Uncovering a Geometric Substructure of Quantum Dynamics

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ABSTRACT. The description of a closed quantum system is extended with the identification of an underlying substructure enabling an expanded formulation of dynamics in the Heisenberg picture. Between measurements a "state point" moves in an underlying multi-dimensional complex projective space with constant velocity determined by the quantum state vector. Following a measurement the point changes direction and moves with new constant velocity along one of several possible new orthogonal paths with probabilities determined by the Born Interpretation of the state vector. From this previously hidden substructure a new picture of quantum dynamics and quantum measurements emerges without violating existing no-go theorems regarding hidden variables. A natural generalisation to a Riemannian substructure is proposed, determined by the entropy of the background environment. This leads to a suggested interaction between the substructure of quantum dynamics and the background gravitational field.

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1 Introduction

There have been many attempts to describe aspects of quantum mechanics in geometric and sometimes also information-theoretic terms. See in particular ([1] - [8]) and references therein. Meanwhile efforts to identify "hidden variables" that clarify the nature of the quantum measurement process have been restricted by powerful "no-go" theorems [9, 10]. The difficuties facing the quantisation of general relativity have also received a great deal of attention over many years – see for example [11] and references therein – with no agreement that any satisfactory resolution has been achieved, leading to the suggestion that rather than trying to quantise general relativity it may be more sensible to "gravitise" quantum mechanics [12, 13].

The description of the measurement process and the associated Born Interpretation of the state vector have been the most contentious features of quantum mechanics since its inception. In the case of a conservative system the strangeness of the orthodox description is seen most clearly in the Heisenberg picture of quantum dynamics [9, 14, 15]. There the state vector $|\psi\rangle$ is a constant unit vector in a Hilbert space $\mathcal{H}$, possibly infinite-dimensional, between measurements at times t_0 and $t_1 > t_0$, while self-adjoint operators $\widehat{A}(t)$, $\widehat{B}(t)$, ... representing observables evolve in time in accordance with Heisenberg's equation of motion

$$i\hbar\frac{d\widehat{A}}{dt} = [\widehat{A}\,,\widehat{H}]\,, \quad etc. \tag{1}$$

Here $\widehat{H}$ is the Hamiltonian operator. Only closed systems for which $\widehat{H}$ is time-independent are considered at this stage.

But in another and very different type of dynamical process that is assumed to reflect the interaction of the system with the measuring apparatus and the observer, a maximal set of commuting observables is chosen for measurement and the state vector is subsequently observed to move into a common eigenvector of that set with a probability determined by Born's Rule [9, 14, 15]. (Complications that arise with observables having partially continuous spectra are set aside here.)

Suppose that at time t_0 a measurement has been made of such a set of commuting observables and $|\psi\rangle$ has been observed subsequently in one of their common eigenvectors. The state vector remains constant for $t_0 < t < t_1$. We may expand it during this time interval in terms of some chosen reference basis, a complete set of orthonormal vectors

$|\varphi_i\rangle$, $i = 1\,,2\,\ldots$, possibly infinite in number, with constant complex expansion coefficients α^i. Then

$$|\psi\rangle = \sum_i \alpha^i |\varphi_i\rangle \quad \text{with} \quad \sum_i |\alpha^i|^2 = \langle\psi|\psi\rangle = 1\,. \tag{2}$$

Now consider another, in general different maximal set of commuting observables $\widehat{A}(t)$, $\widehat{B}(t)$, $\ldots$ evolving in time for $t > t_0$ in accordance with (1) and having a complete orthormal set of common eigenvectors $|\chi_K(t)\rangle$ for $K = 1\,,2\,,\ldots$ with components $\beta_K^{\;i}(t)$ in the same reference basis used for $|\psi\rangle$, so

$$|\chi_K(t)\rangle = \sum_i \beta_K^{\;i}(t)|\varphi_i\rangle\,. \tag{3}$$

(In the special case when $\widehat{H}$ is one of the chosen set then all the observables in the set are constants of the motion and their common eigenvectors $|\chi_K\rangle$ and all their components $\beta_K^{\;i}$ are constant.)

Note that orthogonality of $|\chi_K(t)\rangle$ and $|\chi_L(t)\rangle$ for $K \neq L$ implies

$$\sum_i \overline{\beta_K^{\;i}(t)}\beta_L^{\;i}(t) = 0\,, \quad K \neq L\,, \tag{4}$$

where the overbar indicates complex conjugation.

Suppose that a measurement is made at some time $t_1 > t_0$ of this second set of observables. Immediately prior to this second measurement the state vector can be expressed as

$$|\psi\rangle = \sum_K \langle\chi_K(t_1)|\psi\rangle|\chi_K(t_1)\rangle\,, \tag{5}$$

and the system can be said to be in one of the states $|\chi_K(t_1)\rangle$ in this superposition, which one being indeterminate until the measurement is completed. Immediately following the measurement, for each K there is according to Born's Rule a probability

$$P_K = |\langle\chi_K(t_1)\psi\rangle|^2 = \sum_i |\overline{\beta_K^{\;i}(t_1)}\alpha^i|^2 \tag{6}$$

of an observer finding the system with a new constant state vector equal to $|\chi_K(t_1)\rangle$. If for example the system is observed to be in the state with vector $|\chi_{K'}(t_1)\rangle$ say, then for $t > t_1$

$$|\psi\rangle = |\chi_{K'}(t_1)\rangle = \sum_i \beta_{K'}^{\,i}(t_1)|\varphi_i\rangle \tag{7}$$

and

$$\begin{aligned} \widehat{A}(t_1)\,|\psi\rangle &= \widehat{A}(t_1)|\chi_{K'}(t_1)\rangle = a|\chi_{K'}(t_1)\rangle = a|\psi\rangle\,, \\ \widehat{B}(t_1)\,|\psi\rangle &= \widehat{B}(t_1)|\chi_{K'}(t_1)\rangle = b|\chi_{K'}(t_1)\rangle = b\,|\psi\rangle\,, \quad \ldots \end{aligned} \tag{8}$$

for some corresponding eigenvalues a, b, ...

If no observation is made of the state following the measurement, the system sits in the mixture of pure states $|\chi_K(t_1)\rangle$ for $t > t_1$ with associated probabilities P_K as in (6).

To summarise this standard description of quantum dynamics in the Heisenberg picture: between the measurements at times t_0 and t_1 the operators in the set to be measured at t_1 evolve in time in accordance with (1). Their associated eigenvectors $|\chi_K(t)\rangle$ also evolve accordingly as do their expansion coefficients $\beta_K^{\,i}(t)$, which may be pictured as a group of quantities "rotating" unitarily while remaining orthogonal as in (4) until the measurement at t_1. Then the system is observed to move into a new state with state vector $|\psi\rangle = |\chi_{K'}(t_1)\rangle$ say, and the corresponding coefficients $\beta_{K'}^{\,i}(t_1)$ are selected from the group. Both $|\psi\rangle$ and its expansion coefficients $\beta_{K'}^{\,i}(t_1)$ remain constant thereafter – until another measurement, perhaps.

2 Identifying a substructure

The reader may observe that the dynamical process described above is strongly reminiscent of the behaviour of a free particle travelling with constant velocity between impulsive forces applied at t_0 and t_1. This suggests the association of the state vector $|\psi\rangle$ with the velocity vector of a point moving in a hitherto unidentified underlying space. Accordingly the $V\,\alpha^i$ are now identified with the components v^i of the constant velocity vector of a "state point" moving in an underlying space S say, with complex coordinates z^i so that

$$v^i = \frac{dz^i}{dt}\,, \qquad i = 1\,, 2\,, \ldots \tag{9}$$

(For convenience a constant V with dimensions of velocity LT^{-1} has been inserted here so that each z^i has dimensions of length L.) Note from (2) that

$$\sum_i \overline{v^i} v^i = V^2 \,. \tag{10}$$

Suppose that the state point starts at a location with coordinates ${z_0}^i$ at time t_0. Because the v^i are constants it follows trivially from (9) that

$$z^i(t) = {z_0}^i + v^i\,(t - t_0)\,, \quad t_0 \leqslant t \leqslant t_1\,. \tag{11}$$

For $t > t_1$ the point moves in a new direction determined by which of the eigenvectors $|\chi_K(t_1)\rangle$ results from application of Born's Rule (6) to the measurement at t_1. The possible directions are determined by and associated with the corresponding "velocity vectors" having components $w_K^{\,i}(t_1) = V\,\beta_K^{\,i}(t_1)$ with $\beta_K^{\,i}(t_1)$ as in (3). Note that these directions are orthogonal according to (4) and that each velocity vector is normalized as in (10). Note also that the probabilities associated with the different directions as given by (6) can be viewed as the moduli squared of generalized direction cosines between the velocity vector immediately before the measurement v^i and those possible immediately after the measurement, the $w_K^{\,i}(t_1)$, since

$$\sum_i |\overline{v^i(t_1)} w_K^{\,i}(t_1)|^2 = V^2 \sum_i |\overline{\alpha^i} \beta_K^{\,i}(t_1)|^2 = V^2 P_K\,. \tag{12}$$

For $t_0 < t < t_1$ the vectors $w_K^{\,i}(t)$ evolve in time as noted above and may be pictured as a set of orthogonal velocity vectors rotating unitarily about the state point as it moves along the straight line (11). If the state vector observed following the measurement at $t = t_1$ is $|\chi_{K'}(t_1)\rangle$ then for $t > t_1$ the state point moves with new constant velocity $w_{K'}^{\,i}(t_1)$, so that

$$z^i(t) = {z_0}^i + v^i\,(t_1 - t_0) + w_{K'}^{\,i}(t_1)\,(t - t_1)\,, \quad t > t_1\,. \tag{13}$$

For $t < t_1$ it is indeterminate which of the evolving velocity vectors $w_K^{\,i}(t)$ will result from the measurement at t_1. If no observation is made the new state for $t > t_1$ is a mixture of the eigenvectors weighted by the probabilities given by Born's Rule and the trajectory of the state point belongs to a "fan" of possible trajectories weighted accordingly.

Note that when the choice of reference basis is altered by a unitary transformation

$$|\varphi_i'\rangle = \widehat{U}|\varphi_i\rangle \Rightarrow |\varphi_i'\rangle = \sum_j U_i^{\,j}|\varphi_j\rangle\,,$$

$$U_i^{\,j}(U^\dagger)_j^{\,k} = (U^\dagger)_i^{\,j} U_j^{\,k} = \delta_i^{\,k}\,, \quad (U^\dagger)_i^{\,j} = \overline{U_j^{\,i}}\,, \tag{14}$$

any coordinate basis in S with elements z^i must undergo the corresponding unitary transformation

$$z'^{\,i} = \sum_j U_j^{\,i}\, z^j\,. \tag{15}$$

In addition to $\widehat{U}$, the action of all linear operators on the Hilbert space of state vectors can be extended to action as matrices on the state point and its velocity in S, for example

$$\widehat{A}(t)|\psi\rangle \Rightarrow \sum_j A_j^{\,i}(t)\, v^j\,, \quad A_j^{\,i}(t) = \langle\varphi_i|\widehat{A}(t)|\varphi_j\rangle \tag{16}$$

and if as before $\widehat{A}(t)$, $\widehat{B}(t)$ ... comprise the set of commuting operators measured at $t = t_1$ and the system moves into the state $|\chi\rangle_{K'}(t_1)$ after the measurement, then

$$\sum_j A_j^{\,i}(t_1)\, w_{K'}^{\,j}(t_1) = a\, w_{K'}^{\,i}(t_1)\,,$$

$$\sum_j B_j^{\,i}(t_1)\, w_{K'}^{\,j}(t_1) = b\, w_{K'}^{\,i}(t_1)\,, \ldots \tag{17}$$

corresponding to (3) and (8).

Note also that because any state vector $|\psi\rangle$ can be identified with $e^{i\theta}|\psi\rangle$ for every real θ, the space S can be assumed to have the projective property that any two points with coordinates z^i and $e^{i\theta}z^i$ for all $i = 1\,, 2\,, \ldots$ are to be identified for every real θ.

At this point it is worth emphasizing that the quantum system as described may consist of arbitrarily many interacting particles (or subsystems). Accordingly, the Hilbert space $\mathcal{H}$ could be the tensor product of many Hilbert subspaces.

More generally the tensor product of as many Hilbert subspaces as there are quantum systems can be considered whether interacting with each other or not. The space associated with a particular system of interest can accordingly be considered to be a subspace of a much larger Hilbert space. The underlying space S is enlarged accordingly. What is essential for the present discussion is that the encompassing Hilbert space $\mathcal{H}$ has a countable basis as in (2). The z_i then label points in the enlarged space S.

Unlike the closed system on which attention has been focussed so far some of these extra systems will be open and interacting continuously or intermittently, unitarily or non-unitarily with their environment rather than at isolated points associated with measurements. They enter increasingly complicated mixed states as time passes [16] - [18].

(These extensions of the description to include multiple quantum systems, whether interacting or not and whether open or not, are not pursued here. Furthermore the complications arising from the introduction of quantum fields are not considered.)

The generalised description of one closed quantum system as described above can be thought of as an extension of the “matrix mechanics" formulation of quantum dynamics [19] which is as old as quantum mechanics itself and has its origins in the pioneering work of Heisenberg and Born. Observables are represented there by Hermitian matrices, time-dependent in general, and defined as above. All quantum mechanical calculations can now be carried out in terms of the state point and its velocity. For example the expectation value of an observable $A(t)$ in the state $|\psi\rangle$ between measurements can be expressed as

$$\langle A(t)\rangle = \sum_{i\,,j} \overline{\alpha^j} A_j^{\,i}(t_1)\alpha^i = (1/V^2) \sum_{i\,,j} \overline{v^j} A_j^{\,i}(t) v^i \tag{18}$$

What is new is that the extension describes a previously hidden substructure that provides a different way of thinking about quantum dynamics and the quantum measurement process, as described in the next section.

3 Hidden variables and quantum measurement

Are the hitherto unrecognized variables z^i the much discussed “hidden variables" that resolve long-standing questions about quantum indeterminacy and quantum measurement more generally? The short answer is “No." There has been extensive discussion of these questions since the birth of quantum theory – see for example [9, 15], [21] - [26] and especially the decisive work [10].

As described above, the state point has a fan of possible future directions to choose from at $t = t_1$ which is converted to a mixture of uncertain outcomes by the measurement process in accordance with Born's Rule. The role of the observer [23] is to convert this resultant mixed state into a pure state by identifying which of the possible trajectories the state point follows after the measurement.

Before the measurement the Shannon - von Neumann entropy of the system has the value

$$-\rho \log(\rho) = 0 \tag{19}$$

where ρ denotes the density matrix [9], which has just the one eigenvalue 1 when the system is in a pure state and the state point has a definite trajectory as in (11). As a result of the measurement the entropy increases to the value

$$\sum_K P_K \log(1/P_K) \tag{20}$$

where the P_K are the Born probabilities of the trajectories in the fan as in (6). It decreases to 0 again after the observation of which of those trajectories the state point now follows.

Only simple measurements are considered here as described in [9, 14, 15] and by many others since. More sophisticated descriptions of the measurement process and more generally of a quantum system's possible interactions with its environment have been developed over the years – see in particular [16, 17, 18, 20] – together with more sophisticated measures of the system's entropy [27].

The reader may consider that the behaviour of the state point before and after a measurement is analogous to that of a macroscopic object floating down a horizontal stream that forks into two such streams at right angles. The square of the direction cosine between the direction of either fork and that of the original stream may be considered a first estimate of the probability that the object will float down that particular fork. For example if the two forks are at angles of $\pi/6$ and $\pi/3$ with the original stream, with associated direction cosines $\sqrt{3}/2$ and $1/2$ respectively, then the associated probabilities are $3/4$ and $1/4$.

The critical difference between this classical behaviour and that associated with the quantum measurement is that the indeterminacy in the behaviour of the classical object prior to reaching the fork can in principle be reduced arbitrarily greatly by more refined observation of the system so that it becomes more certain which fork will be followed. Except in special cases [10] this is not possible in the quantum case. In short, the classical indeterminacy prior to the fork is arbitrarily reducible in principle whereas in general the quantum indeterminacy prior to the measurement – associated with the set of velocity vectors rotating unitarily about the trajectory of the state point – is irreducible.

The quantum measurement itself is now to be regarded as an interaction of the quantum system with its macroscopic environment at the point in S reached at time t_1. The state point moves between such points associated with measurements at times t_0 and t_1.

4 A suggested generalization

Several questions suggest themselves. What is the nature of the points in S associated with measurements? More generally what interpretation can be given to the space S in which the process underlying quantum dynamics occurs? Why does the state point move in a straight line between measurements?

It is convenient to address these questions in the context of a natural generalizarion of the dynamical substructure described so far and it is now proposed that the space underlying quantum dynamics as described above is actually a locally flat subspace of a more general space S, a complex Riemannian manifold with associated Hermitian metric tensor

$$g_{i\,j}(\bar{z}\,,z) = \overline{g_{j\,i}(\bar{z},\,z)}\,. \tag{21}$$

Here z denotes the point in S with coordinates z^i, and $\bar{z}$ its complex conjugate. Infinitesimal distance-squared on the manifold is then defined as

$$ds^2 = g_{i\,j}\,d\bar{z}^i dz^j = \overline{ds^2}\,, \tag{22}$$

where the summation convention has now been introduced.

In general the points in S associated with measurements of a closed quantum system can be described as local singularities or "stagnation points" in S associated with the location in space-time where the measurements take place, being typically the location of measuring devices in a meta-stable state [19] – think of a cloud chamber, a Geiger counter or a photographic plate, for example.

As to the meaning of S, it is proposed that it represents the entropy content (equivalently, the information content) of the physical environment within which quantum systems evolve, including any measuring devices. This implies that the structure of S changes when the entropy content of the environment changes. For example when a photographic plate is exposed during a quantum measurement it is clear that the entropy of the neighbouring environment increases abruptly as the associated singularity in S disappears. More generally "measurements" may simply refer to interactions between quantum systems and their environment whether continuous or stochastic, at singular points in S or non-locally. In the absence of observations quantum systems move after such interactions into more and more complicated mixed states with higher entropy. It is suggested that while such interactions increase the entropy of the quantum systems involved and their environment, they also provide the (only) source for changes in the structure of S.

One possible description of that structure would be provided by supposing that it is a Kähler manifold [7, 28] with real potential $B(\bar{z}, z)$ such that

$$g_{i\,j}(\bar{z}\,, z) = \frac{\partial^2 B(\bar{z}, z)}{\partial \bar{z}^i \partial z^j}\,, \tag{23}$$

and by supposing further that B is a measure of the entropy/information content of th environment in which quantum systems evolve. It would remain to determine a mathematical description of how changes in B arise from interactions of those systems with their environment. Note that (22) and (23) would imply

$$ds^2 = \frac{\partial^2 B(\bar{z}, z)}{\partial \bar{z}^i \partial z^j}\, d\bar{z}^i\, dz^j$$

and

$$\frac{\partial g_{i\,j}}{\partial z^k} = \frac{\partial g_{i\,k}}{\partial z^j}\,, \quad \frac{\partial g_{i\,j}}{\partial \bar{z}^k} = \frac{\partial g_{i\,k}}{\partial \bar{z}^j}\,. \tag{24}$$

Returning to the behaviour of the state point of a closed system between measurements, straight line motion as in (11) is naturally generalized to motion along a geodesic between the locations at P and Q say, of quantum measurements, so minimizing the distance travelled. This may be regarded as an analogue of the principle of least action that leads to geodesic motion of a mass point (a "test particle") in space-time [30] and leads here to a variational condition in the familiar form [29]

$$0 = \delta \int_P^Q ds = \int_{u_P}^{u_Q} \left(g_{i\,j}\bar{p}^i p^j\right)^{1/2} du\,, \quad \bar{p}^i = \frac{d\bar{z}^i}{du}\,, \quad p^j = \frac{dz^j}{du}\,, \tag{25}$$

where u is a parameter measuring distance along the geodesic.

It then follows by a generalization from the real [29] to the complex case that

$$g_{i\,j}\,\frac{dp^j}{du} - \frac{\partial g_{k\,l}}{\partial z^i}\,\bar{p}^k p^l + \frac{\partial g_{j\,i}}{\partial z^k}\bar{p}^j p^k + \frac{\partial g_{i\,j}}{\partial z^k}p^j p^k + \quad \text{c. c.} = 0\,. \tag{26}$$

Supposing that the metric on S is non-singular with inverse $g^{i\,j}$ such that

$$g^{i\,j}g_{j\,k} = \delta^i_k = g_{k\,j}g^{j\,i}\,, \tag{27}$$

an absolute derivative of p^m with respect to u along a geodesic can be defined from (26) by

$$\begin{aligned}\frac{\delta p^m}{\delta u} &= g^{m\,i}\left(g_{i\,j}\,\frac{dp^j}{du} - \frac{\partial g_{k\,l}}{\partial z^i}\,\bar{p}^k p^l + \frac{\partial g_{j\,i}}{\partial z^k}\bar{p}^j p^k + \frac{\partial g_{i\,j}}{\partial z^k}p^j p^k\right)\\ &= \frac{dp^m}{du} - g^{m\,i}\left(\frac{\partial g_{k\,l}}{\partial z^i}\,\bar{p}^k p^l - \frac{\partial g_{j\,i}}{\partial z^k}\bar{p}^j p^k - \frac{\partial g_{i\,j}}{\partial z^k}p^j p^k\right)\end{aligned} \tag{28}$$

together with its complex conjugate. The vanishing of $\delta p^m/\delta u$ along a geodesic then leads to $z^m(u)$ by integration given (25).

The vector (with components) v^i corresponding to the the state vector $|\psi\rangle$ is now to be considered as parallel transported along the geodesic traced by $z^i(t)$, where u is now replaced by elapsed time t along that geodesic and $p^i(u)$ $(= dz^i/du)$ is replaced by $v^i(t)$ $(= dz^i/dt)$. The state vector and corresponding velocity vector v^i are no longer constant between measurements. Instead v^i has vanishing absolute derivative along the geodesic as defined from (28), so that

$$0 = \frac{\delta v^m}{\delta t} = \frac{dv^m}{dt} - g^{m\,i}\left(\frac{\partial g_{k\,l}}{\partial z^i}\,\bar{v}^k v^l - \frac{\partial g_{j\,i}}{\partial z^k}\bar{v}^j v^k - \frac{\partial g_{i\,j}}{\partial z^k}v^j v^k\right)\,. \tag{29}$$

The Hilbert space $\mathcal{H}$ is now taken to be the tangent space to the geodesic followed by $z^i(t)$, obtained by integrating (28). Vectors in $\mathcal{H}$, including the quantum state vector $|\psi\rangle$, are parallel transported along the geodesic, preserving lengths and orthogonality relations.

It is natural to assume further that the Hamiltonian operator $\widehat{H}$ has vanishing absolute derivative along the geodesic, obtained by regarding its matrix representation $H_j{}^i$ as a mixed tensor and generalizing (29) accordingly [29], while the governing differential equation for other operators representing time-dependent observables includes an extra term generalizing (1).

As before, following a measurement the state vector moves into a new vector among the eigenvectors of the set of commuting operators being measured in accordance with Born's Rule and accordingly $z^i(t)$ embarks along a new geodesic.

The differences between the simple substructure described in the preceding sections and the generalized substructure of the quantum dynamics proposed here can be expected to have implications for the outcome of quantum measurements and for quantum dynamics more generally, not least because of possible effects of the curvature of S. More analysis is necessary to determine the nature of such implications.

Consideration of the space S suggests a further generalization with possibly greater consequences for physics, as we discuss in the next section.

5 Interaction with the gravitational field

As mentioned in the Introduction there has been extensive discussion over many years of attempts to quantise the theory of general relativity, and more recently of the possibility of "gravitizing" quantum theory as an alternative approach to resolving the disconnect between the two theories, each of which boasts major successes in its own domain. The identification of a Riemannian space underlying quantum dynamics suggests a different resolution of this problem, one which treats quantum theory and the theory of relativity on a more equal footing, and leads to the following final proposals:

• The geometric space S underlying quantum systems can be considered jointly with space-time carrying the local gravitational field, with combined coordinates (z^i, x^μ), for $i = 1, 2, \ldots$ and $\mu = 0, 1, 2, 3$, and combined metric tensor and infinitesimal distance-squared

$$g_{i\,j\,\mu\,\nu}(\bar{z}, z, x)\,, \quad d\sigma^2 = g_{i\,j\,\mu\,\nu} d\bar{z}^i dz^j dx^\mu dx^\nu\,. \tag{30}$$

Here x^μ are the usual space-time coordinates.

• Interactions of quantum systems with their environment are labelled not only by the coordinates z^i corresponding points in S but also by the space-time coordinates x^μ of the point or points in space-time at which such interactions occur.

• The metric tensor is not in general a simple product

$$g_{i\,j\,\mu\,\nu} \neq g_{i\,j} g_{\mu\,\nu}\,, \tag{31}$$

in particular during measurements and perhaps also during more general interactions between quantum systems and their environment. This implies that the local gravitational field interacts with measurement processes in particular and may influence quantum dynamics more generally – see for example [13] for related discussions in other contexts. It implies also that the changing entropy content of the space S during such processes can alter the local gravitational field. In short, changing entropy at the quantum level can be an unexpected source of gravitational field strength – a kind of “dark energy." How this new type of interaction affects Einstein’s equation for the gravitational field and equations governing change to the structure of S must be the subject of further study.

6 Concluding remarks

The simple substructure identified in Sec. 2 provides a new way of thinking about quantum dynamics and measurements without suggesting any new observable effects. On the other hand, the generalizations suggested in the following sections would surely have far-reaching and important implications for physics. Further study is encouraged.

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