Semiclassical models for experiments testing Bell's inequalities with pairs of "entangled particles"

Federico Comparsi

I.I.S. Albert Einstein
via Adda 6, 20871 Vimercate (MB), Italy

"A theory may appear in which such conspiracies inevitably occur, and these conspiracies may then seem more digestible than the non-localities of other theories. When that theory is announced I will not refuse to listen, either on methodological or other grounds. [...] However that may be, long may Louis de Broglie continue to inspire those who suspect that what is proved by impossibility proofs is lack of imagination." (J.S. Bell)

ABSTRACT. In this paper we want to bring back on track the unfulfilled Einstein-de Broglie-Schrödinger program [1, 3], recently taken up by Barut and others [4–16]. This program was born before matrix mechanics and it is currently believed to be impossible due to Bell's theorem, since it is a local model on space-time. We will show that it is possible to build toy models in accordance with this program that might be able to roughly reproduce the correlations observed in Bell tests. I want to thank Emilio Santos who privately provided me with some material and useful information to write this paper.

RÉSUMÉ. Dans ce papier, je veux relancer le programme inachevé de Einstein-de Broglie-Schrödinger, récemment reprise par Barut et autres. Ce programme a eté créé avant la mécanique des matrices, et est actuellement considéré comme impossible en raison du théorème de Bell, puisqu'il s'agit d'un modèle local dans l'espace-temps. Je vais démontrer qu'il est possible de construire des modèles jouets en accord avec ce programme qui sont capable d'expliquer approximativement les corrélations observées dans les expériences de type EPR. Je tiens à remercier Emilio Santos qui m'a fourni en privée du matériel et des informations utiles pour écrire ce papier.

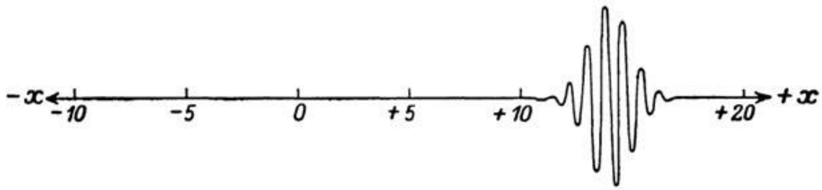

Figure 1: **Illustration of the Gaussian wave packet considered by Schrödinger in 1926 [40].**

1 Introduction

The two loopholes that are naturally involved in the models compatible with the semiclassical theory of matter waves by Einstein-de Broglie-Schrödinger [17–23] are the memory loophole and the detection loophole [24]. In this paper we show that simple models exist that, by exploiting "non-ergodic" [25, 26] and "enhancement" [27] effects, are able to recover the EPR correlations for EPR-Bohm-type experiments in a not too conspiratorial way.

In 1926 Erwin Schrödinger, the great founder of wave mechanics, already suggested us the way to understand the new quantum phenomena: "taking seriously the de Broglie-Einstein undulatory theory of moving particles, according to which the particles are nothing but a kind of 'wave crest' on a background of waves." [28]. Indeed, the very foundations of quantum mechanics reside in the important works by Einstein and de Broglie [29–32] in 1923-1925, which Schrödinger used as a starting point [28]. Starting with relativistic considerations [33, 34] and after discovering the Klein-Gordon equation in 1925, Schrödinger in 1926 fell back on a non-relativistic wave mechanics in configuration space due to some temporary difficulties in his original relativistic treatment. However, as de Broglie emphasized and even Schrödinger later recognized, a fundamental treatment of elementary processes must be relativistic and expressed ultimately in space-time [35–40].

As remarked by Georges Lochak, de Broglie believed in a particle represented as a localized bump in the wave. He knew that such a stable feature can occur only in certain nonlinear equations, and he often quoted as an example the solitary waves: actually his bumplike wave was a soliton. Yet, on the other hand, it may be considered as a sophisticated

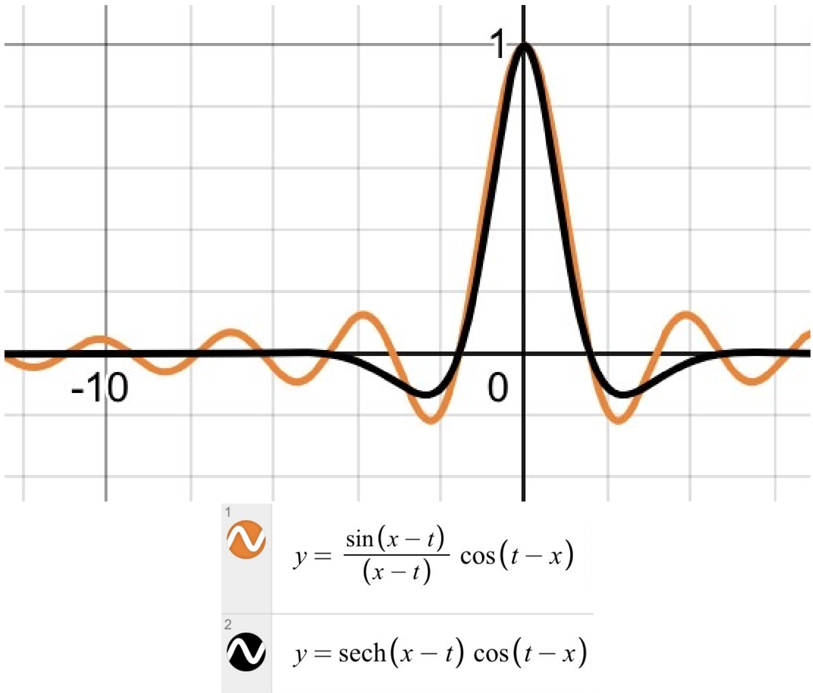

Figure 2: **Plot of a breather soliton solution (black) phase-locked in a de Broglie wavelet (orange). The latter introduced by de Broglie in 1925 [5] [32].**

form of Schrödinger's idea of a wave packet [41]. According to de Broglie the wave is a substantial part of the particle.

We want to clarify that, for the rest of the paper, when we talk about particles we will actually be referring to soliton or soliton-like solutions of the wave equations. This is why we used the quotation marks in the title: there is no real "entanglement" (with absurd instantaneous actions at a distance, quoting Isaac Newton) and there are no real point particles.

Despite trying to account for the necessary non-linearity by adding the self-gravitational[1] potentials in the massive wave equation [17,44], de

[1] Broglie's original considerations were intrinsically relativistic, so the extention of

Figure 3: **Frame of a video of a walking droplet.**

Broglie initially considered as a good representation of a quantum scalar and neutral particle the so called "de Broglie wavelet", a solution of the massless wave equation [29]. This solution has a central peak describing a relativistic particle which moves in space oscillanting in phase with a de Broglie plane wave. Therefore, it can be seen as a linear analog of the breather soliton solutions of non-linear wave equations [45], which can be shown to remain phase-locked in a de Broglie wavelet [6,12]. This result is somewhat reminiscent of the hydrodynamic analogies of Bjerknes [32] as well as of the hydrodynamic experiments carried out by J. W. M. Bush et al. [46] and shown in figure 3.

In this context it should be emphasized that the single particle wave equations in physical space [18, 47] are always non-linear due to the inclusion of the self-fields [48,49] and of the external and environmental interactions [50–54]. With such a theory the localization of the particle in the detector is explained without recurring to instantaneous collapse of the wave function or to the many-worlds interpretation: the particle is always a localized bump in its wave field. It is known that there are

---

the wave equations to the general relativistic case was a necessity for him. Indeed, since the matter wave fields were considered by de Broglie as real physical fields, they should have an associated energy-momentum density and so they would be a source of gravitational field. De Broglie later hoped that the non-linearity introduced through this gravitational self-interaction could permit a derivation of the guidance law just from the wave equation, in a similar way as Einstein would have liked to derive the geodesic equation for classical particles from the gravitational field equations alone. [42, 43]

some localized non spreading solutions of Maxwell equations moving as relativistic particles and also toroidal vortex solutions in vacuum [55–57]. However, at very high intensities (photons with very high frequencies) Maxwell's equations should be modified by introducing the effects of the polarization of the quantum vacuum and in this case there would be additional non-linear terms [58]. The same is true if we consider the Einstein-Maxwell system [59].

In this paper we want to follow exactly Schrödinger's suggestion, showing how the theory of space-time matter waves naturally leads to non-ergodic and enhancement (linked to variable detection probabilities) effects, which can produce EPR correlations in Bell-like tests. The crucial point to understand how this local theory can be compatible with Bell's theorem is to consider that in these kinds of models we have either a violation of statistical independence [60–62] or a violation of the quantum mechanical predictions [63, 64]. This occurs because, during the experimental runs defining the statistical ensemble, the distribution of the set of hidden variables is restricted in such a way that it gives rise to the emergence of properly correlated pairs of particles depending on the previous orientations of the two pieces of apparatus. In the context of the Einstein-de Broglie-Schrödinger unfulfilled program ("non-linear wave mechanics") these effects could be due to reflected empty waves, which influence the preparation (in a way similar to the Purcell effect in "spontaneous" emission) and detection (enhancement effects) of the subsequent particles. Obviously, as correctly understood and pointed out by de Broglie[2] [64] and Bell[3], these models should have different predictions from quantum mechanics in specific experimental circumstances and this implies that these models could be experimentally tested. Indeed, if a local hidden variable theory had the same identical predictions as quantum mechanics in every possible experimental situation, then it would be a superdeterministic model, which requires very specific global initial conditions for each run of the experiment, turning out to be quite implausible. The same is true for models accepting statistical independence, but assuming finite-speed (not instantaneous) superluminal influences:

[2]De Broglie underlined that to consider the problem in an adequate way, it is necessary to precisely indicate all the details of the experimental apparatus and he insisted both on the importance of the dependence of the distribution of the hidden variables on the entire context and on the importance of not confusing the eigenvalues of quantum observables with the real values of the dynamical variables [17, 65].

[3]He wrote[66]: "But if his extension [the double solution theory] is local it will not agree with quantum mechanics".

this kind of models (which reject EPR-locality, but that are nevertheless compatible with the locality of classical field theory) should disagree with quantum mechanical predictions for some experimental contexts [67].

These kinds of effects are particularly relevant when the set of possible configurations for the analyzers is restricted to a few choices (usually $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$), considering that every configuration must obviously be chosen many times during the experiment to collect a sufficiently large sample. We think that the models exploiting these loopholes should not be underestimated, especially since attacks on secure quantum communication protocols based on Bell's inequalities violation is a real threat if loopholes are not definitely closed [68]. Consequently, it would be necessary to do a new kind of "Big Bell Test", where each element of the statistical ensemble (each run of the experiment or each "single event", using Barut's nomenclature [4, 69, 70]) is collected in a distinct laboratory, well separated in space and time from the others and in such a way that for each of these measurements, the free choice is chosen for the first time.

The main phenomenon of coherent resonance of the background wave field can occur in two distinct regions of space in the experimental set-up shown in figure 4 and it can lead to two independent effects. The first region is the space between the analyzers and the detectors, where the resonant empty wave field can lead to the phenomenon of "enhancement" of the probability that an incoming particle is detected if its polarization is near the angles where the analyzer was oriented during the whole experiment. On the contrary, there is suppression of detection probability if the polarization is orthogonal to these angles. The second region is the space between the analyzers and the source of the entangled particles, where the resonant empty wave field can lead to a Purcell-like effect [72], in such a way that the apparently stochastic "spontaneous" emission of a pair of particles (through a pair production/annihilation or a parametric down-conversion process), is actually a hidden deterministic emission process "stimulated" by the "vacuum waves" [73]. In this way the distribution of the hidden polarizations of the pairs of entangled particles is not uniform, but peaked around the most probable angles parallel to the orientation directions of the analyzers (the so called "free choices"). In other words, there is some kind of "memory" in the source of entangled particles. Another kind of loophole is due to possible memory effects in the polarizers (analyzers) or detectors [75, 76].

There is indeed the fundamental question concerning the interpretation of "empty branches" of the ψ-wave after scattering processes. In the usual interpretation when these empty waves collapse they have no subsequent physical effects [74]. But according to de Broglie this is untenable: "it is difficult to conclude otherwise that the wave is not a physical phenomenon in the old sense of the word". We will show that exactly these empty waves could be responsible for the local explanation of the EPR correlations usually explained with non-locality.

Some degree of conspiracy is required to explain current experimental data in a local way. This has to be expected due to Bell's theorem. As F. Laloe said: "no experiment in physics is perfect, and it is always possible to invent ad hoc scenarios where some physical processes, for the moment totally unknown, "conspire" in order to give us the illusion of correct predictions of quantum mechanics [...] One of them is usually called the "conspiracy of the polarizers" (actually, "conspiracy of the analyzers" would be more appropriate; the word polarizer refers to the experiments performed with photons, where the spin orientation of the particles is measured with polarizing filters; but there is nothing specific of photons in the scenario, which can easily be transposed to massive spin 1/2 particles)"[77]. We note that since in de Broglie's theory a complete symmetry exists between matter and radiation, that is to say both photons and electrons are excitations of their respective wave fields, then these effects are present in all the EPR-like quantum experiments.

The fair sampling assumption [78] (alternatively, the no-enhancement assumption) is used in regard to the detection loophole. It states that the ensemble of detected particles pairs is representative of the whole ensemble of pairs emitted from the source. There is no way to experimentally test whether a given experiment does fair sampling, since the number of emitted but undetected pairs is by definition unknown. This is why these hidden variables models have a significant chance to explain direct experimental data. Indeed, the key point of variable detection probability models is that new phenomena emerge when the particles are entangled. This is not as strange as it seems, because we know that particles, when maximally entangled, do not exhibit the same interference pattern as single particles in a double-slit experiment. According to the fair sampling assumption, ensembles of photon pairs that have passed through polarization beam splitters and been identified as pairs using the time window have the same detection properties, regardless of the orientations of the splitters. However, there are no reasons to assume that these pairs of particles have always identical statistical properties[79]. As a consequence, if we have unfair sampling, then the correlation is higher for the particles actually detected than for the average pairs produced in the source.

In a classical gas there could be a small yet positive probability that we have N-body collisions, with $N > 2$. In a similar way, in wave mechanics there is a small yet positive probability to have rogue waves. If we consider that the background fields where the particles are moving is composed of real physical "empty waves" [80–83], then we should consider that this background fields can have physical consequences on the detection of the particles (the peaks of the wavelets or the solitons embedded in them).

Real measurements are always accompanied by some noise. There are three main sources of noise: the internal noise of the detector, environmental thermal noise and zero temperature vacuum fluctuations. The internal noise of the detector is large for weak measurements, but its effect can be reduced by averaging the results of many independent detectors. In this paper we focus in particular on vacuum fluctuations in the limit of zero-temperature. Such a vacuum noise can be much larger than the signal, for example when detecting a single particle, the vacuum noise can make it impossible to distinguish between the presence and absence of the particle [86]. It follows that local hidden variable theories cannot be excluded once noise and the imprecision in real world experiments is taken into account. This does not, of course, imply that such theories are very plausible, but they are possible. These "waves fluctuations" produce unavoidable noise in threshold detectors [87], but more importantly empty waves in the background could give rise to coherent phenonena that could change the probability that a "particle" is detected. Indeed according to E. Nelson: "Nature is (perhaps) described by a family of random fields on space-time, a chaotic family arising from a classical local relativistic field equation. [...] The random fluctuations of the field are the quantum fluctuations. They are as real as thermal fluctuations and may eventually prove to be observable." [84].

Consequently, in these models involving variable detection probabilities, we typically have undetected events of the kind: $P_{\pm,0}$, $P_{0,\pm}$ and $P_{0,0}$ in addition of the usual joint probabilities $P_{\pm,\pm}$ used to calculate the correlations. However, since the number of undetected pairs of entangled particles $P_{0,0}$ is by definition unknown (just like the total number of emitted pairs, which are emitted by stochastic processes) and since we can't experimentally distinguish the enhanced dark counts from the predicted joint probabilities $P_{\pm,0}$ and $P_{0,\pm}$, then we can have a full class of local models agreeing with observed data. Furthermore, as remarked by Santos, if we attempt to minimize the false negative we should increase the sensitivity of the detectors, but this would increase the probability of false positives. Thus in detectors there is a trade-off between high efficiency and dark rate, that may be associated to vacuum fluctuations. Indeed, the detector may record a count either due to the arrival of a signal or the arrival of an eventual intense vacuum fluctuation (a kind of rogue wave) [85].

In the following sections of the paper we will mathematically describe two toy models that exploit the two above-described independent effects caused by empty waves, namely memory effects at the source (which cause a non-ergodic distribution of hidden variables) and enhancement (which causes the variable detection probability phenomenon). We will show that these kinds of models can lead to the experimentally observed EPR correlations in two distinct yet compatible ways, i.e., there could also exist models that exploit both the memory and the detection loopholes simultaneously. We emphasize that these two effects are automatically and naturally implied by the Einstein-de Broglie-Schrödinger theory of matter waves, although they seem to conspire for roughly reproducing the EPR correlations observed in the usual experimental contexts.

2 Quantum predictions

We shall comsider the simplest case of photon pairs with maximal entanglement.

$$state: \psi = \frac{1}{\sqrt{2}} \left(|V\rangle|V\rangle + |H\rangle|H\rangle \right)$$

$$observables: \hat{P}_\alpha, \hat{P}_\beta,$$

where $\hat{P}_\alpha$ and $\hat{P}_\beta$ are projectors onto the directions α, β, and $|V\rangle$ and $|H\rangle$ are the vertical and horizontal polarization states. The quantum prediction for the probability of a single count by Alice, P_a, and Bob, P_b, are:

$$P_a = \langle \psi \,| P_\alpha |\, \beta \rangle = \frac{1}{2} = P_b, \tag{1}$$

and the probability of a joint detection is:

$$P_{ab} = \langle \psi \left| P_\alpha P_\beta \right| \psi \rangle = \frac{1}{2} \cos^2 \left(\alpha - \beta \right). \tag{2}$$

This applies to ideal detectors. For detectors with efficiency $\eta < 1$ the probabilities are:

$$P_a = P_b = \frac{\eta}{2}, P_{ab} = \frac{\eta^2}{2} \cos^2 \left(\alpha - \beta \right). \tag{3}$$

As it is well known [89, 90], Bell´s inequalities may be violated if the efficiency is $\eta > 0.83$. In this context we can assume either that the efficiency is the same for the two detectors and that it is constant or we could say that it changes as a consequence of the influence of the the background field [27, 76].

3 The simplest classical model consistent with Malus's law

If we take the validity of the Malus's law and a uniform distribution for the (hidden) polarizations of the two photons, we will obtain the following joint probabilities for the various combinations of horizontal/vertical outcomes given the two "free measurements" of polarization $(\mathbf{a}, \mathbf{b})$ carried out by Alice and Bob respectively:

$$P(H, H|\mathbf{a}, \mathbf{b}) = \frac{1}{\pi} \int_0^\pi cos^2(\alpha - \lambda) cos^2(\beta - \lambda)\, d\lambda = \frac{1 + \frac{1}{2} cos[2(\alpha - \beta)]}{4} \tag{4}$$

This classical model gives joint probabilities differing from those given by quantum mechanics, but they have a similar form. This is clear if we note that:

$$\frac{cos^2(\alpha - \beta)}{2} = \frac{1 + cos[2(\alpha - \beta)]}{4}$$

4 A simple non-ergodic model with reflected "empty" waves

We present here a "toy model" compatible with the Einstein-de Broglie theory, to explain what a "non-ergodic model" is and how it rejects the condition of statistical independence through memory effects. Probably

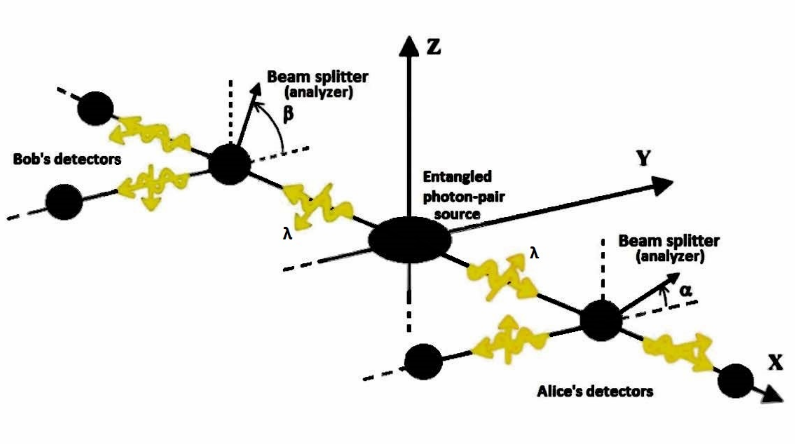

Figure 4: **Illustration of the experimental setup of an EPR-type experiment.**

this model is not compatible with all the EPR experiments so far carried out, however it is a simple model useful to explain how a natural effect, automatically included in the 3+1 matter wave theory, can lead to a violation of Bell's inequalities for certain pairs of "free choices". Let us consider a source of pairs of particles in a Bell state: they can be two photons, of which we measure the vertical or horizontal linear polarization, or two particles emitted by a radioactive process, of which we measure the position and momentum; the situation is conceptually the same. In both cases the process of creation of the particles is a decay process caused by quantum fluctuations in the vacuum and as such it is an unpredictable stochastic process [88]. Let us suppose we have a succession of N pairs of maximally entangled particles heading towards their respective measurement apparatuses, as shown in figure 4.

We now associate the respective polarization vectors to the individual photons of the various pairs, which in this case assume the role of hidden variables[4]. The two photons of each pair will have the same polarization

[4]In order to further explain the observed statistics, that is to justify Malus's law, further hidden variables could be considered, such as the phase of the pilot wave and the location of the central peak representing the particle, but also the complex microscopic configuration of the apparatus. In this paper we limit ourselves to a local probabilistic hidden variable model which is in a sense a minimal extension of

if they propagate in the opposite direction along the x axis and if the initial state of the atom from which they are produced is spherically symmetrical. This is a consequence of the symmetries of the problem and of the fact that electromagnetic interactions conserve parity. We are therefore considering an entangled state for photon pairs of the type:

$$|\psi_+\rangle = \frac{|V\rangle|V\rangle + |H\rangle|H\rangle}{\sqrt{2}}$$

where $|V\rangle$ is the pure state with vertical polarization along z and $|H\rangle$ is the pure state with horizontal polarization along y. The state $|\psi_+\rangle$ is symmetrical (it is even for spatial reflections) and can be obtained from the de-excitation of an atom in an excited state with $S = J = 0$ which decays with two successive electric dipole transitions, that is: $(J = 0) \rightarrow (J = 1) \rightarrow (J = 0)$ with a process called atomic cascade SPS. Alternatively, we could consider the antisymmetric state

$$|\psi_-\rangle = \frac{|V\rangle|H\rangle - |H\rangle|V\rangle}{\sqrt{2}}$$

which is obtained from the annihilation of positronium in two photons and whose ground state has intrinsically negative parity. In this case the polarizations of the two photons will be orthogonal to each other [93].

We suppose as well that the law of interaction between photon and polarizer is the same as in the case of single photons prepared with a defined polarization, namely Malus's law: if a photon propagates along the x axis with a vector of polarization $\vec{\lambda}$ which lies on the plane orthogonal to the direction of propagation and subsequently enters a polarizer oriented along the vector β, the probability for the photon to pass is $P = cos^2(\theta)$, where θ is the angle between λ and β.

This is what we would obtain with a one-channel polarizer, but in modern experiments we have two-channel polarizers (beam splitters), which transmit one polarity and reflect the other one, emulating a Stern-Gerlach device for charged particles. Once a photon has passed, its polarization will change to the polarizer angle. If it doesn't pass, its polarization will change to the perpendicular to the polarization angle of the polarizer.

The hypothesis of the model is that, when the wave-particle pairs interact with the polarizers, the guiding wave packet separates into one of

orthodox quantum mechanics [91] [92].

the two components parallel or orthogonal to the direction along which the polarizer is placed, that is to say the α and β angles [65] [5]. The reflected parts of these waves on the polarizer will return to the particle source and affect the process of "spontaneous" emission of the new pairs of particles. The peak of the wave, that is the particle, will remain confined within one of the two components of the transmitted part, with vertical or horizontal polarization relative to the orientation of the polarizer, with the relative probabilities P and $1 - P = sin^2(\theta)$. Hence for the λ at the source there will be a probability distribution which will be equally distributed[5] around the most probable values: α, β, $\alpha + \pi/2$ and $\beta + \pi/2$. In other words, we assume that the so-called stochastic processes, such as the emission of particles or the choices of pseudo-random generators, are actually determined by a hidden deterministic ("sub-quantum") dynamics. In this model, the physical situations in which the correlations predicted by quantum mechanics occur are analogous to states of thermodynamic equilibrium, towards which out-of-equilibrium states rapidly tend due to the fast hidden dynamics, which is significantly influenced by the procedures of preparation and measurement. As it can be verified through the formula:

$$P(A, B|\mathbf{a}, \mathbf{b}) = \int_\Lambda P_A(\alpha, \lambda)\, P_B(\beta, \lambda)\, \rho(\lambda|\alpha, \beta)\, d\lambda \tag{5}$$

where in this case we will have $\rho = \rho(\lambda, \alpha, \beta)$, integrating on the distribution of the hidden variables λ we obtain the joint probabilities predicted by quantum mechanics:

$$\begin{cases} P_{H,H} = P_{V,V} = \frac{cos^2(\alpha-\beta)}{2} \\ P_{H,V} = P_{V,H} = \frac{sin^2(\alpha-\beta)}{2} \end{cases} \tag{6}$$

The $\rho(\lambda)$ distribution should be modified if two different angles are considered for each measurement on A and B (α, α', β and β'), as in experimental tests designed to close the "locality loophole" [94]. If $\alpha' = \alpha + \frac{\pi}{2}$ and $\beta' = \beta + \frac{\pi}{2}$ the predictions of the model are still the same as those of quantum mechanics, but there will be particular angles for which, with appropriate experimental precautions, the predictions will differ and therefore it is a falsifiable model. For example in the case $\alpha' = \alpha + \frac{\pi}{4}$

[5] It is assumed that every possible change in the polarization of the reflected waves occurs for quantities that are multiples of $\pi/2$, which would be reasonable to assume provided that only total internal reflections or reflections on mirrors are involved.

and $\beta' = \beta + \frac{\pi}{4}$ we have the same predictions of the simple classical model of the previous section with half the correlations of those predicted by quantum mechanics. So this effect alone is not able to explain all the experiments with fast-switching of the analyzers for arbitrary choice of α' and β'.

We note that in this toy model the memory loophole is not invoked as a consequence of a path-memory induced by an underlying medium (the Lorentz-Dirac aether)[25, 95], but as a consequence of the continuously propagating empty waves in the background physical space.

5 A simple semiclassical model with variable detection probability

Let us consider two correlated signals arriving to Alice and Bob with polarization $\vec{\lambda}$ perpendicular to the direction of propagation.

After crossing polarizers placed with polarizations in directions at angles α, β with respect to the horizontal, the amplitudes arriving at the respective detectors will be:

$$\begin{aligned} Alice : A &\equiv \Big(|\vec{\lambda}| cos\lambda \cos\alpha + |\vec{\lambda}| sin\lambda \sin\alpha\Big) \\ Bob : B &\equiv \Big(|\vec{\lambda}| cos\lambda \cos\beta + |\vec{\lambda}| sin\lambda \sin\beta\Big) \end{aligned} \tag{7}$$

and the field intensities will be the squares of the amplitudes. Actually, the relevant amplitudes should be the averages over a time interval $[0, T]$, or rather the Fourier transforms of the time-dependent amplitudes.

In this simple model we assume that the detection probabilities are proportional to the intensities, a plausible hypothesis as shown in Appendix A. Of course, the amplitudes in equation (7) will need to be added to the amplitudes due to the background field.

We have already seen in the previous chapter how the influence of the reflected waves on the polarizers can change the distribution of hidden variables on the source, in such a way as to give rise to quantum mechanical probabilities. Now we want to see how the same result can be obtained considering that empty waves of the background field can enhance the detection probabilities for some particles and reduce the detection probabilities of other particles. Let us therefore assume, as de Broglie did, that "the empty wave of one particle is able to affect the full wave of another".

Let us consider that once the particle has passed the polarizer it has a certain probability of being detected depending on the alignment of its polarization with the background field. Let us suppose that this probability is equal to $cos^2(\alpha - \lambda)$ or $sin^2(\alpha - \lambda)$ depending on whether the particle passed with one polarization or with the other perpendicular.

In this case using formula 4 we obtain for the joint probability:

$$P(H, H|\mathbf{a}, \mathbf{b}) = \frac{1}{\pi}\int_0^\pi cos^4(\alpha - \lambda) cos^4(\beta - \lambda)\, d\lambda = \\ = \frac{18 + 16cos[2(\alpha - \beta)] + cos[4(\alpha - \beta)]}{128} \quad (8)$$

Now let us consider that each probability of detection is enhanced by a normalizing factor $\frac{4}{3}$. The physical meaning of this factor greater than one is that if we exclude from the statistical ensemble the particles emitted but not detected from the favorable cases we should as well remove them from the total cases. We think that variable detection probabilities (and more generally "hidden probabilities" [96]) is the only rational way to interpret quasiprobabilities [69]. The detailed calculations for all the other joint probabilities are reported in Appendix B with additional explanations.

With this correction applied to the previous calculations we find for the joint detection probability:

$$P(H, H|\mathbf{a}, \mathbf{b}) = \frac{1 + \frac{8}{9}cos[2(\alpha - \beta)] + \frac{1}{18}cos[4(\alpha - \beta)]}{4} \quad (9)$$

which is only slightly different from quantum mechanical predictions.

The correlation coefficient $\mathbf{e}$ is proportional to:

(number of concordant pairs) – (number of discordant pairs)

which in terms of probabilities gives:

$$\mathbf{e} = P_{H,H} + P_{V,V} - P_{H,V} - P_{V,H}$$

By using quantum mechanical joint probabilities we find that

$$\mathbf{e} = cos^2(\alpha - \beta) - sin^2(\alpha - \beta) = cos[2(\alpha - \beta)]$$

Considering the enhanced variable detection probability model we find $\mathbf{e} = \frac{8}{9}cos[2(\alpha - \beta)]$, which is only slightly different from quantum mechanical correlations.

In this context we should also count the enhanced dark counts which were not emitted by the source. The phenomenon of enhancement of dark counts is supposed to be caused by the effects of the background empty field on the fluctuations in the detectors. Indeed, some detected coincidences might be due to accidental counts (i.e., two detections mistakenly interpreted as an entangled pair), so the rate of these accidental coincidences should be subtracted. However, in this simple toy model we will neglect this complication. A more realistic one should also account for noise. In this case, the noise could depend on the background field. As such, there is a wider range of possible variable detection probability models.

Taking into account the basic variable detection probability model without enhancement, that is to say the one used in equation (4), but with the non-uniform distribution of hidden variables used in the non-ergodic model, we find the same correlations of quantum mechanics since: $\mathbf{e} = cos^4(\alpha - \beta) - sin^4(\alpha - \beta) = cos[2(\alpha - \beta)]$. This is a very interesting fact, since it means that we can have local models that exploit a combination of the two effects, that is to say exploiting at the same time the memory and detection loopholes.

Santos has shown that the memory loophole alone cannot account for the EPR correlations for any given pair of measurement settings (analyzer orientations). We agree with this analysis [85], however, to the best of our knowledge, no model has ever been proposed that simultaneously exploits both the detection and the memory loopholes. Our conjecture is that such a model could explain the EPR correlations for any choice of measurement settings, provided we exploit the two loopholes in a complementary manner.

To clarify what we mean when we say that the effects of the aforementioned mechanisms should act in a complementary manner, we suggest, for example, that memory effects primarily occur when the angle between the pairs of chosen measurement bases is close to integer multiples of $\pi/2$, while enhancement effects primarily occur when the angle is closer to 0. We do not currently have such a model, but we consider its existence to be plausible.

Of course many experiments have been done in the past to close the locality loophole [97–99] and also the detection loophole[100], but to our knowledge no experiment has ever been conducted in which all the experimental precautions are simultaneously taken to completely close all the loopholes. We emphasize that they must be closed without invoking indirect mechanisms that may introduce other additional loopholes, as it happens for example with the experiments using the technique called entanglement swapping[101].

We understand that we should give a detailed list of the experimental precautions to be taken to exclude such a class of models rejecting both fair sampling and statistical independence, but these questions go beyond the scope of this paper and we reserve the right to address these complex issues in a subsequent paper, where a specific model will be presented. However, we have proposed in the introduction a way to design this new "Big Bell test" where, in addiction to the fast-switching of the analyzers and the avoidance of post-selection/data rejection, there is also the caution to choose the free choices for the first time and so to collect the statistics in different measuring apparatuses. We understand also that this would be very expensive.

To conclude, we ask ourselves what it means for this model to be considered "conspiratorial". From a mathematical perspective, it implies a violation of the condition of statistical independence or the condition of fair sampling (conspiracy of the detectors). However, from a physical standpoint, the apparent conspiracy is merely a consequence of the fact that we did not expect a mechanism, naturally implied by the wave theory of matter, acting in such a way as to produce those correlations. After all, the mechanism of the instantaneous collapse of the wave function is certainly no less strange from a physical point of view.

6 Conclusion

We have seen that current experiments used to test Bell-like inequalities are not conclusive in order to exclude all local models. We have provided some physical local mechanisms, compatible with the matter wave theory by Einstein-de Broglie-Schrödinger, which are able to roughly reproduce EPR-Bohm correlations. This suggests that there is no reason to think that quantum mechanics cannot be modified in the way envisioned by de Broglie, Einstein and Schrödinger [2, 36, 102–104], in such a way as to be recovered as an approximation through a sub-quantum hidden thermodynamics of fast wave fields excitations. Indeed, it is unfortunate that today most Bohmians have forgotten what even Bohm later recognized [105, 106], that is to say the importance of finishing the double solution program by de Broglie, where particles are nothing but classical solutions of non-linear wave equations defined on space-time. The basic starting point of de Broglie was that these non-linear terms in the wave equations must not be introduced ad hoc, but they come from relativistic terms of interaction and self-interaction (environmental and general relativistic effects) between the local excitations of the fields and the rest of the universe [105, 106]. The self-fields of the particles are always present since the particles have an extended structure [48, 49].

We conclude by noting that these local models have remarkable implications for recent applications of quantum theory to computation and cryptography. As a consequence, since these models can be experimentally tested, we suggest that we should pay more attention to the possibility that there could be a local model (not excessively conspiratorial) that is able to explain the correlations in measurements carried out in typical EPR-type experiments. As for this point, we would like to quote Feynman who reminded us that Nature has always got better imagination than we have: "A very interesting question is the origin of the probabilities in quantum mechanics. Another way of puttings things is this: we have an illusion that we can do any experiment that we want. We all, however, come from the same universe, have evolved with it, and don't really have any "real" freedom. For we obey certain laws and have come from a certain past. Is it somehow that we are correlated to the experiments that we do, so that the apparent probabilities don't look like they ought to look if you assume that they are random. There are all kinds of questions like this, and what I'm trying to do is to get you people who think about computer-simulation possibilities to pay a great deal of attention to this, to digest as well as possible the real answers of quantum mechanics, and see if you can't invent a different point of view than the physicists have had to invent to describe this. In fact the physicists have no good point of view."[107].

7 Appendix A

Here we report a naive model of a detector suggested by E. Santos [85, 108] and consisting of an harmonic oscillator driven by an external time-dependent force. The equation of the model with obvious notation is:

$$\ddot{x} + \omega^2 x = f(t). \tag{10}$$

We assume that the probability of detection during the time interval $[0, T]$ is proportional to the energy transferred to the oscillator that is initially at $x(0) = 0$. The energy will be the time integral of the power, that is:

$$W = \int_0^T \dot{x} f(t) dt \tag{11}$$

The solution of the ODE eq.(10) with the appropriate initial condition is:

$$\begin{aligned} x(t) &= \frac{1}{\omega} \int_0^t \sin\left[\omega\left(t - t'\right)\right] f\left(t'\right) dt' \\ \Rightarrow \dot{x}\left(t\right) &= \int_0^t \cos\left[\omega\left(t - t'\right)\right] f\left(t'\right) dt' \end{aligned}$$

whence eq.(11) gives:

$$\begin{aligned} W &= \int_0^T f\left(t\right) dt \int_0^t \cos\left[\omega\left(t - t'\right)\right] f\left(t'\right) dt' \\ &= \frac{1}{2} \int_0^T dt \int_0^T \cos\left[\omega\left(t - t'\right)\right] f\left(t\right) f\left(t'\right) dt' \\ &= \frac{1}{2} \left| \int_0^T \exp\left[i\omega t\right] f\left(t\right) dt \right|^2 \end{aligned} \tag{12}$$

We may identify the force on the charged oscillator with the product of the charge times the electric field of the light beam arriving at it, that is to say $f = eE$. Hence a detection model might be obtained assuming that a detection event takes place whenever W reaches some fixed value (the threshold). However, we may simplify the model postulating that the probability of a count is proportional to W, i.e. to the square of the amplitude of the wavelet arriving at the detector, provided that the frequency of the arriving signal belongs to the band width of the detector.

8 Appendix B

Here we report the details of the calculations for the variable detection probability model firstly suggested by Delgado [27] and futher developed in chapter 5, where we gave a physical interpretation of the model in terms of interference effects between the resonant background field and the incoming wavelets. In figure 8 we reported the spreadsheet with all

B1 =1/128*(16*COS(2*A12)+COS(4*A12)+18)

	A	B	C	D	E	F	G
1	P(H,H)	0,0799403	- ->	0,14211614			
2	P(H,V)	0,1919587	- ->	0,341259969			
3	P(V,V)	0,0799403	- ->	0,14211614			
4	P(V,H)	0,1919587	- ->	0,341259969			
5	P(H,0)	0,1031009	multiply by normalizing constant				
6	P(V,0)	0,1031009					
7	P(0,H)	0,1031009					
8	P(0,V)	0,1031009					
9	P(00)	0,0437981					
10							
11	angle(a-b)	SUM(B1;B9)		SUM(D1;D4)			
12	45	1		1			

the calculations for all the joint probabilities analogous to the one in equation 8, for example $P(H,V|\mathbf{a},\mathbf{b}) = \frac{1}{\pi}\int_0^{\pi} cos^4(\alpha-\lambda)sin^4(\beta-\lambda)$ and so on for all the combinations.

The normalizing constant is required in the case of data rejection because the probabilities of all events must sum to 1. The central point here is how the detection efficiency is usually defined for detectors, i.e. $\eta = \frac{N_{detected}}{N_{incident}}$. Of course this definition requires that we can control exacly the number of emitted particles and that this efficiency is a constant property of the detector. However, in the case of a Bell test we do not control the emitted particles and it could be that the detection probability is not a constant, but it could depend on the configuration of the background field. This is why the operations of post-selection or data rejection are very problematic. We know that even by using detectors with high efficiency, undetected events can occur, and in any case the probability P_{00} cannot be controlled. Indeed, the basic idea of variable detection probability is that some quantum probabilities are different for different individual quantum systems, so that new physical features arise only for two (or more) correlated systems[24]. This hypothesis is not so strange if we consider the fact that in a double-slit experiment conducted with maximally entangled particles, the interference pattern is not observed on the screen.

References

[1] L. de Broglie, *Interference and Corpuscular Light*, Nature **118**, 441–442 (1926)

[2] L. de Broglie, *Strong processes and Transient States*, Found. Phys. **4**, 321-333 (1974)

[3] A. O. Barut, *There are Two Wave Mechanics — Not One*, in *Bell's Theorem and the Foundations of Modern Physics*, Edited by A. van der Merwe, F. Selleri and G. Tarozzi, World Scientific, Cesena, 1993

[4] A. O. Barut, *The fallacy of the arguments against local realism in quantum phenomena*, in *Waves and Particles in Light and Matter*, Edited by A. van der Merwe and A. Garuccio, Plenum Press, New York, 1994

[5] P. Gueret, J. P. Vigier, *De broglie's Wave Particle Duality in the Stochastic Interpretation of Quantum Mechanics: A testable Physical assumption*, Foundations of Physics **12**, 1057-1083 (1982)

[6] A. O. Barut and P. Rusu, *On the wave-particle-like solutions of nonlinear equations*, Can. J. Phys. **67**, 100-105 (1989)

[7] B.V. Baby and A.O. Barut, *Nonlinear de Broglie waves. 1. On nonlinear equations admitting soliton solutions with a 'de Broglie phase'. 2. The relation between relativistic and nonrelativistic solitons and kinks.*, IC/88/163, 1-33 (1988)

[8] T.W.B. Kibble, *Relativistic models of nonlinear quantum mechanics*, Commun. math. Phys. **64**, 73-82 (1978)

[9] E.W. Mielke, *Outline of a Nonlinear, Relativistic Quantum Mechanics of Extended Particles*, Fortschritte der Physik **30**, 551-585 (1981)

[10] Yi-Fang Chang, *Double Solutions of Some Nonlinear Equations with Soliton and Chaos, New Type of Soliton Equation Described Some Statistical Distributions and Nonlinear Equations Unified Quantum Statistics*, Int. J. Modern Math. **8**, 185-194 (2013)

[11] A. Maccari, *Nonlinear Field Equations and Solitons as Particles*, EJTP **3**, 39–88 (2006)

[12] Y.P. Rybakov and B. Saha, *Soliton Model of Extended Quantum Particles*, Annales de la Fondation Louis de Broglie **26**, 381-388 (2001)

[13] S. Depaquit, P. H. Guéret, J. P. Vigier, *Classical Spin Variables and Classical Counterpart of the Dirac-Feynman-Gell-Mann Equation*, International Journal of Theoretical Physics **4**, 19-32 (1971)

[14] A. Drezet, *Whence Nonlocality? Removing Spooky Action-at-a-Distance from the de Broglie Bohm Pilot-Wave Theory Using a Time-Symmetric Version of the de Broglie Double Solution*, Symmetry**16**, 1-33 (2024)

[15] A. Drezet, *Quantum solitodynamics: Non-linear wave mechanics and pilot-wave theory*, Found. Phys.**53**, 31 (2023)

[16] D. Darrow and J.W.M. Bush, *Revisiting de Broglie's Double-Solution Pilot-Wave Theory with a Lorentz-Covariant Lagrangian Framework*, Found.Phys.**16**, 1-31 (2024)

[17] L. de Broglie, *Non-linear wave mechanics - A causal interpretation*, Elsevier Publishing Company, Amsterdam London New York Princeton, 1960

[18] S. Colin, T. Durt and R. Willox, *de Broglie's double solution program: 90 years later*, Annales de la Fondation Louis de Broglie **42**, 19-71 (2017)

[19] D. Fargue, *Louis de Broglie's "double solution": a promising but unfinished theory*, Annales de la Fondation Louis de Broglie **42**, 9-17 (2017)

[20] A. O. Barut, *Coherent states versus de Broglie-wavelets*, Proceedings of the Conference on Coherent States (1993)

[21] A. O. Barut, *The Revival of Schrödinger's Interpretation of Quantum Theory*, Found. of Physics Letters **1**, 47-56 (1988)

[22] D. W. Sciama, *On the interpretation of the Einstein-Schrödinger unified theory*, J. Math. Phys. **2**, 472-477 (1961)

[23] J. Winogradzki, *Le groupe relativiste de la théorie unitaire d'Einstein-Schrödinger*, J. Phys. Radium **16**, 438-443 (1955)

[24] F. Selleri, *Quantum Mechanics Versus Local Realism: The Einstein-Podolsky-Rosen Paradox*, Plenum Press, New York, 1988

[25] V. Buonomano, *A limitation on Bell's Inequality*, Ann. Inst. Henry Poincaré **29**, 379-394 (1978)

[26] A. Khrennikov, *Buonomano against Bell: Nonergodicity or nonlocality?*, International Journal of Quantum Information**15**, 1740010 (2017)

[27] R. Risco-Delgado, *The variable detection approach: a wave particle model*, Found. Phys. Lett. **6**, 399-428 (1993)

[28] E. Schrödinger, *Zur Einsteinschen Gastheorie*, Phys. Z. **2**, 95-101 (1926)

[29] L. de Broglie, *Ondes et Quanta*, C. R. Acad. Sci. Paris **177**, 507-510 (1923)

[30] A. Einstein, *Quantentheorie des einatomigen idealen Gases (Quantum theory of the monoatomic ideal gas)*, Sitzungsber. Preuss. Akad. Wiss., 261-267 (1924) and 3-14 (1925)

[31] L. de Broglie, *Recherches sur la théorie des Quanta*, PhD Thesis, Masson, Paris, 1924

[32] L. de Broglie, *Sur la fréquence propre de l'électron*, Comptes Rendus **180**, 498-500 (1925)

[33] E. Schrödinger, *Dopplerprinzip und Bohrsche Frequenzbedingung*, Physikalische Zeitschrift **23**, 301-303 (1922)

[34] E. Schrödinger, *Über eine bemerkenswerte Eigenschaft der Quantenbahnen eines einzelnen Elektrons*, Zeitschr. für Phys. **12**, 13–23 (1923)

[35] E. Schrödinger, *Diracsches Elektron im Schwerefeld I.*, Sitzungsber. Preuss. Akad. Wiss., Phys.-Math. Kl. **105** (1932)

[36] E. Schrödinger, *Are there quantum jumps?* part I and II, The british Journal for the philosophy of science **3**, 109–123, 233-242 (1952)

[37] E. Schrödinger, *The meaning of Wave Mechanics*, Original Typoskript For the July Colloquium, Dublin, 1952

[38] E. Schrödinger, *Sur la théorie relativiste de l'électron et l'interprétation de la mécanique quantique*, Annales de l'I. H. P. **2**, 269-310, (1932)

[39] E. Schrödinger, *Diracsches Elektron im Schwerefeld I*, Preuss. Akad. d. Wissenschaft, 105–128 (1932)

[40] E. Schrödinger, *Der stetige Obergang von der Mikro-zur Makromechanik (The continuous Transition from Micro-to Macro-Mechanics*, Naturwissenschaften **14**, 664-666 (1926) Translated into English in *Collected papers on Wave Mechanics* (Translated from the second german edition), Blackie and Sons Limited, London and Glasgow, 1928

[41] G. Lochak, *Convergence and divergence between the ideas of de Broglie and Schrödinger in wave mechanics*, Found Phys **2**, 1189–1203 (1987)

[42] A. Einstein, *Do gravitational fields play an essential part in the structure of the elementary particles of matter?*, Preussische Akademie der Wissenschaften (Berlin), 349-356 (1919)

[43] A. Einstein, L. Infeld, B. Hoffmann, *The Gravitational Equations and the Problem of Motion*, Annals of Mathematics **39**, 65-100 (1938)

[44] L. de Broglie, *L'universe a cinq dimensions et la mécanique ondulatoire*, J. Phys. Radium **16**, 65-73 (1927)

[45] U. Enz, *The Sine-Gordon breather as a moving oscillator in the sense of de broglie*, Foundations of Physica D **17**, 116-119 (1985)

[46] M. Durey and J. W. M. Bush, *Hydrodynamic Quantum Field Theory: The Onset of Particle Motion and the Form of the Pilot Wave*, Front. Phys., **8**, 1-14 (2020)

[47] A. O. Barut, *On the Status of Hidden Variable Theories in Quantum Mechanics*, Apeiron **2**, 97-98 (1995)

[48] I. Acikgoz, A. O. Barut, J. Kraus, N. Ünal, *Self-field quantum electrodynamics without infinities. A new calculation of vacuum polarization*, Phys. Rev. A **198**, 126-130 (1995)

[49] D. Hestenes, *Quantum Mechanics from Self-Interaction*, Foundations of Physics **15**, 63–87 (1983)

[50] H. Breuer and F. Petruccione , *Radiation Damping and Decoherence in Quantum Electrodynamics*, In: Heinz-Peter Breuer and Francesco Petruccione (Eds.), *Relativistic quantum measurement and decoherence*, Lecture Notes in Physics 559, pp. 31-65 (Springer, Berlin, 2000)

[51] J. Andrade e Silva, *La théorie des systèmes de particules dans l'interprétation causale de la Mécanique ondulatorie*, Annales de l'I. H. P. **16**, 289-359 (1960)

[52] T. Norsen, D. Marian, X. Oriols, *Can the wave function in configuration space be replaced by single-particle wave functions in physical space?*, Synthese **192**, 3125-3151 (2015)

[53] T. Norsen, *Bohmian Conditional Wave Functions (and the status of the quantum state)*, J. Phys.: Conf. Ser. **701**, 1-11 (2016)

[54] R. van Geleuken and A.V. Martin, *Conditional wave theory of environmental interactions with a quantum particle*, Phys. Rev. Research **2**, 1-9 (2020)

[55] G. N. Borzdov, *Localized Electromagnetic Fields in Complex Media and Free Space*, International Conference on Electromagnetics of Complex Media, 59-62 (2000)

[56] A. O. Barut, *Quantum particle-like configurations of the electromagnetic field*, Found. Phys. Lett. **3**, 303-309 (1990)

[57] N. Papasimakis, V. A. Fedotov, V. Savinov, T. A. Raybould, N. I. Zheludev, *Electromagnetic toroidal excitations in matter and free space*, Nature Materials **15**, 263–271 (2016)

[58] M. Soljacic and M. Segev, *Self-trapping of electromagnetic beams in vacuum supported by QED nonlinear effects*, Phys. Rev. A **62**, 1–6 (2000)

[59] A. Garate and R.J. Gleiser, *Einstein-Maxwell breathers*, Class. Quantum Grav. **12**, 119-131 (1995)

[60] G. Lochak, *Has Bell's Inequality a General Meaning for Hidden-Variable Theories?*, Found. Phys. **6**, 173-184 (1976)

[61] L. de la Pena, A.M. Cetto and T.A. Brody, *On hidden-variable Theories and Bell's inequality*, Lettere al Nuovo Cimento **5**, 177-181 (1972)

[62] D. J. Bohm, B. J. Hiley, *Nonlocality in Quantum Theory Understood in terms of Einstein's Nonlinear Field Approach*, Foundations of Physics **11**, 529-546 (1981)

[63] L. Vervoort, *Are Hidden-Variable Theories for Pilot-Wave Systems Possible?*, Foundations of Physics **48**, 803–826 (2018)

[64] L. de Broglie, *Sur la réfutation du théorème de Bell*, Comptes Rendus de l'Academie des Sciences B **278**, 721-722 (1974)

[65] S. Diner, D. Fargue, G. Lochak, F. Selleri (eds.), *The Wave-Particle Dualism: A Tribute to Louis de Broglie on his 90th Birthday*, D. Reidel Publishing Company, Dordrecht, 1984

[66] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge, 1987

[67] J-D. Bancal, S. Pironio, A. Acín, Y-C. Liang, V. Scarani and N. Gisin, *Quantum non-locality based on finite-speed causal influences leads to superluminal signalling*, Nature Physics **8**, 867–870 (2012)

[68] S. Sajeed, N. Sultana, C. Ci Wen Lim and V. Makarov, *Bright-light detector control emulates the local bounds of Bell-type inequalities*, Nature Physics **8**, 867–870 (2012)

[69] A. O. Barut, *Towards a Theory of Single Events in Spin Correlation Experiments*, Ann. N.Y. Acad. Sci. **480**, 393-399 (1986)

[70] A. O. Barut, *How to Avoid "Quantum Paradoxes"*, Found. Phys. **20**, 137-142 (1992)

[71] A.M. Cetto and L. de la Pena, *Environmental Effects on Spontaneous Emission and Lamb Shift, according to Stochastic Electrodynamics*, Physica Scripta **21**, 27-32 (1988)

[72] M.V. Rybin, S.F. Mingaleev, M.F. Limonov and Y.S. Kivshar, *Purcell effect and Lamb shift as interference phenomena*, Sci. Rep. **6**, 1-9 (2016)

[73] B. Fain *Spontaneous Emission vs Vacuum Fluctuations*, Il Nuovo Cimento **68**, 73-78 (1982)

[74] A. Drezet, *A causal and continuous interpretation of the quantum theory: About an original manuscript by David Bohm sent to Louis de Broglie in 1951*, Annales de la Fondation Louis de Broglie **46**, 169-195 (2021)

[75] S. Pascazio, *Time and Bell-type inequalities*, Phys. Lett. A **118**, 47-53 (1986)

[76] E. Santos, *Critical analysis of the empirical tests of local hidden-variable theories*, Phys. Rev. A **46**, 3646-3656 (1992)

[77] F. Laloe, *Do we really understand quantum mechanics?*, American Journal of Physics **69**, 655-701 (2001)

[78] E. Santos, *Proposed optical test of Bell's inequalities not resting upon the fair sampling assumption*, Phys. Lett. A **327**, 33-37 (2004)

[79] A. Khrennikov, *Bell's inequality: Physics meets Probability*, Information Science **179**, 492-504 (2009)

[80] R. Folman and Z. Vager, *Empty wave detecting experiments: a comment on auxiliary "hidden assumptions"*, Found. Phys. Lett. **8**, 55-61 (1995)

[81] M. Schmidt and F. Selleri, *Empty wave effects on particle trajectories in triple-slit experiments*, Found. Phys. Lett. **4**, 1-17 (1991)

[82] J.R. Croca, A. Garuccio, V.L. Lepore and R.N. Moreira, *Quantum-optical predictions for an experiment on the de Broglie waves detection*, Found. Phys. Lett. **3**, 557-564 (1990)

[83] P. Holland, *The roads not taken: empty waves, wavefunction collapse and protective measurement in quantum theory*, in *Protective Measurement and Quantum Reality*, ed. S. Gao, Cambridge University Press, 2014

[84] E. Nelson, *Field theory and the future of stochastic mechanics*, 438–469 (1986) In: Albeverio, S., Casati, G., Merlini, D. (eds) *Stochastic Processes in Classical and Quantum Systems*, Lecture Notes in Physics**262**, Springer, Berlin, Heidelberg

[85] E. Santos, *Realistic Interpretation of Quantum Mechanics*, Cambridge Scholars Publishing, 2022

[86] A. Bednorz and W. Belzig *Effect of relativity and vacuum fluctuations on quantum measurement*, Phys. Rev. D **105**, 1-15 (2022)

[87] A. Khrennikov, *Measurement problem: from de Broglie to theory of classical random fields interacting with threshold detectors*, J. Phys.: Conf. Ser. **442**, 1-13 (2013)

[88] C. C. Gerry, P. L. Knight, *Introductory Quantum Optics*, Cambridge University Press, 2005

[89] A. Cabello, J. Larsson and D. Rodriguez, *Minimum detection efficiency required for a loophole-free violation of the Braunstein-Caves chained Bell inequalities*, Physical Review A. Atomic, Molecular, and Optical Physics **79**, 1-4 (2009)

[90] M. Ruzbehani, *Simulation of the Bell inequality violation based on quantum steering concept*, Sci Rep. **11** 1-11 (2021)

[91] A. van der Merwe, M. Ferrero, *New developments on fundamental problems in quantum physics*, Springer Science + Business Media, Dordrecht, 1997

[92] G. Hunter, *Photons as solitons and EPR-Bohm*, Am. Inst. Phys. **461**, 295–298 (1999)

[93] A. Peres, *Quantum Theory: Concepts and Methods*, Kluwer Academic Publishers, 2002

[94] A. Aspect, *Bell's Theorem: The Naive View of an Experimentalist*, in R. A. Bertlmann, A. Zeilinger (eds.), *Quantum (Un)speakables - From Bell to Quantum Information*, Springer, Berlin, 2002

[95] L. Vervoort, *Are Hidden-Variable Theories for Pilot-Wave Systems Possible?*, Found. Phys. **48**, 803-826 (2018)

[96] L. de Broglie, G. Lochak, J. A. Beswick, J. V. Pereira, *Present, Predicted, and Hidden Probabilities*, Found. Phys. **6**, 3-14 (1976)

[97] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter and A. Zeilinger, *Violation of Bell's Inequality under Strict Einstein Locality Conditions*, Phys. Rev. Lett.**81**, 5039-5043 (1998)

[98] W. Tittel, *Violation of Bell inequalities by photons more than 10 km apart*, Phys.Rev.Lett.**81**, 3563-3566 (1998)

[99] J. Yin et al., *Satellite-based entanglement distribution over 1200 kilometers*, Science**356**, 1140-1144 (2017)

[100] B. Hensen et al., *Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres*, Nature**526**, 682–686 (2015)

[101] H. Price and K. Wharton, *Entanglement Swapping and Action at a Distance*, Found. Phys.**105**, 1-24 (2021)

[102] E. Schrödinger, *Die gegenwärtige Situation in der Quantenmechanik*, Naturwissenschaften **23**, 807–812, 823–828, 844–849 (1935); J. D. Trimmer, *The Present Situation in Quantum Mechanics: A Translation of Schrödinger's 'Cat Paradox' Paper*, Proceedings of the American Philosophical Society **124**, 323–338 (1980)

[103] A. Einstein, B. Podolsky and N. Rosen, *Can Quantum-Mechanical Description of Physical Reality be Considered Complete?*, Phys. Rev. **47**, 777-780 (1935)

[104] A. Einstein and N. Rosen, *The Particle Problem in the General Theory of Relativity*, Phys. Rev. **48**, 73-77 (1935)

[105] M. Flato, Z. Maric, A. Milojevic, D. Sternheimer, J. P. Vigier, (eds.), *Quantum Mechanics, Determinism, Causality, and Particles - An International Collection of Contributions in Honor of Louis de Broglie on the Occasion of the Jubilee of His Celebrated Thesis*, Springer Netherlands, 1976

[106] D. J. Bohm, B. J. Hiley, *The de Broglie Pilot Wave Theory and the Further Development of New Insights Arising Out of It*, Foundations of Physics **12**, 1001-1016 (1982)

[107] R. P. Feynman, *Simulating physics with computers*, International Journal of Theoretical Physics**21**, 467-488 (1981)

[108] E. Santos, *Stochastic Interpretation of Quantum Mechanics Assuming That Vacuum Fields Are Real*, Foundations**2**, 409-442 (2022)

(Manuscrit reçu le 15 février 2025)