{r}, t, q, q) = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 \tag{23}$$

with

$$\boldsymbol{F}_1 = q^2\,\mu_0\,\omega^2\ \frac{s_1\ \cos\left(\omega\left(t - \frac{2\,|r_x|}{c}\right)\right)}{4\,\pi\,|r_x|}\ \boldsymbol{e}_z, \tag{24}$$

$$\boldsymbol{F}_2 = a_-\,q^2\,\mu_0\,\omega^3\ \frac{s_1\ \sin\left(\omega\left(t - \frac{2\,r_m}{c}\right)\right)}{4\,\pi^2\,c\,r_m^4} \begin{pmatrix} r_x\,(r_z - L) \\ r_y\,(r_z - L) \\ r_x^2 + r_y^2 \end{pmatrix} \tag{25}$$

and

$$\boldsymbol{F}_3 = a_+\,q^2\,\mu_0\,\omega^3\ \frac{s_1\ \sin\left(\omega\left(t - \frac{2\,r_p}{c}\right)\right)}{4\,\pi^2\,c\,r_p^4} \begin{pmatrix} r_x\,(L - r_z) \\ r_y\,(L - r_z) \\ r_x^2 + r_y^2 \end{pmatrix}. \tag{26}$$

The auxiliary quantities $r_m := \|\boldsymbol{r} + L\,\boldsymbol{e}_z\|$ and $r_p := \|\boldsymbol{r} - L\,\boldsymbol{e}_z\|$ are introduced to shorten the notation.

Comparing the term $\boldsymbol{F}_1$ with $\boldsymbol{F}_2$ or $\boldsymbol{F}_3$, we notice that $\boldsymbol{F}_1$ decreases more for short distances than $\boldsymbol{F}_2$ or $\boldsymbol{F}_3$, because $\boldsymbol{F}_1$ contains a $1/\text{distance}$ dependence, whereas $\boldsymbol{F}_2$ or $\boldsymbol{F}_3$ has a $1/\text{distance}^2$ dependence. Moreover, the terms $\boldsymbol{F}_2$ or $\boldsymbol{F}_3$ gain importance when a_- or a_+ – i.e., the areas of the openings in the reflecting surface – are increased. Moreover, figures 5 and 6 show that the contribution of $\boldsymbol{F}_1$ plays a role but is dominated by the interference.

For this reason, we simplify the subsequent calculation and approximate the integral (21) with

$$\phi \approx \frac{\kappa_q}{T} \int\limits_0^T (\boldsymbol{F}_2 + \boldsymbol{F}_3) \cdot (\boldsymbol{F}_2 + \boldsymbol{F}_3)\ \mathrm{d}t. \tag{27}$$

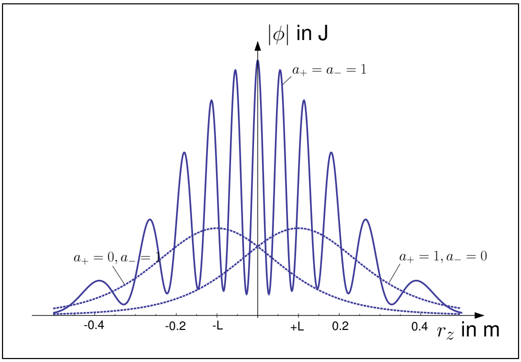

Figure 7: The absolute value of the ponderomotive potential ϕ for a frequency of 5 GHz at a distance of 35 cm in front of a reflecting metal surface with two openings at -10 cm and $+10$ cm. The dashed lines show the absolute value of the potential when one of the two openings is closed.

This integral can be solved, and for the ponderomotive potential ϕ, we obtain the equation

$$\phi = \frac{\kappa_q \, \mu_0^2 \, q^4 \, s_1^2 \, \omega^6 \left(r_x^2 + r_y^2\right)}{32 \, \pi^4 \, c^2} \left(\frac{a_-^2}{r_m^6} + \frac{a_+^2}{r_p^6} + \frac{2 \, a_- \, a_+ \left(r^2 - L^2\right) \cos\left(\frac{2\,\omega}{c}\left(r_m - r_p\right)\right)}{r_m^4 \, r_p^4} \right). \quad (28)$$

The formula of the ponderomotive potential (28) clearly describes interference. If one of the two openings, for example the upper one, is closed by setting $a_+ = 0$, the term with the cosine disappears. The same is true for $a_+ = 0$. Figure 7 gives an example showing the absolute value of the ponderomotive potential of a Hertzian dipole of frequency 5 GHz, which is located 35 cm in front of a metallic surface in which openings are located at $r_z = -10$ cm and $r_z = +10$ cm. The solid line in Figure 7 corresponds to the absolute value of the ponderomotive potential for the case in which both openings are open. If one of the two openings is closed, the interference disappears, as indicated by the dashed lines.

Incidentally, equation (27) shows that attaching polarization filters in front of or behind the openings can disturb or even completely eliminate the interference. For example, if $\boldsymbol{F}_2$ and $\boldsymbol{F}_3$ are orthogonal to each other, then

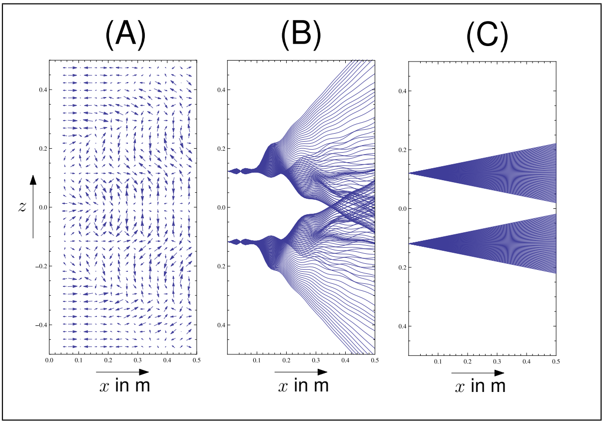

Figure 8: (A) shows the field $\boldsymbol{F} \cdot x^2$, where $\boldsymbol{F}$ is the force that a Hertzian dipole with a frequency of 3 GHz exerts on itself when a double slit with openings at $z = -12\,\text{cm}$ and $z = +12\,\text{cm}$ is located to its left, at $x = 0$. (B) shows the trajectories of Hertzian dipoles with correspondingly small masses in this field. (C) illustrates how the Hertzian dipoles would move if they were significantly heavier. As can be clearly seen, a pronounced interference pattern is formed in (B).

$(\boldsymbol{F}_2 + \boldsymbol{F}_3) \cdot (\boldsymbol{F}_2 + \boldsymbol{F}_3) = F_2^2 + F_3^2$, because the relevant term for the interference $2\,\boldsymbol{F}_2 \cdot \boldsymbol{F}_3$ is zero.

Figure 7 and equation (28) clearly demonstrate that a single Hertzian dipole close in front of or behind a double slit does not behave similarly to a billiard ball, but prefers certain locations and avoids others. If a Hertzian dipole is brought so close to a double slit that the ponderomotive force can become effective, and the distance to the double slit is fixed with suitable measures, the Hertzian dipole gradually moves to a position where it is attracted by the ponderomotive force. If the location of the Hertzian dipole is measured after a certain time, the results are recorded in tabular form, and the experiment is repeated again at another random location close to the double slit, after many repetitions, the recorded locations in the table will be found to be not completely random but to form an interference pattern. The probability distribution for finding the Hertzian dipole at a certain location in the plane in front of the double slit corresponds to the ponderomotive potential ϕ normalized to 1.

Remarkably, this effect disappears when one of the two slits is closed. Furthermore, the Hertzian dipole interestingly does not need move through one of the slits at all, because the ponderomotive force acts both in front of and behind the double slit. Thus, the interference arises from the mere existence of the double slit. However, interference would also occur if the Hertzian dipole were moving and passing through one of the two openings.

Figure 8 provides an example, which was generated by means of *openWME*[2]. The partial figure (A) shows the force that a single 3GHz Hertzian dipole would exert on itself if it were located to the right of a double slit, which has an opening at $x = 0$ and $z = -12\,\text{cm}$ and also at $x = 0$ and $z = +12\,\text{cm}$. The amplitudes of the forces are very small and in the Nanonewton range for a transmitter with an ERP of approx. 100 W. The mass of such transmitters would have to be correspondingly small so that they would move on serpentine trajectories in this field. In the middle, at (B), a simulation of such trajectories was calculated, assuming that the transmitters move through the slit at a speed of 1000 m/s. The direction of the velocity was varied gradually. Subfigure (C) shows the trajectories that would have occurred without the force. Note that the forces in the x-direction, i.e. the magnetic effects, were neglected (see Figure 4).

The simulation demonstrates that if one were able to construct very strong electromagnetic transmitters with extremely small mass, these transmitters would behave very strangely. If one would shoot such transmitters one by one through a double slit, they would, on the one hand, move through the openings according to the laws of classical physics or, if they miss the openings, would bounce off. On the other hand, they would be influenced by their self-generated ponderomotive force and would move in front of and behind the double slit on serpentine trajectories.

If a detector plane were installed behind the double slit, not every location would be equally likely to be reached by the particles, and the corresponding probability distribution would show an interference pattern. This finding was previously believed to be unexplainable through Newtonian mechanics. As now becomes evident, this belief is a fallacy, because in classical physics, macroscopic objects are in principle also capable of interfering with themselves, provided they emit strong radio waves and nearby structures reflect the waves in a suitable manner.

[2] Source code and results can be found here.

6 Conclusions and summary

The article has shown that even ordinary everyday objects can in principle interfere with themselves when they emit radio waves and move through a double slit made of highly reflective material. The high similarity of this type of interference to the interference of electrons or neutrons in the quantum mechanical double-slit experiment is striking, because it occurs with only a single Hertzian dipole, given that it interacts with the field that it itself emits and is reflected back to it by the environment.

The consequence of this self-interaction is that a Hertzian dipole does not always move as would be expected from a tangible, solid object, because such an object usually moves in a straight line and changes its direction only when it collides with another object. Hertzian dipoles, in contrast, can also move along serpentine trajectories. Whether they do so depends, as shown by the calculations and estimates in this article, on the nature of the environment and the frequency of the Hertzian dipole. Because of this strange behavior, Hertzian dipoles behind a double slit do not reach every location with equal probability, and a typical interference pattern emerges after many repetitions.

As clarified in this article, this finding is due to the ponderomotive potential, which interferes at the double slit. Consequently, real classical electromagnetic forces appear in front of and behind the double slit, thus influencing the Hertzian dipoles such that an interference pattern of impact locations is ultimately created. This is not a quantum effect but an effect of classical physics, whose explanation requires only Newtonian mechanics and classical electrodynamics. Thus, filming the trajectory of a Hertzian dipole with a high speed camera should be possible and would reveal that the Hertzian dipole is approaching the detector on a serpentine path. The extent to which this model is related to the de Broglie–Bohm theory, particularly to de Broglie's original double solution approach (e.g. [27]), remains to be clarified.

The mechanism described herein actually differs from that in an experiment with electrons or neutrons only in the order of magnitude. In particular, the interference disappears if one of the slits is closed, two orthogonal polarizers are placed in front of the slits, or if the double slit is heated so that electromagnetic noise is generated, or if the oscillation and field of the Hertzian dipole are subjected to interference with other electromagnetic waves, because, in that case, the coherent standing wave of the Hertzian dipole with its environment is destroyed. These are the same disturbances that are known to destroy quantum entanglement.

Consequently, a question arises as to whether all types of elementary particles are small plasma droplets, which, when excited to oscillate by an external field, e.g., by a particle accelerator, surround themselves with an oscillating classical electromagnetic field. A loss of energy does not necessarily occur, because the wave equation of Weber-Maxwell electrodynamics indicates that classical electromagnetic waves have no energy and no momentum, but only mediate energy and momentum between particles of matter [28]. The hypothesis that all elementary particles and photons are plasma droplets implies that light cannot be an electromagnetic wave in the strict sense. However, photons can be interpreted as electrically and gravitationally[3] neutral plasma droplets. They then could serve as the carriers of energy and momentum and the cause of the photoelectric effect. Simultaneously, as Hertzian dipoles, they would surround themselves with a classical electromagnetic wave, which in turn would be responsible for the wave properties of light.

This hypothesis, although certainly unusual, can explain many open questions, such as why electrons and photons behave so similarly, and why a pair comprising an electron and a positron can become two photons and vice versa. Regardless of whether one agrees with this hypothesis, Hertzian dipoles are always subject to ponderomotive forces that arise from reflections of the radiated wave. The mathematics associated with this type of classical mechanics is highly complex, but the basic mechanism is simple to understand and deeply logical.

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